On the enumeration of plane bipolar posets and transversal structures

Éric Fusy, Erkan Narmanli, and Gilles Schaeffer

Summary

Maps, introduction

1. Specialization of the KMSW bijection

a. Bipolar orientations, KMSW bijection
b. Plane bipolar posets
c. Transversal structures
d. Plane bipolar posets by vertices

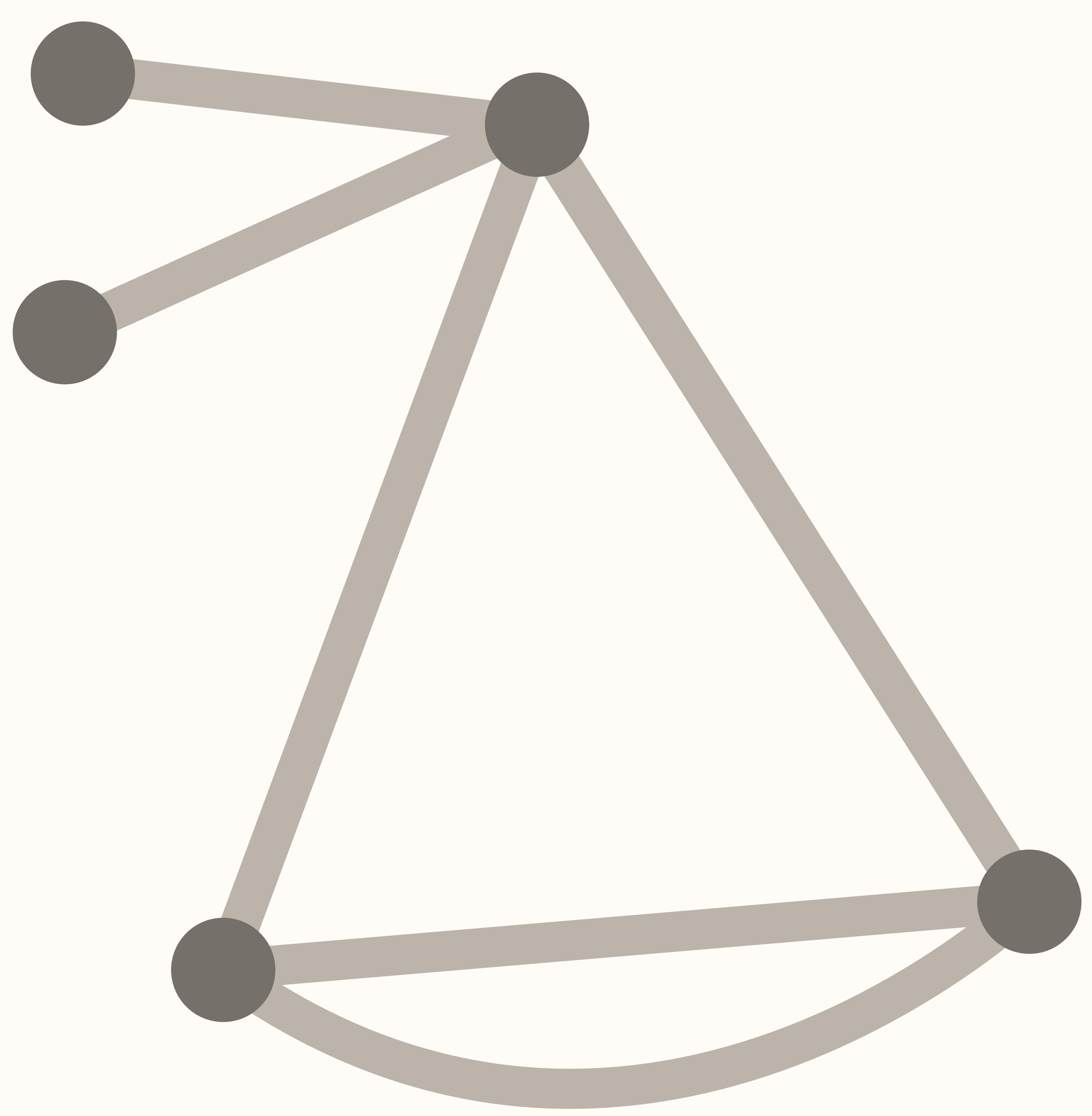
2. Asymptotic counting results

3. Link with plane permutations

a. Plane permutations
b. Bijection with posets by vertices

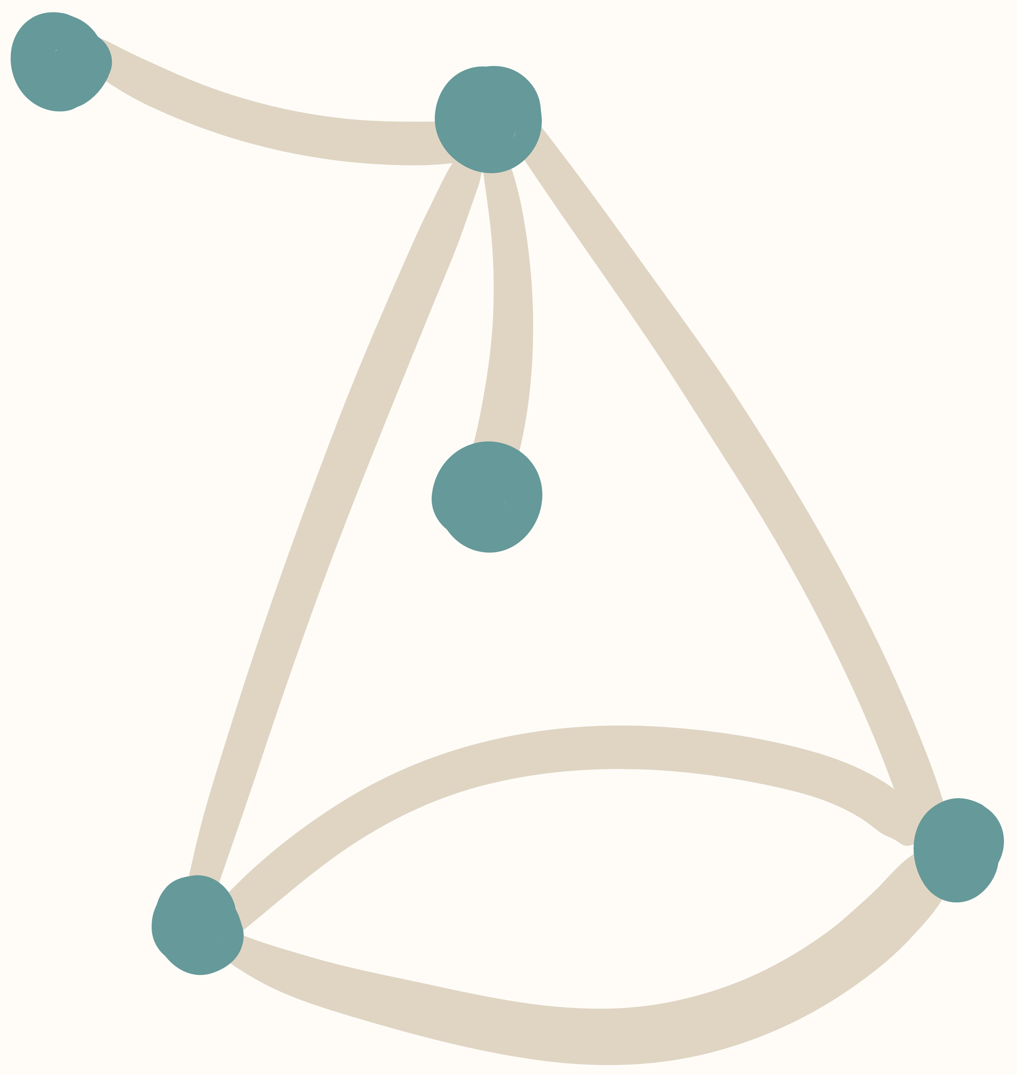
Maps

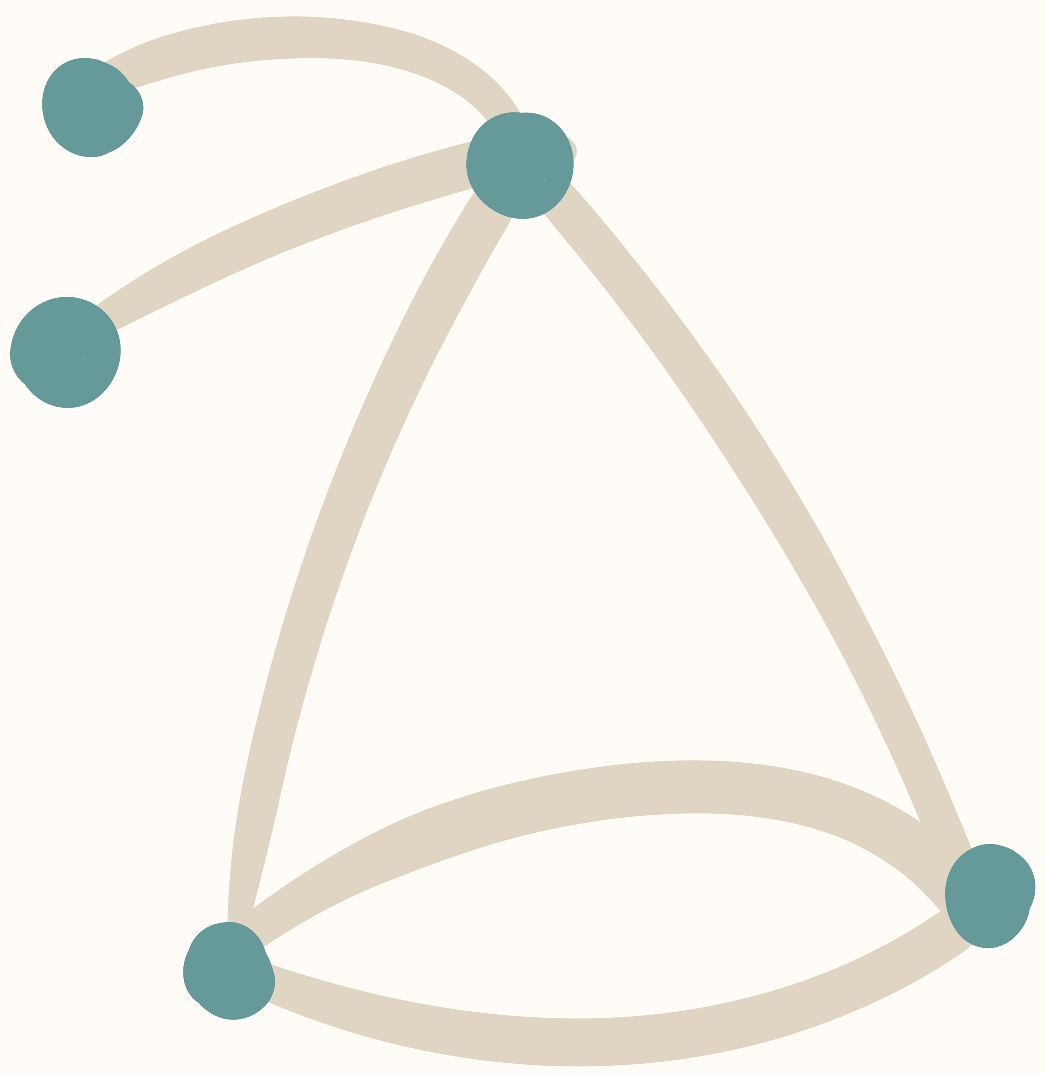
Graphs



Maps

embeddings in the plane





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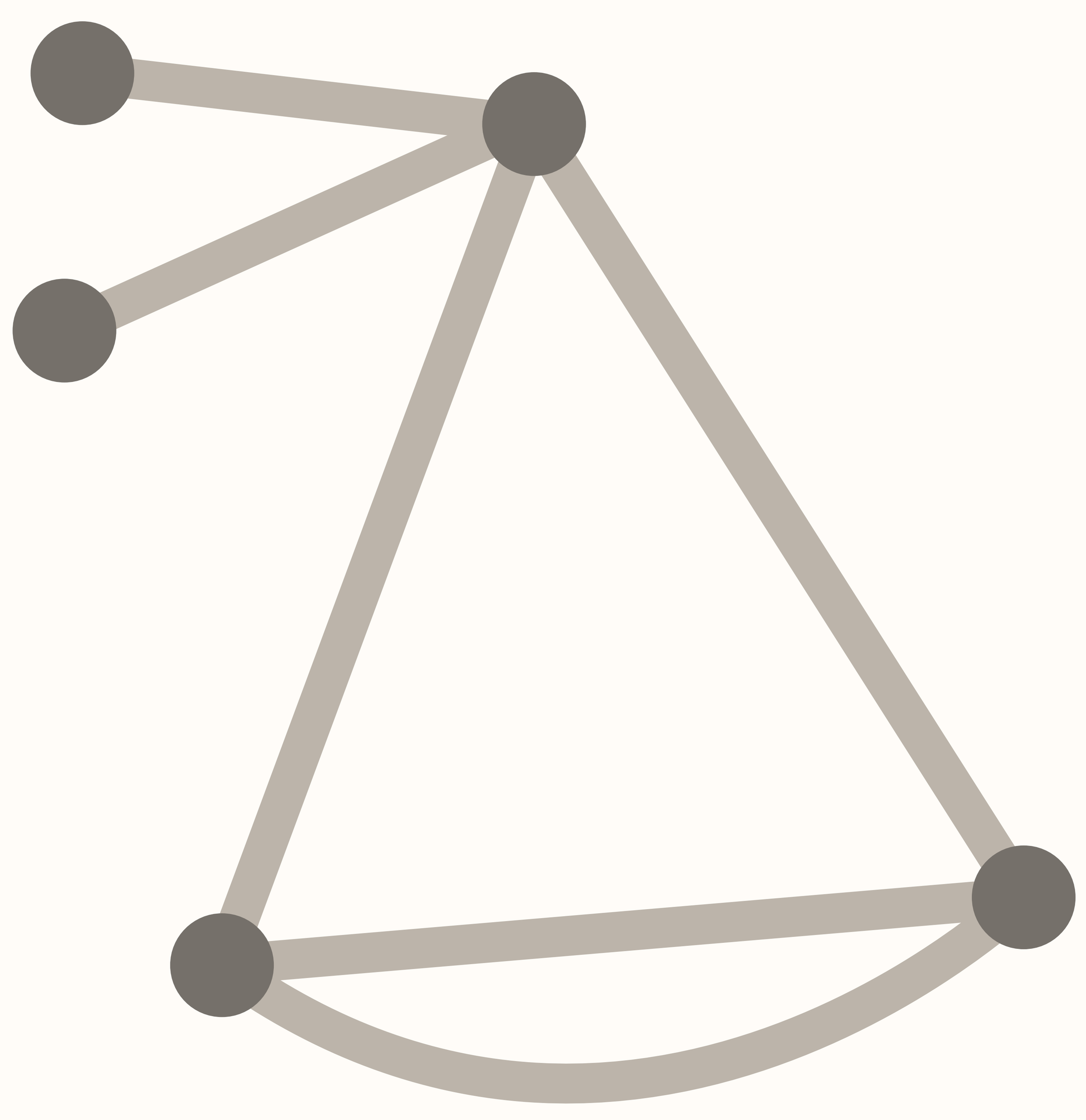

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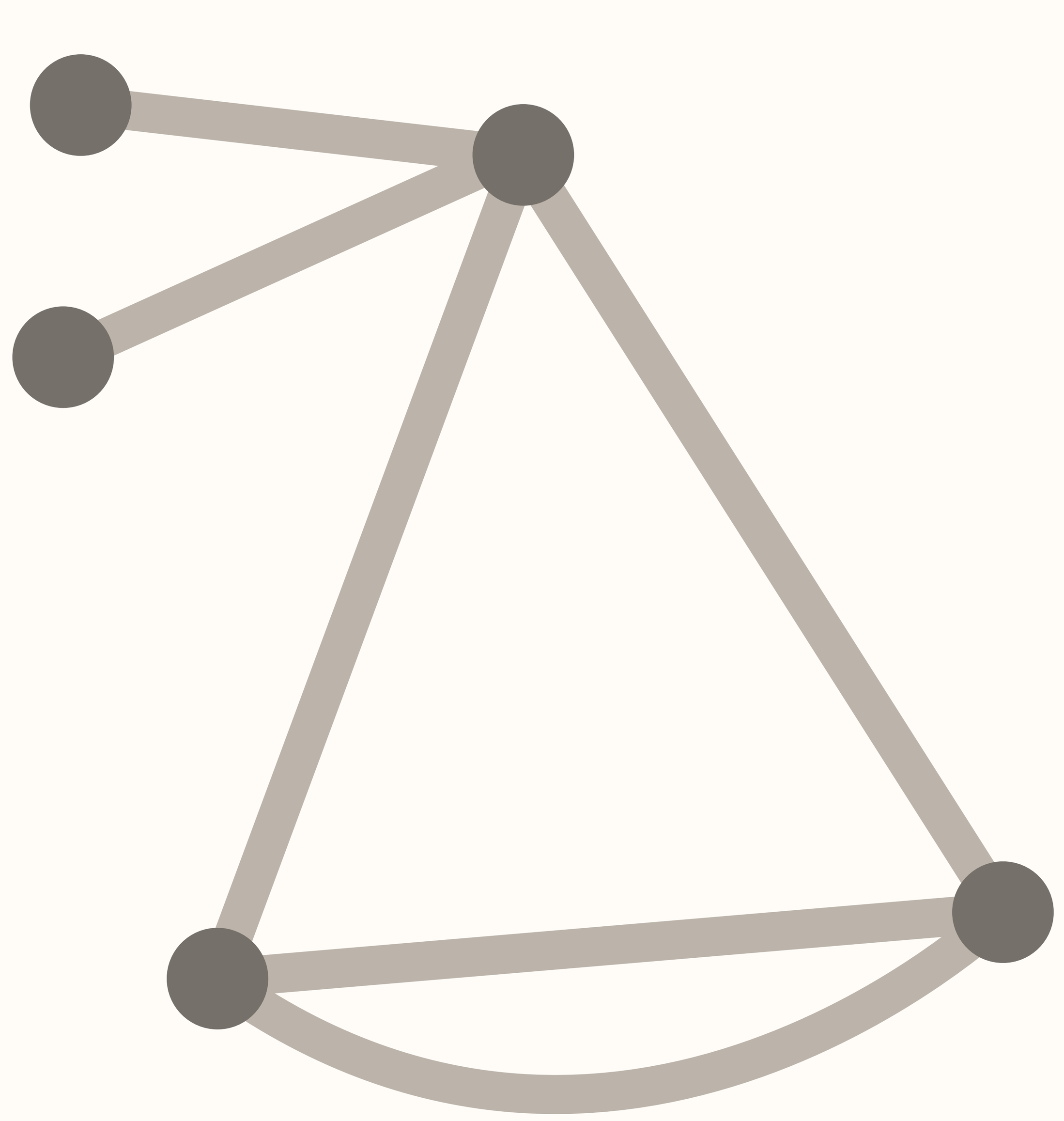

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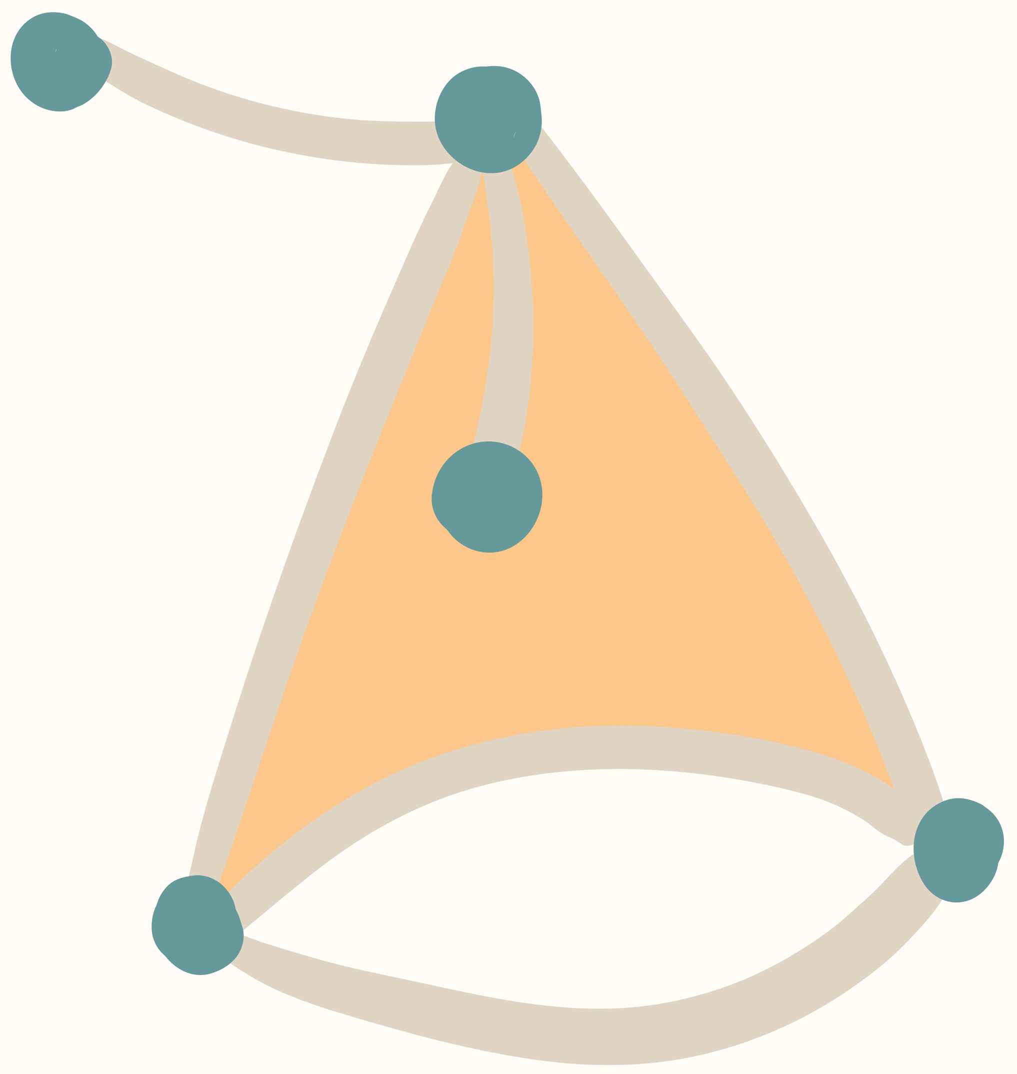

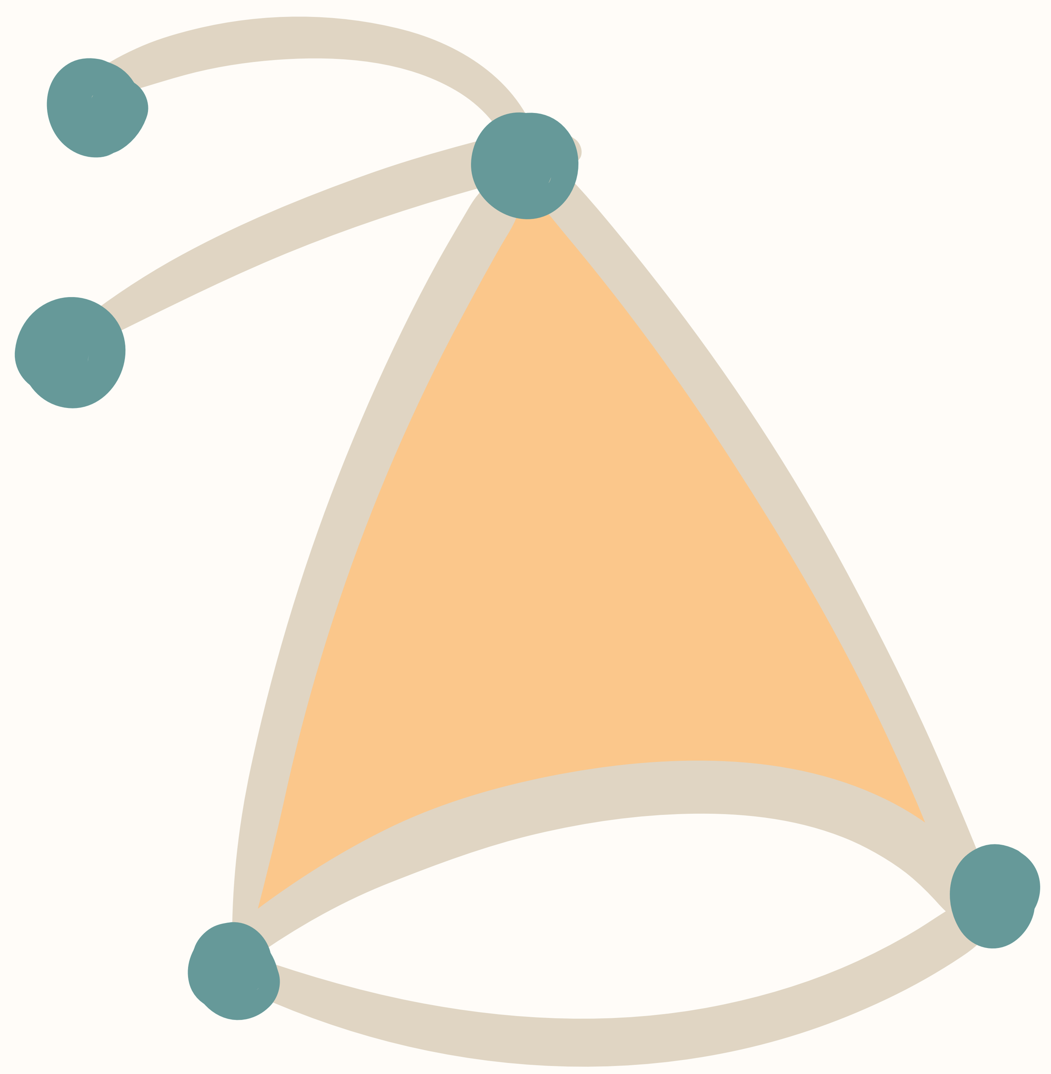

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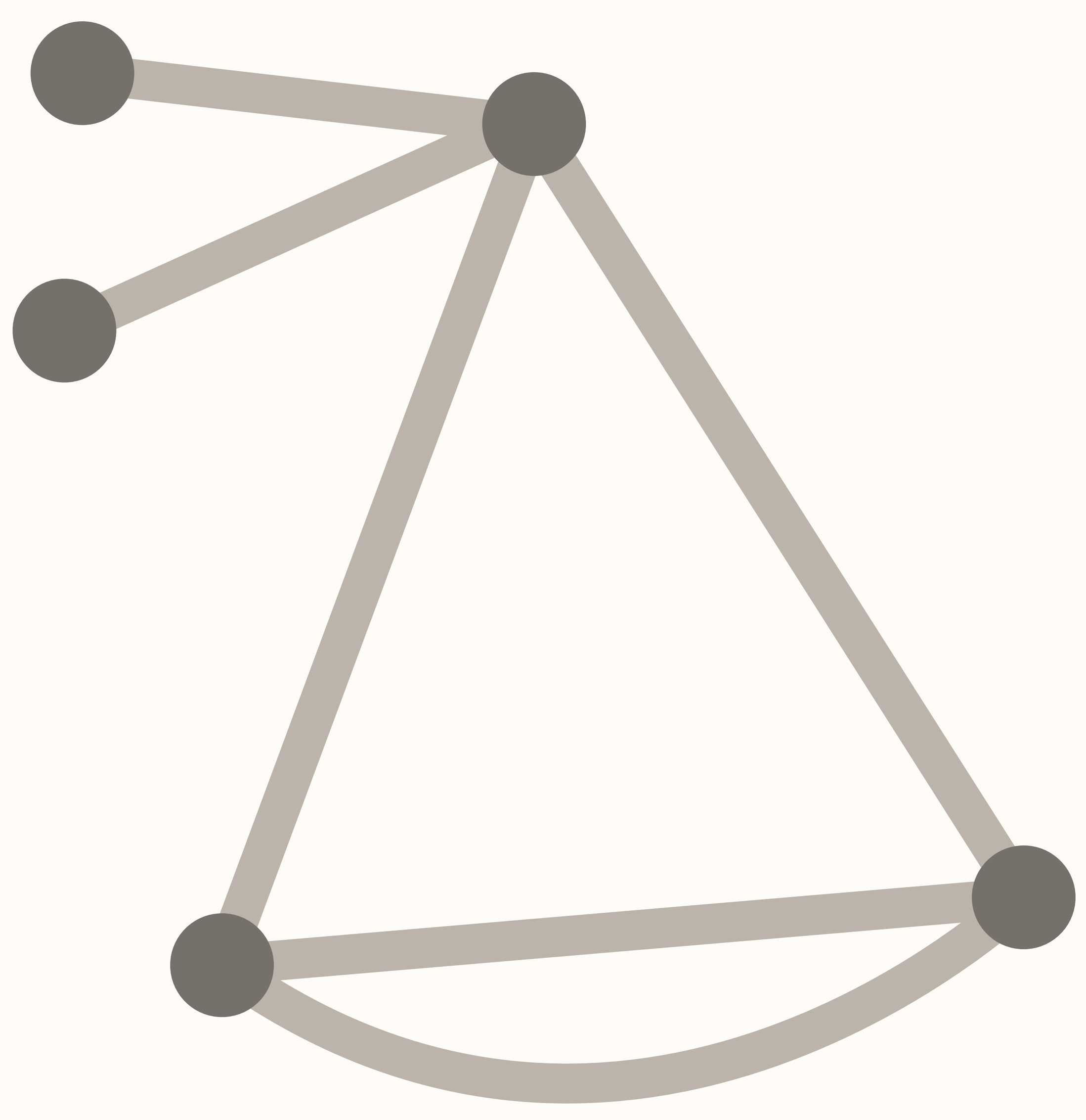

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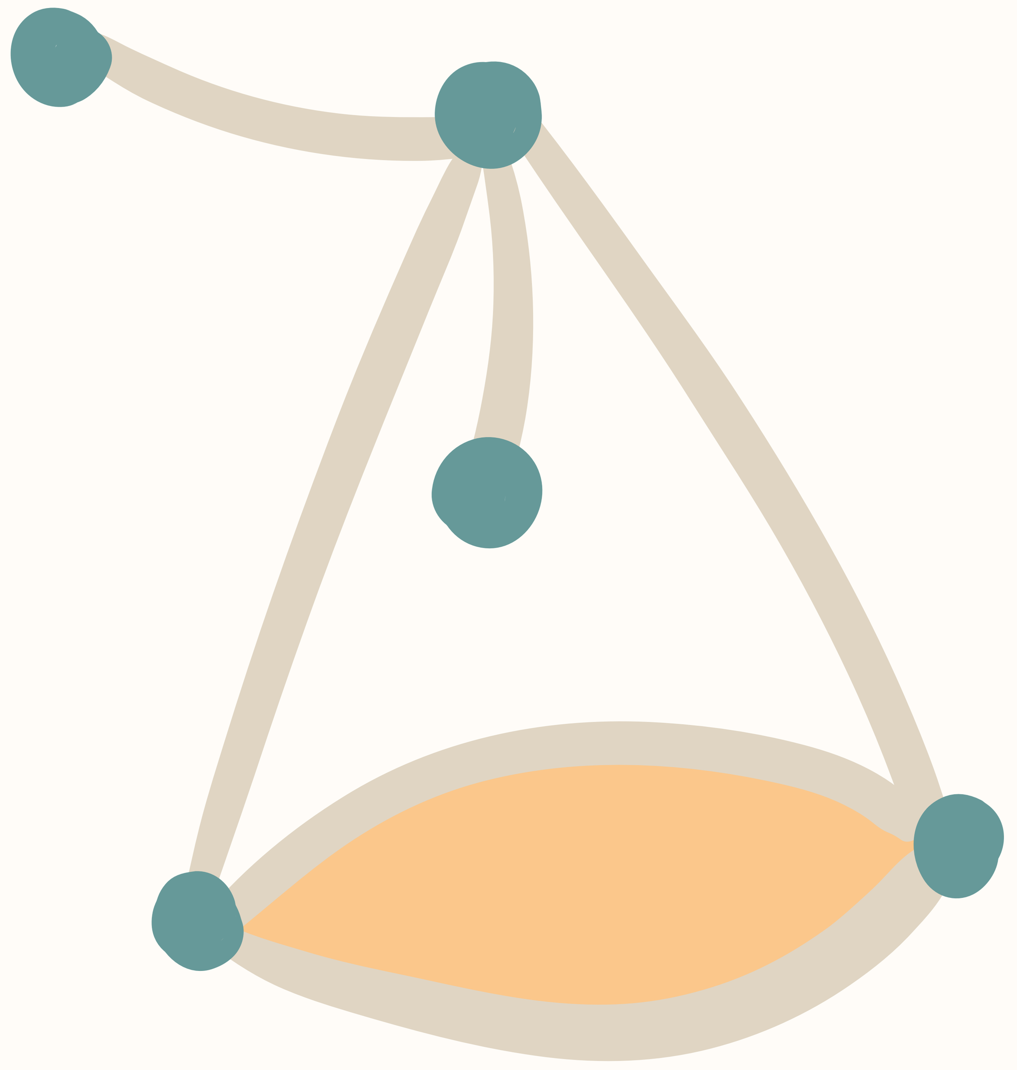

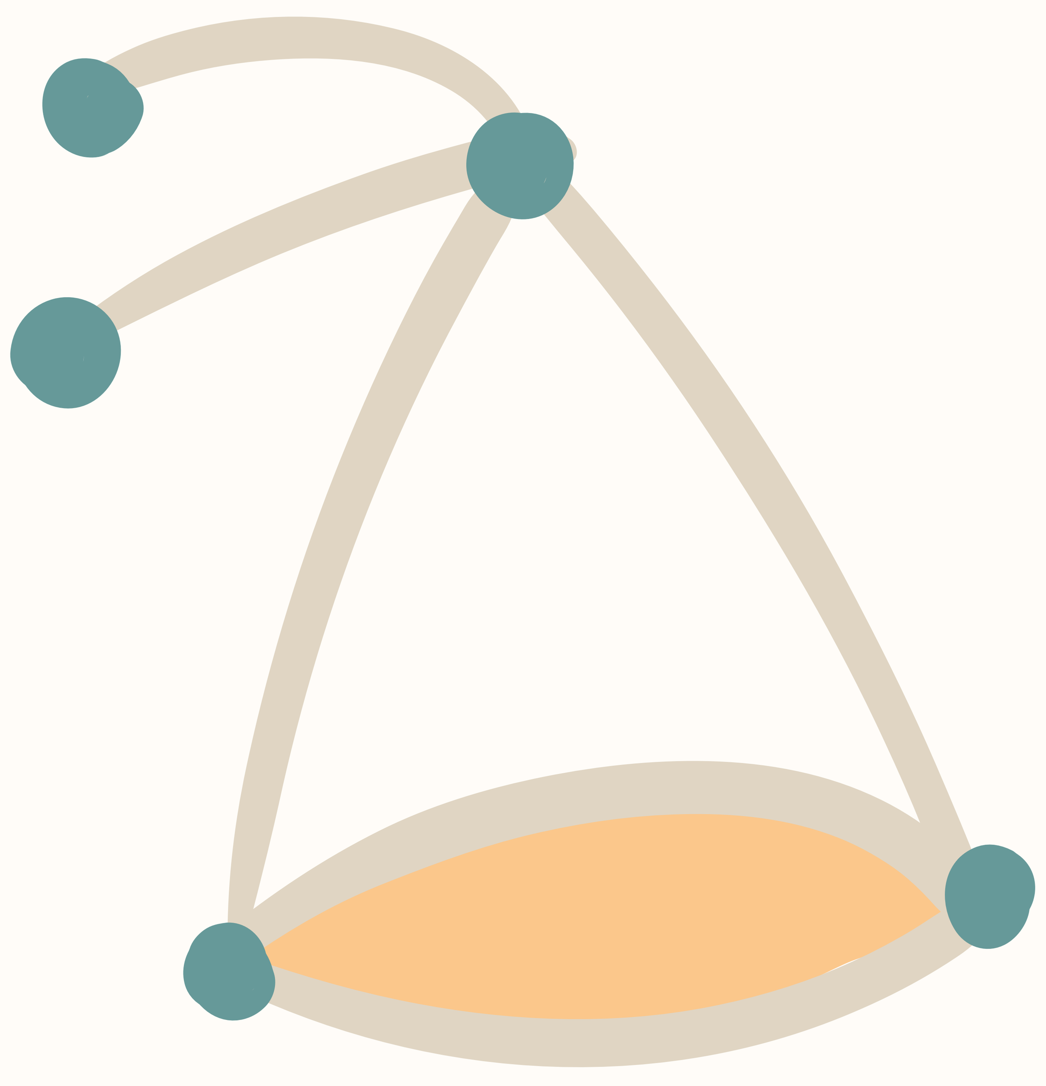

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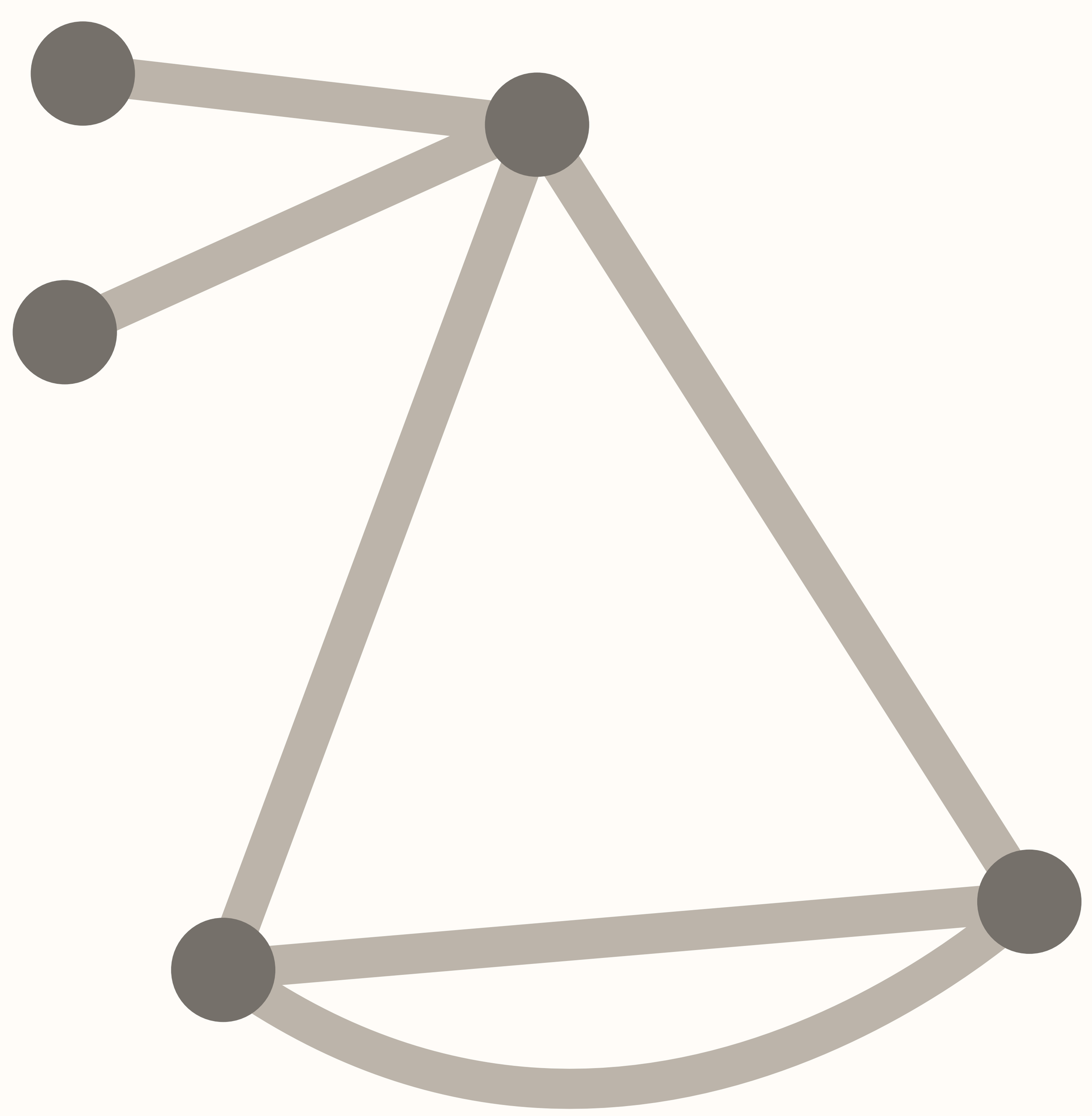

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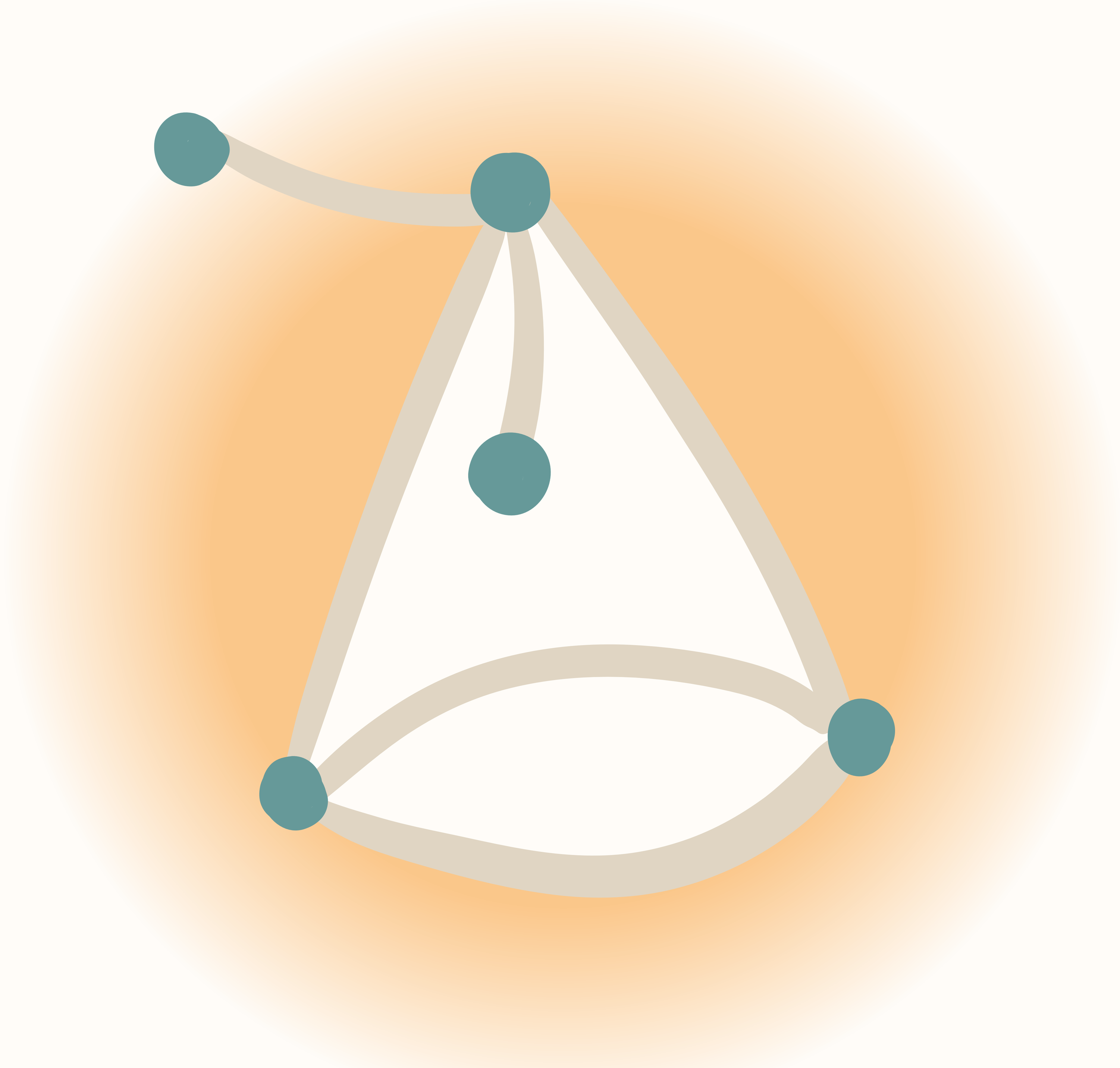

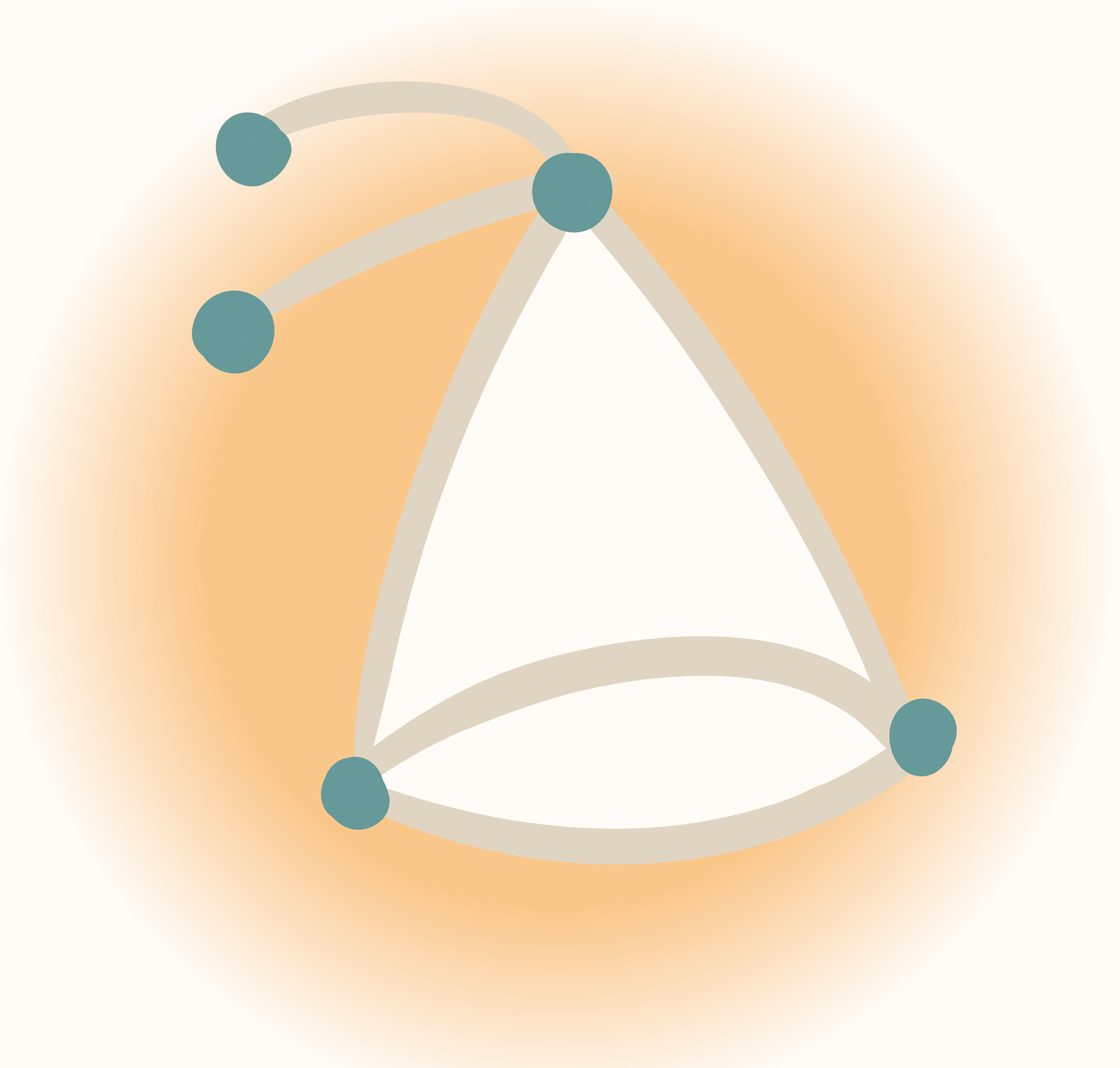

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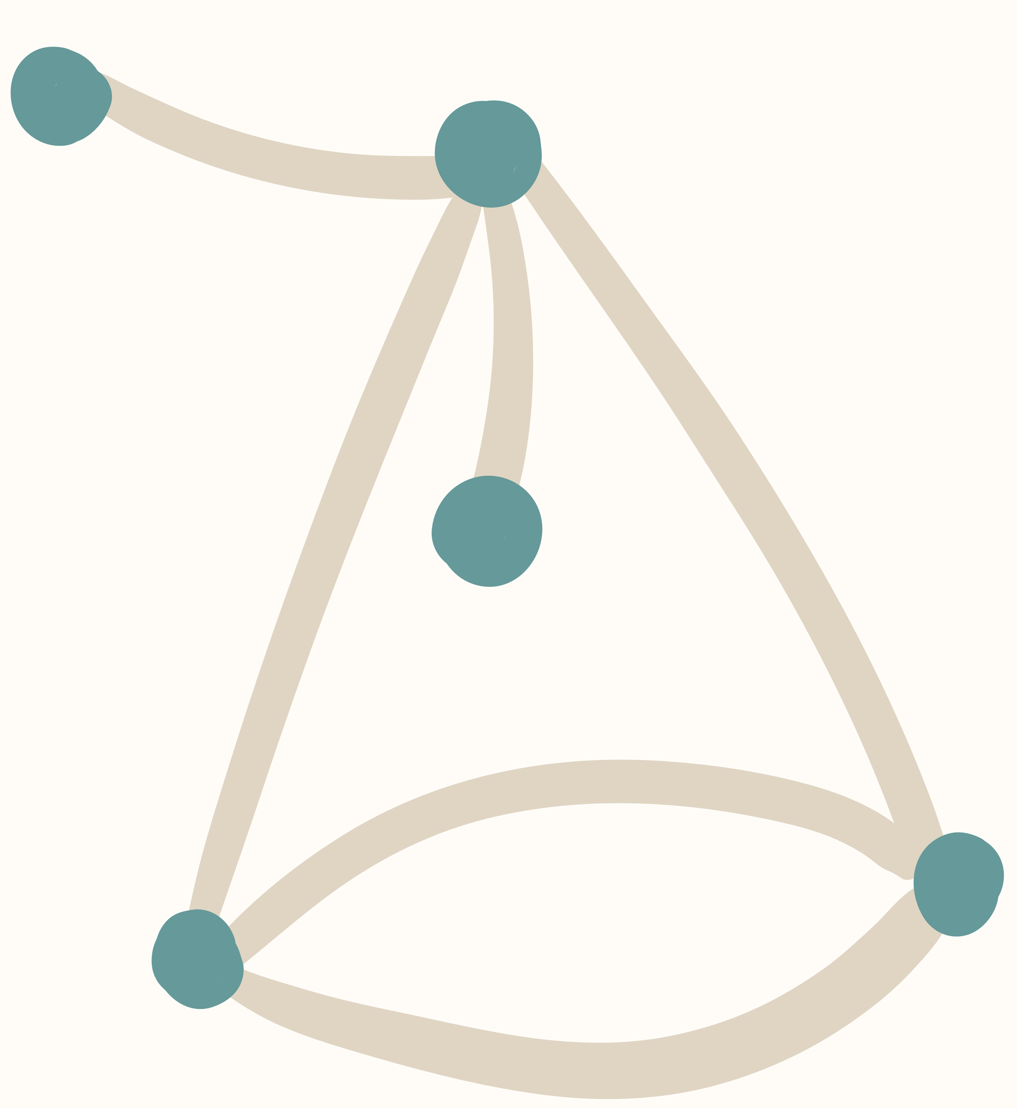

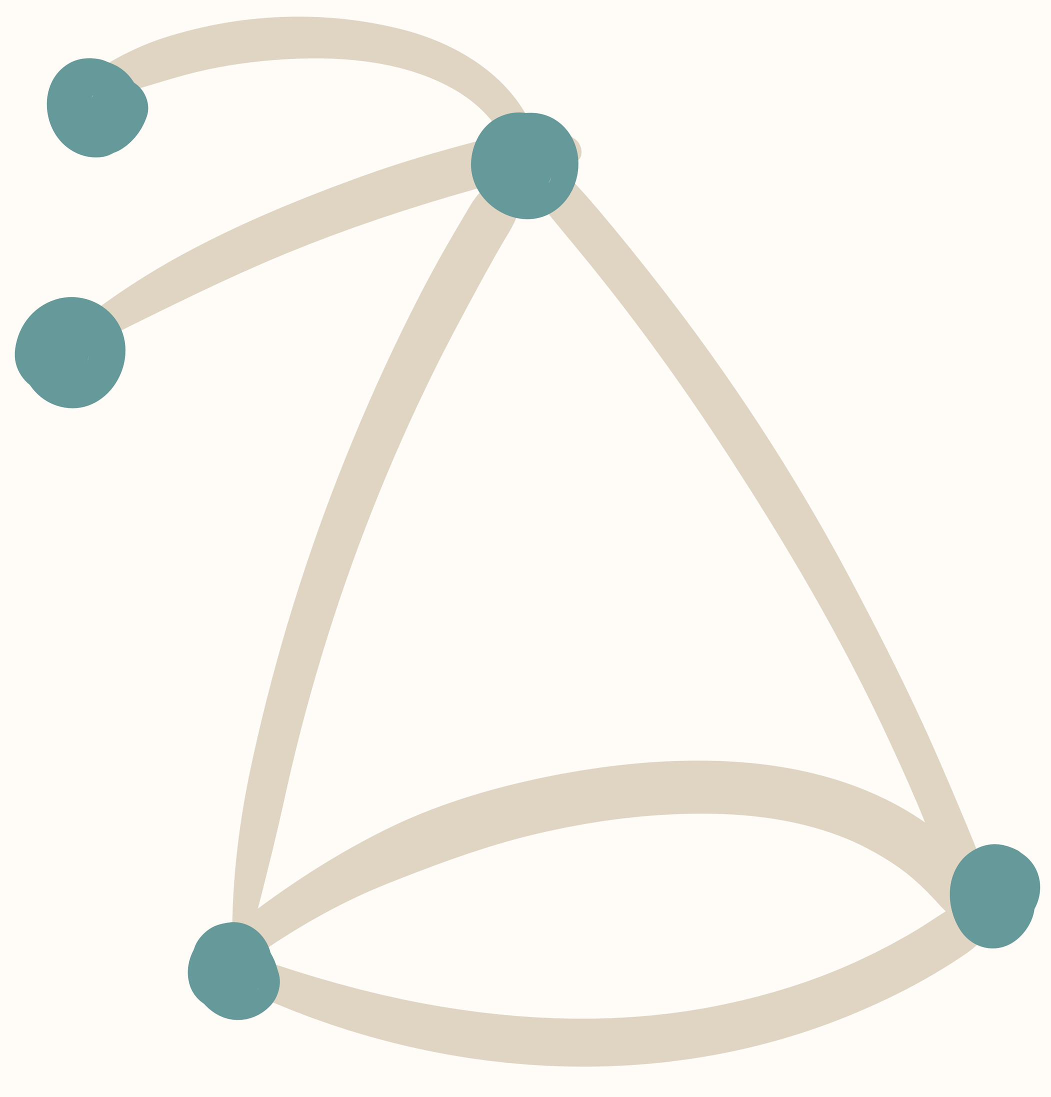

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→ *exact enumeration formulas*

→ *universal asymptotic exponent :*

\# maps with n edges $= \varkappa \cdot \gamma^n n^{-5/2}$

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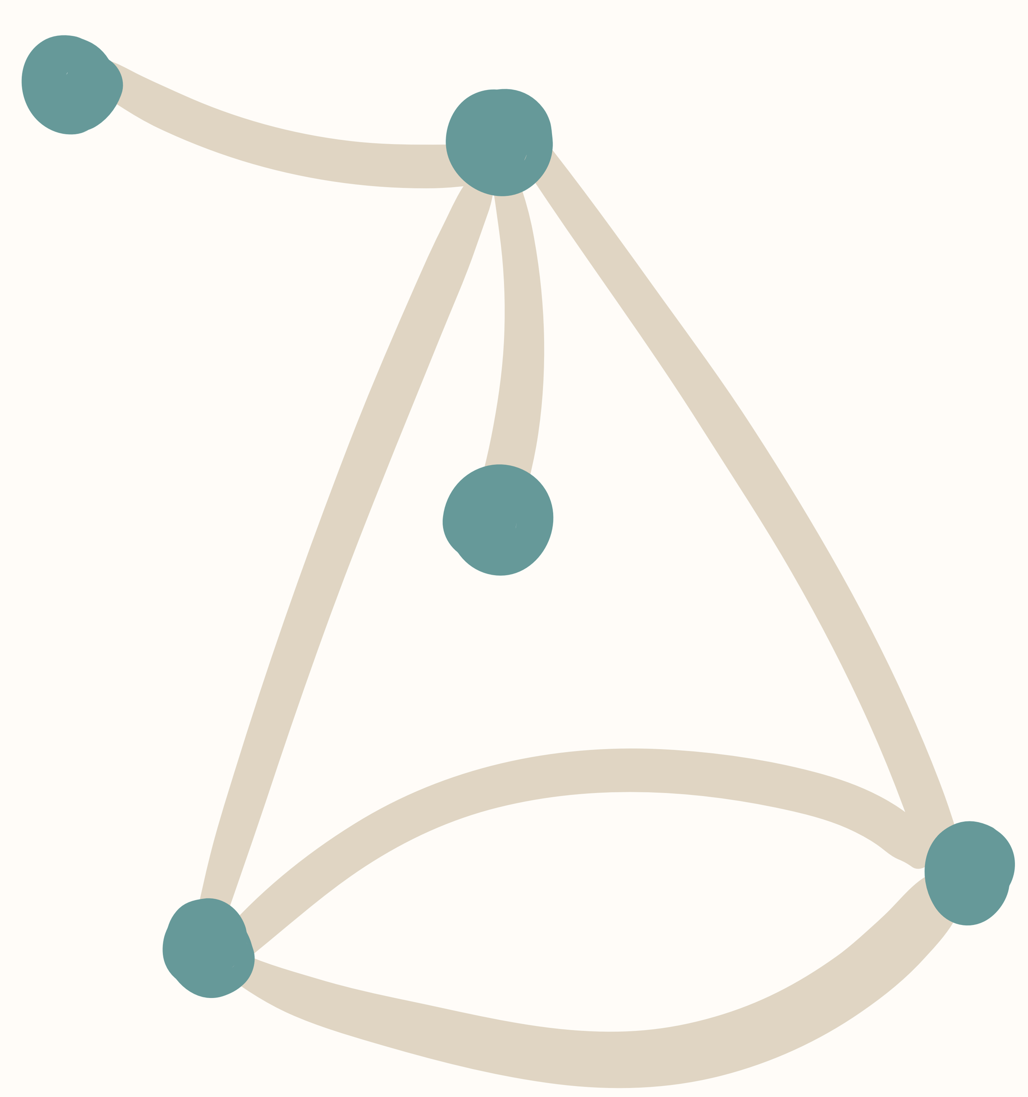

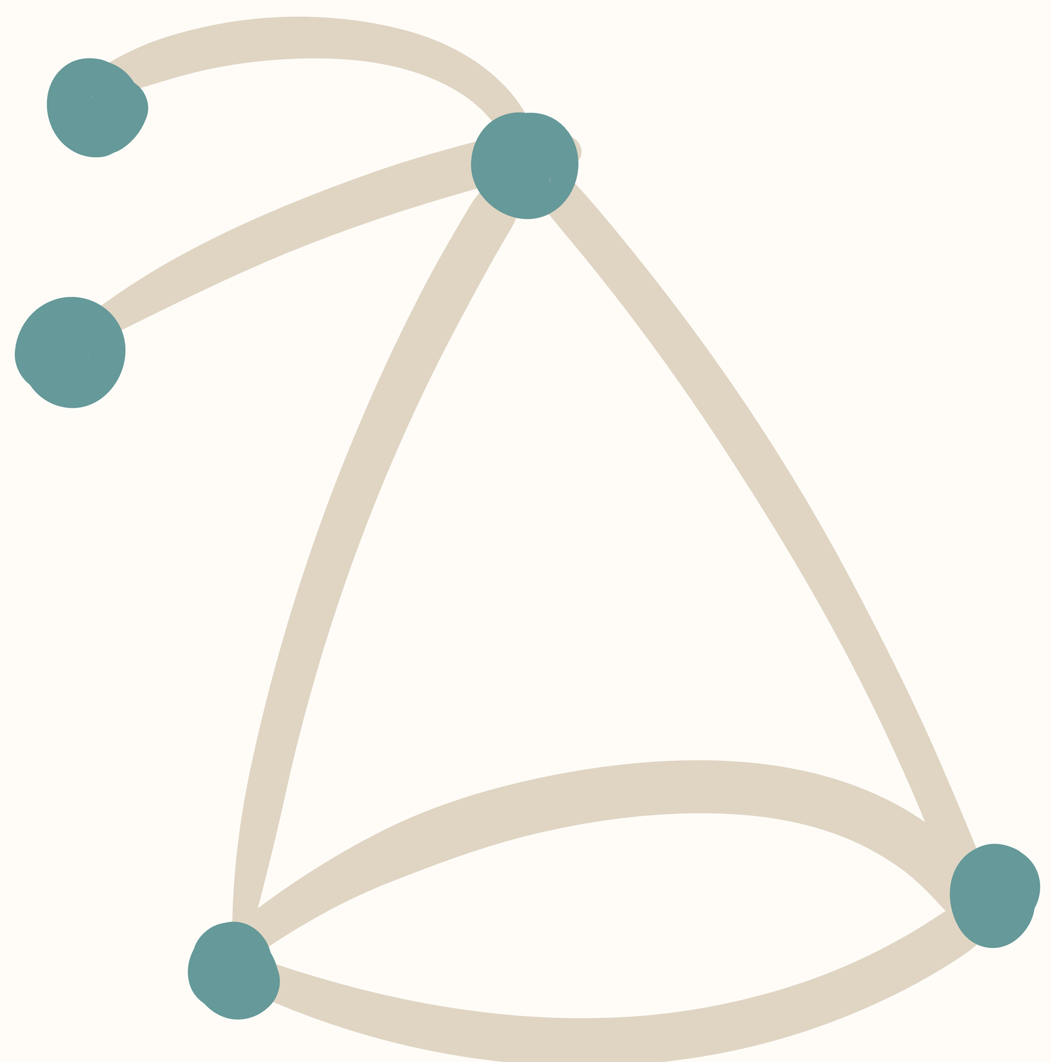

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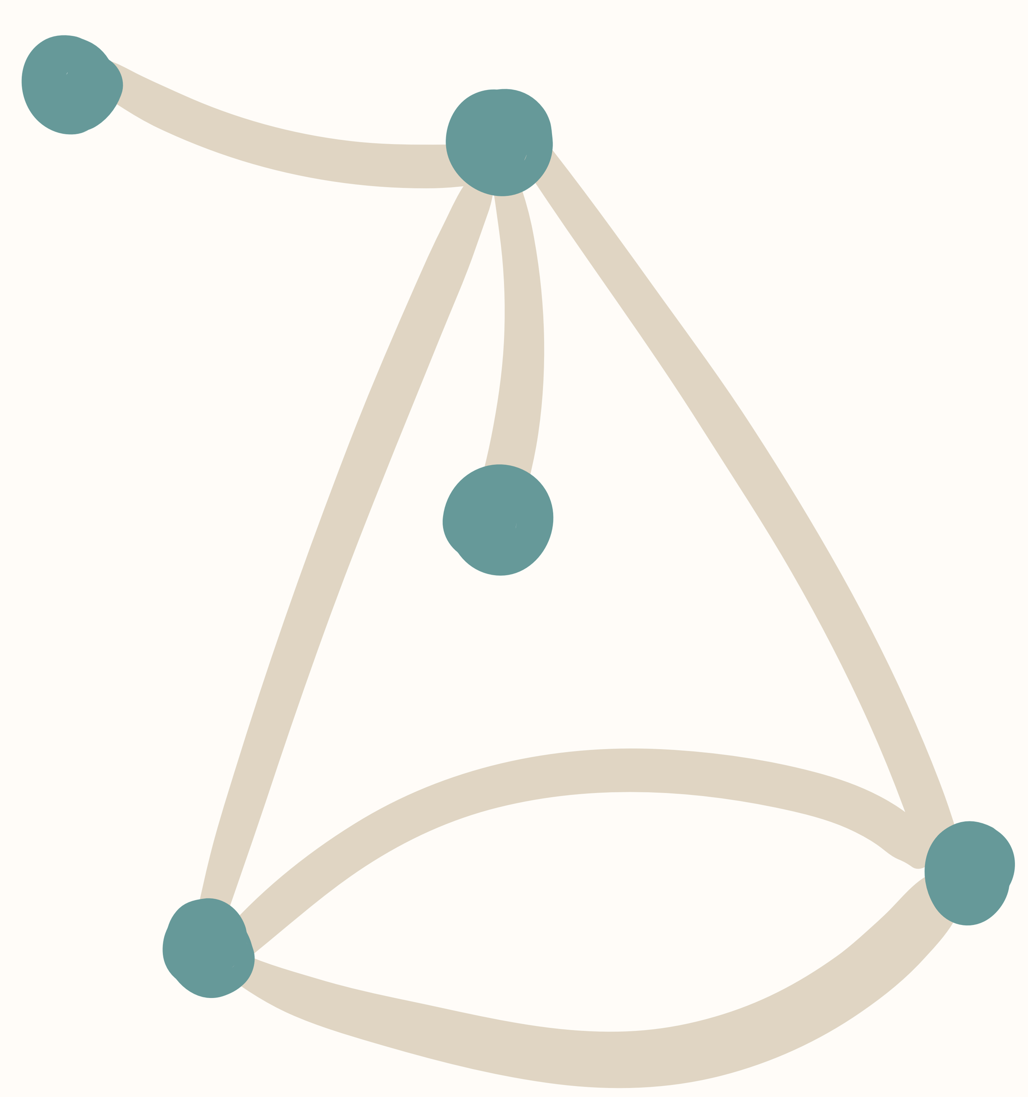
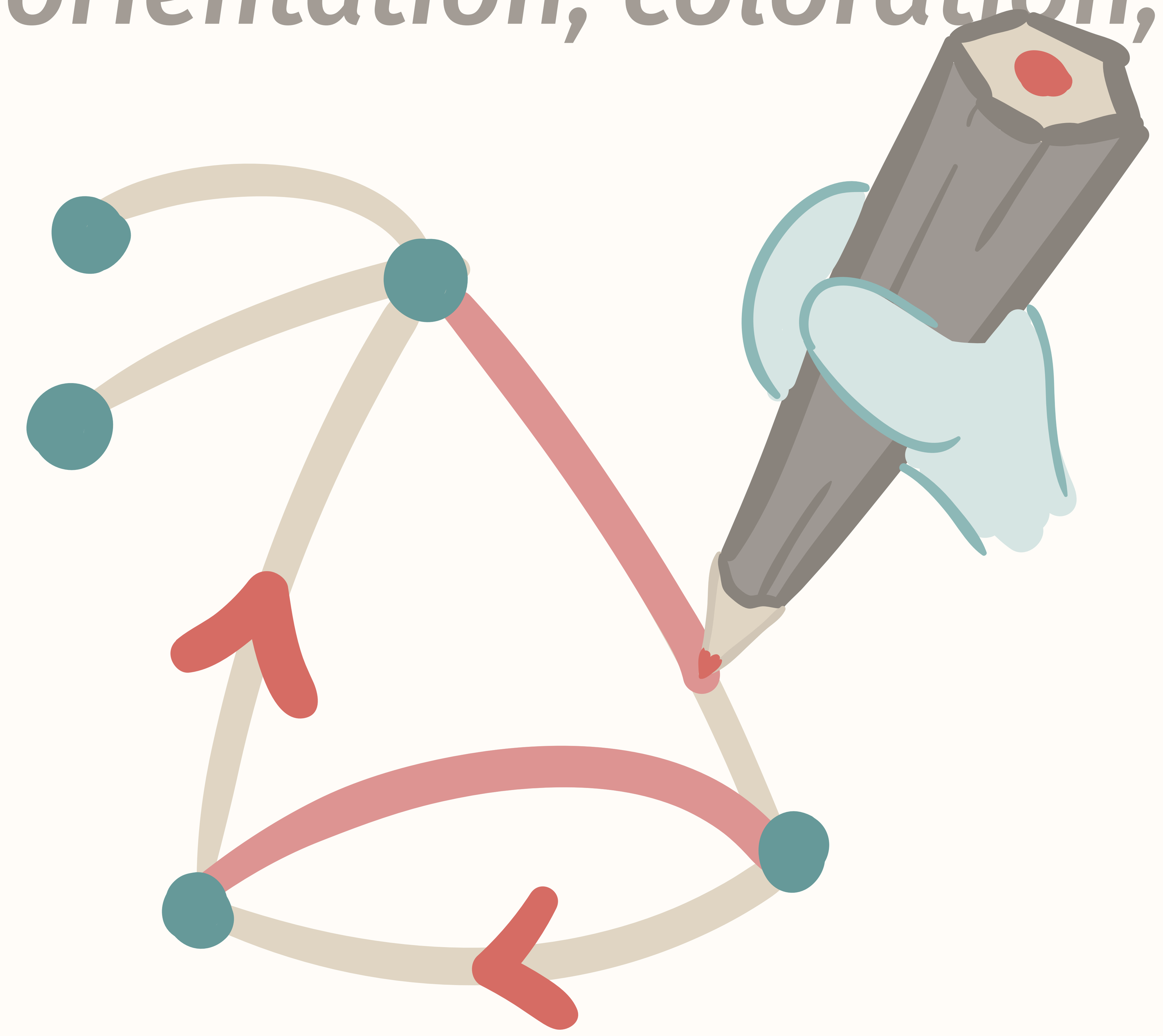


Decorated maps

orientation, coloration, etc.



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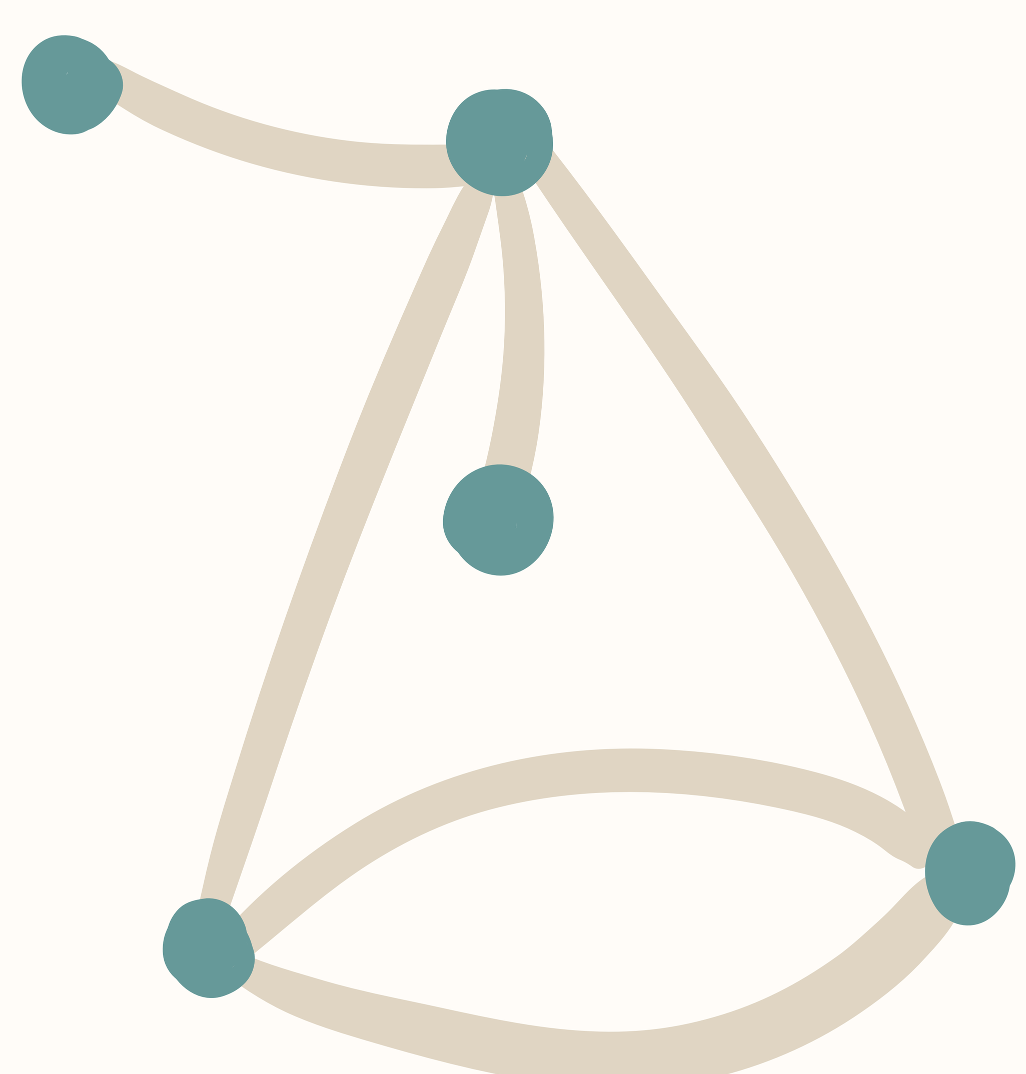
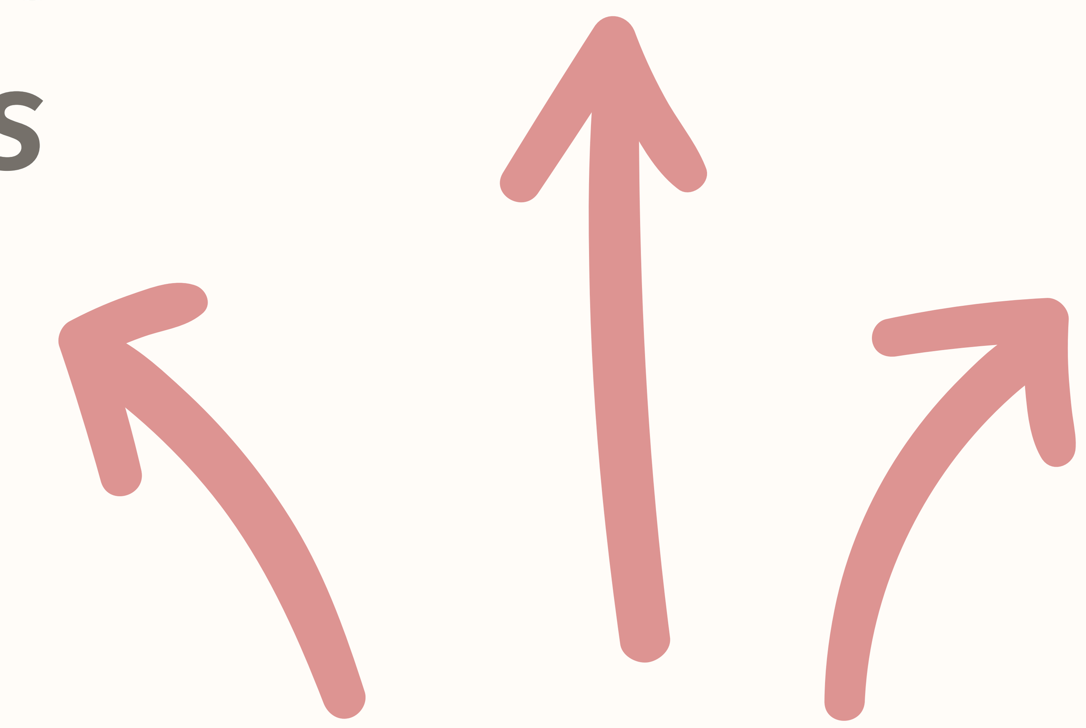


transversal structures

Schnyder Woods

Plane bipolar posets



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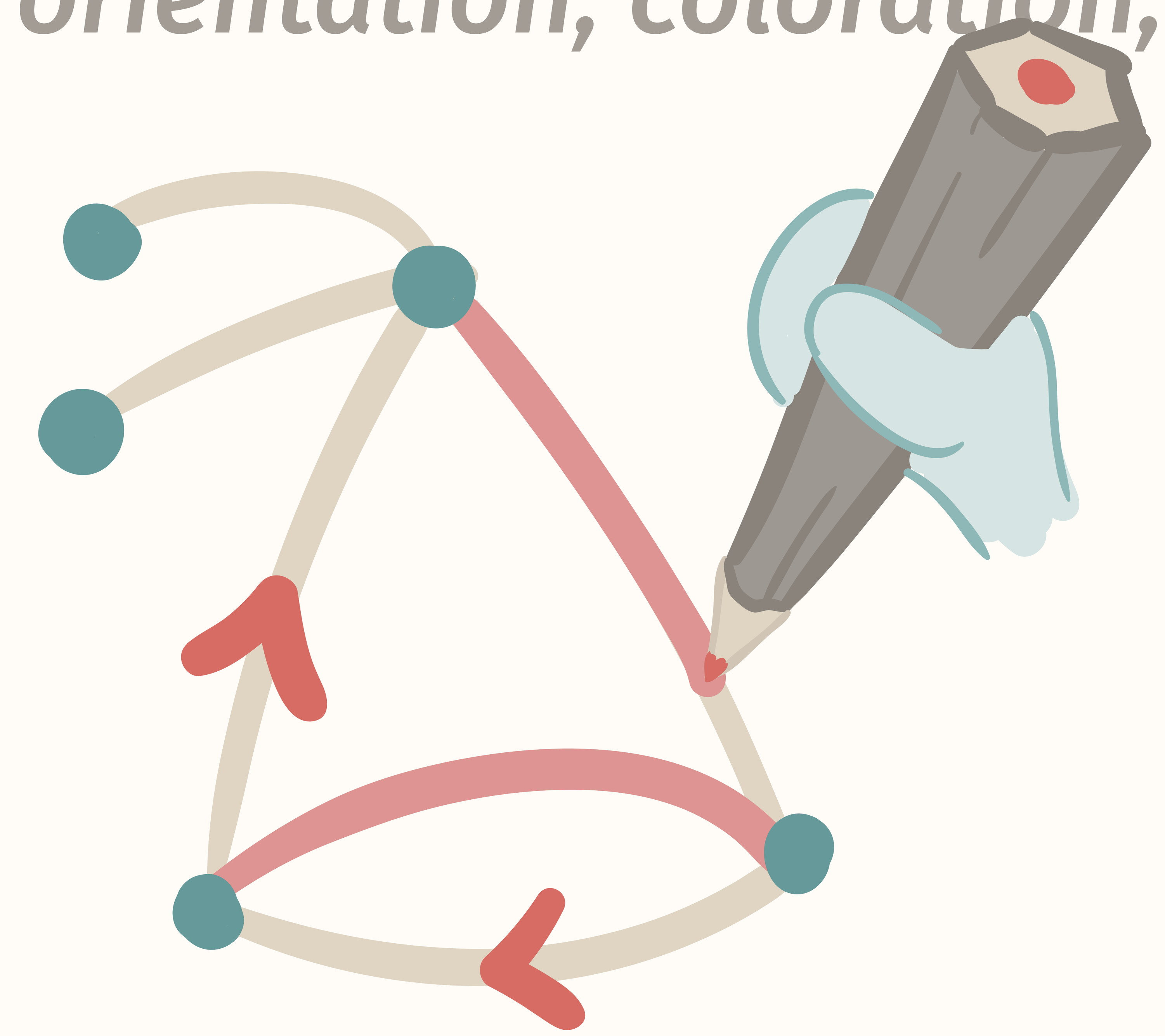

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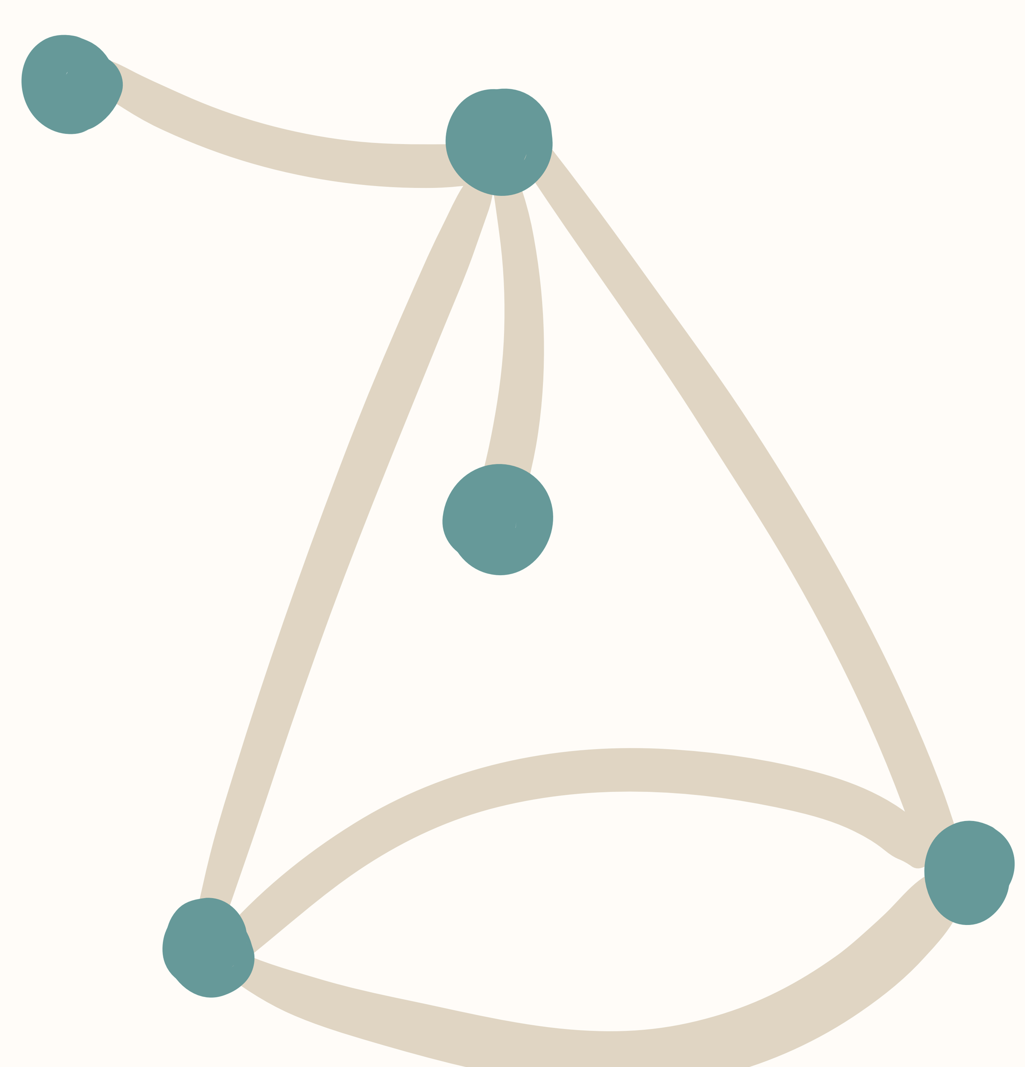

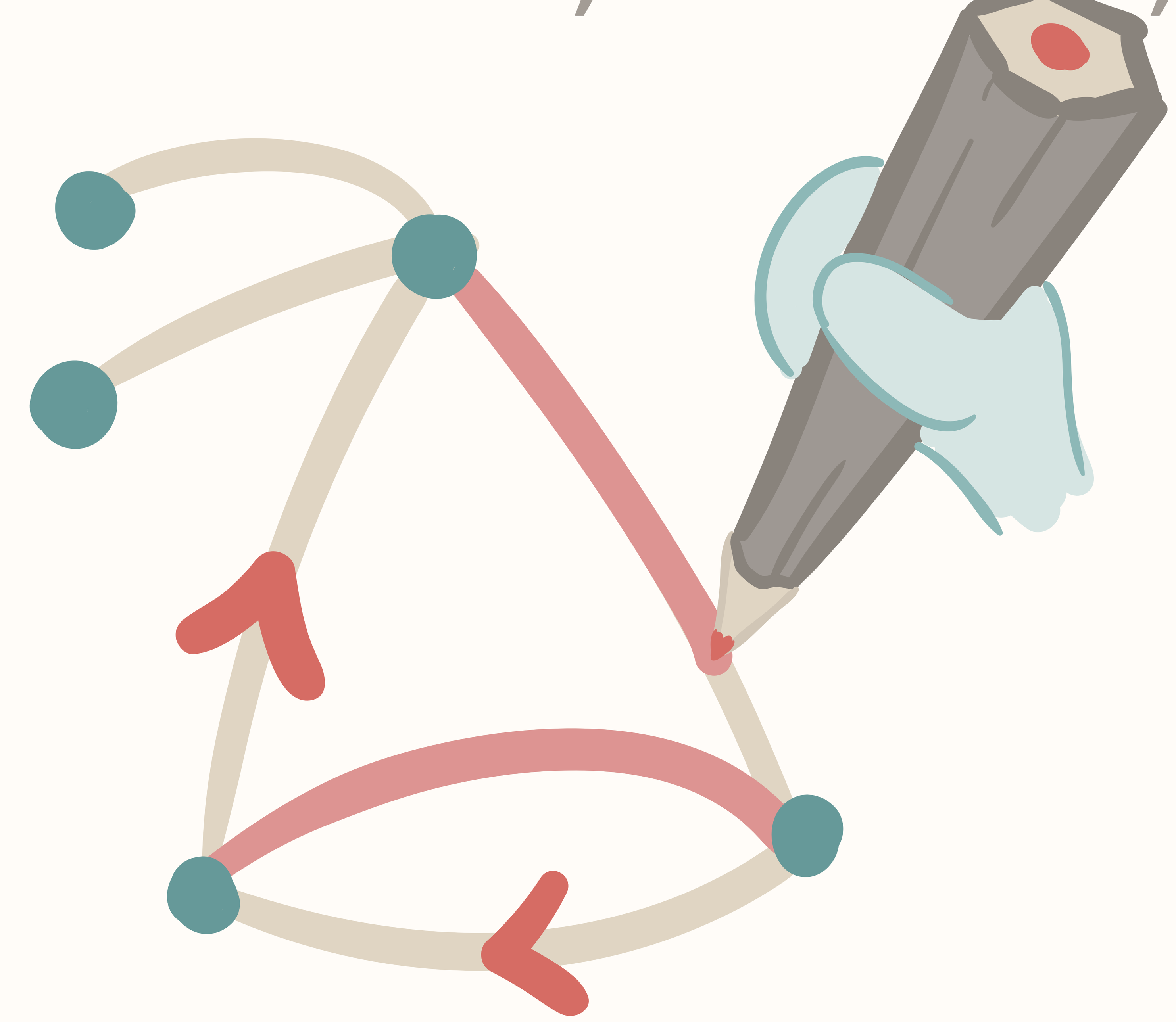

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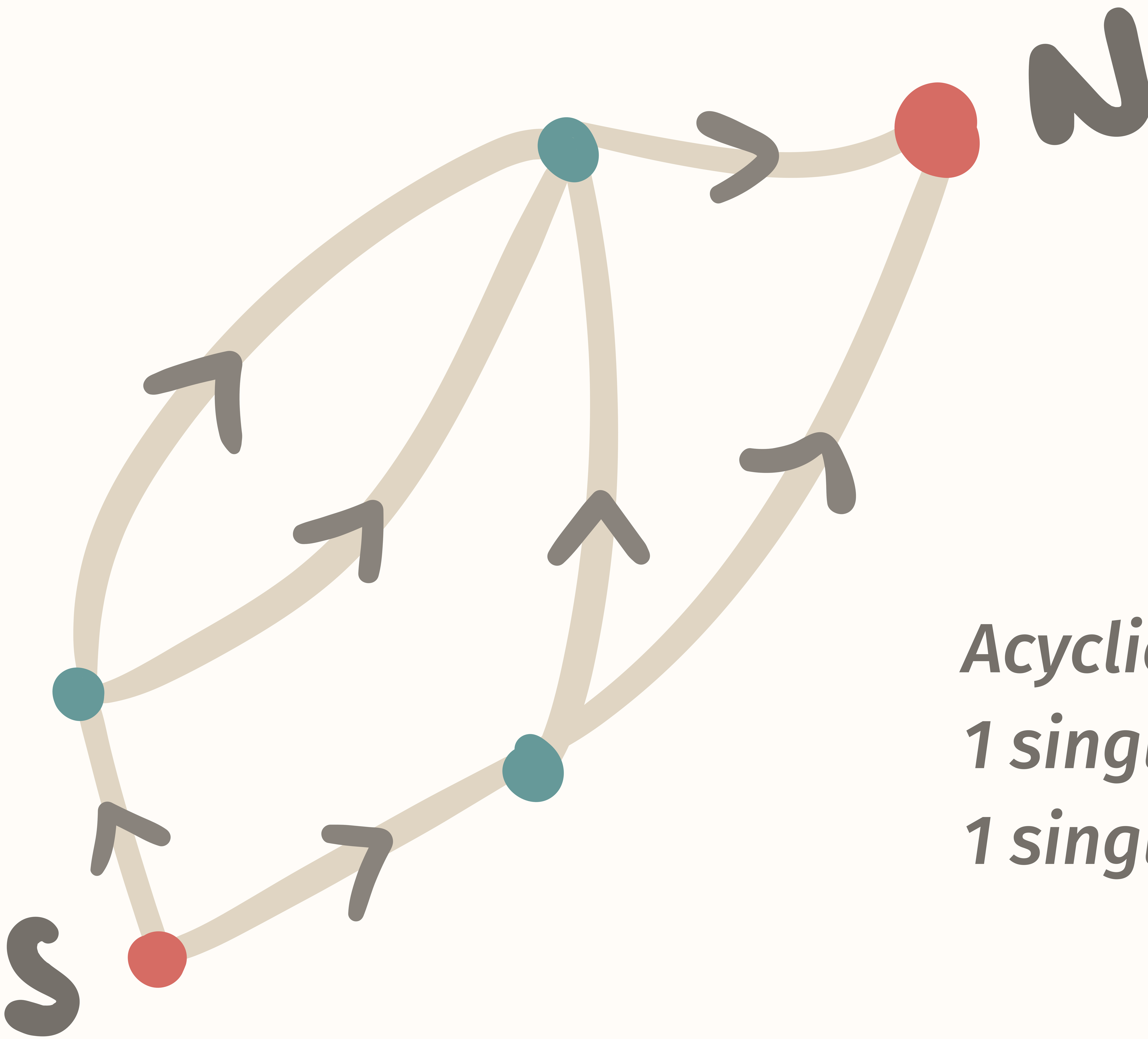
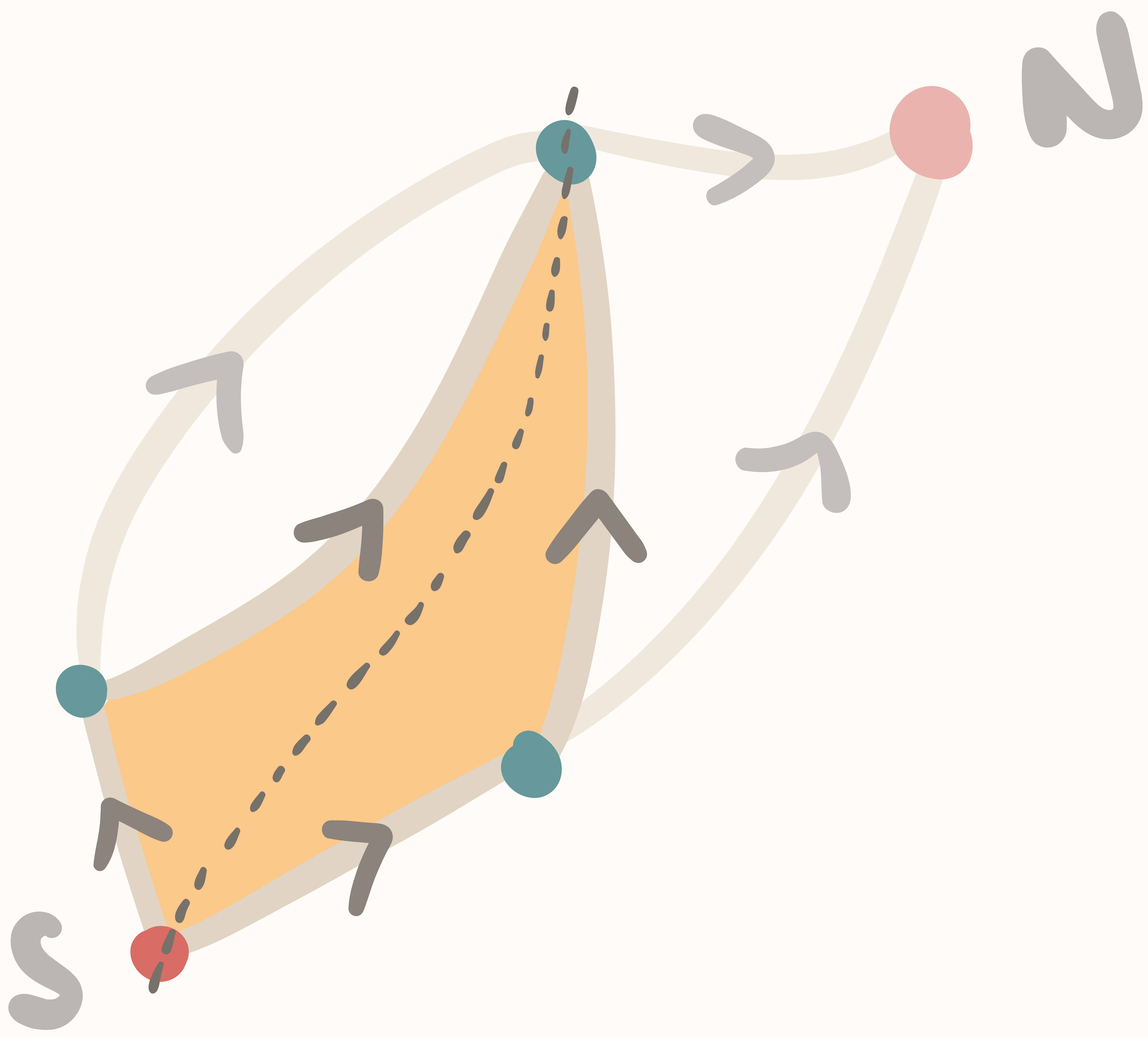


Acyclic
1 single source S
1 single sink N

Plane bipolar orientation



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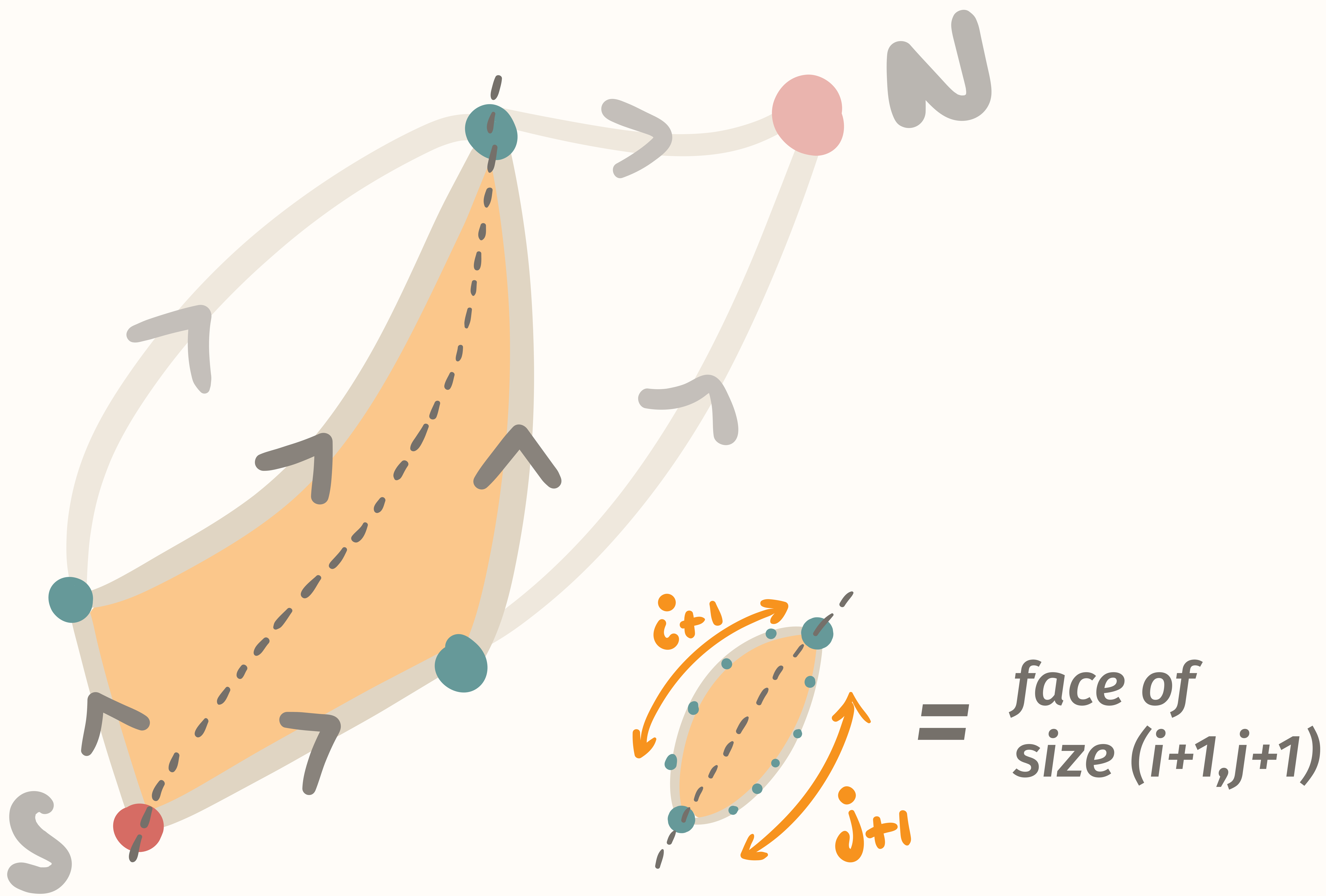
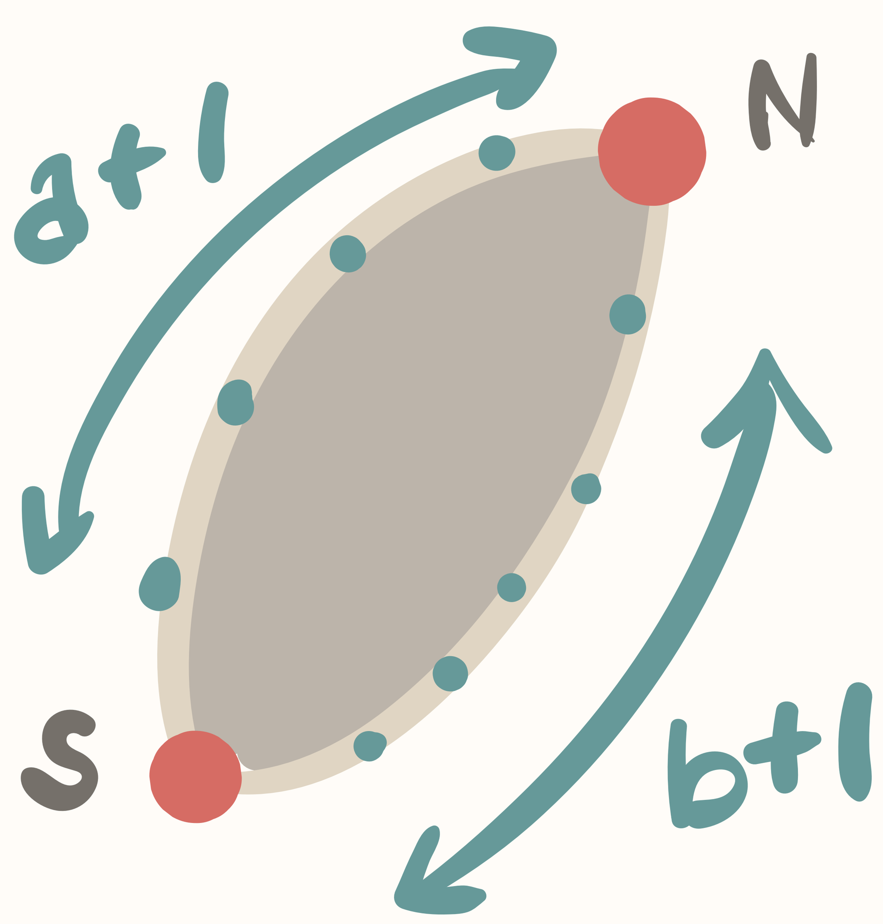


The KMSW bijection

Plane bipolar orientations $\longleftrightarrow$ *tandems walks in the quarter plane*

The KMSW bijection

Plane bipolar orientations ⟷ *tandems walks in the quarter plane*



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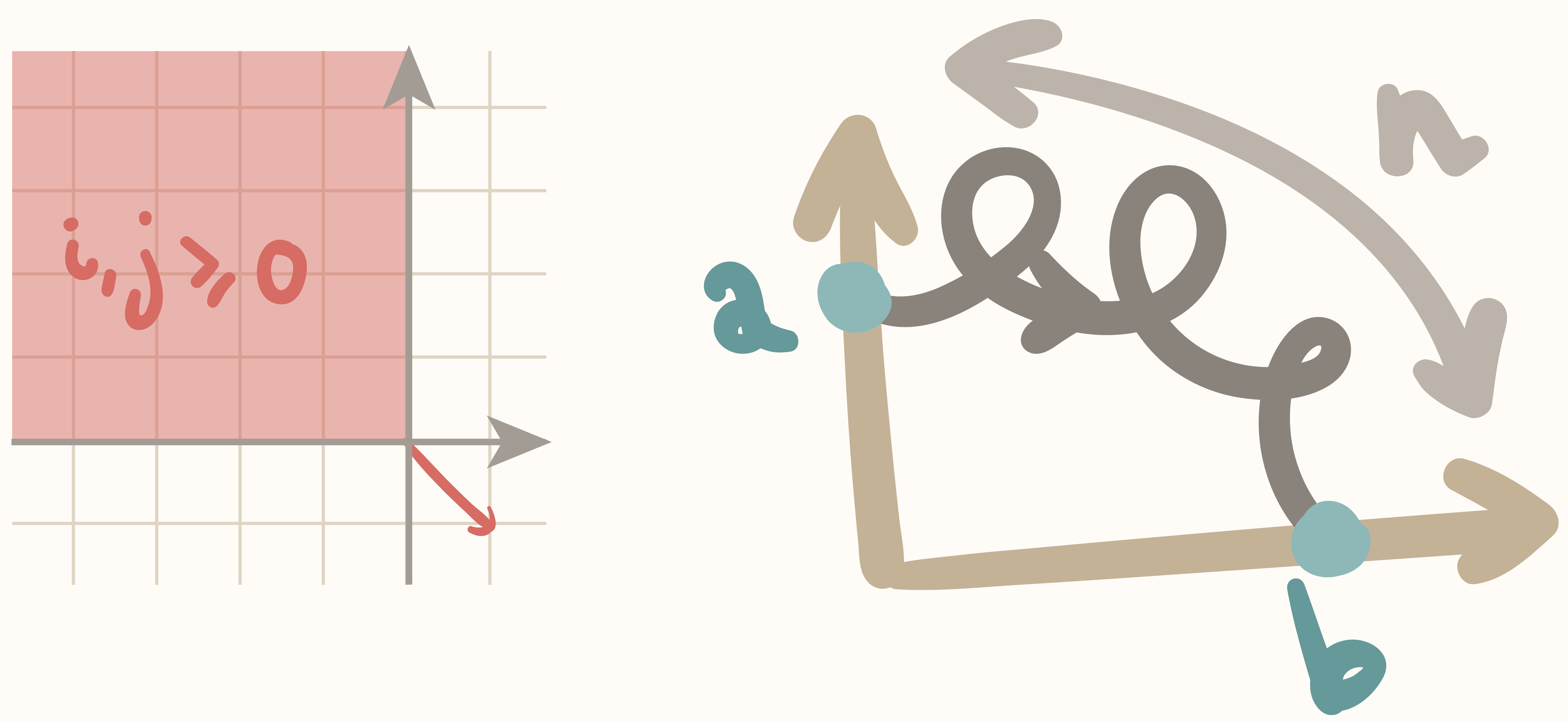

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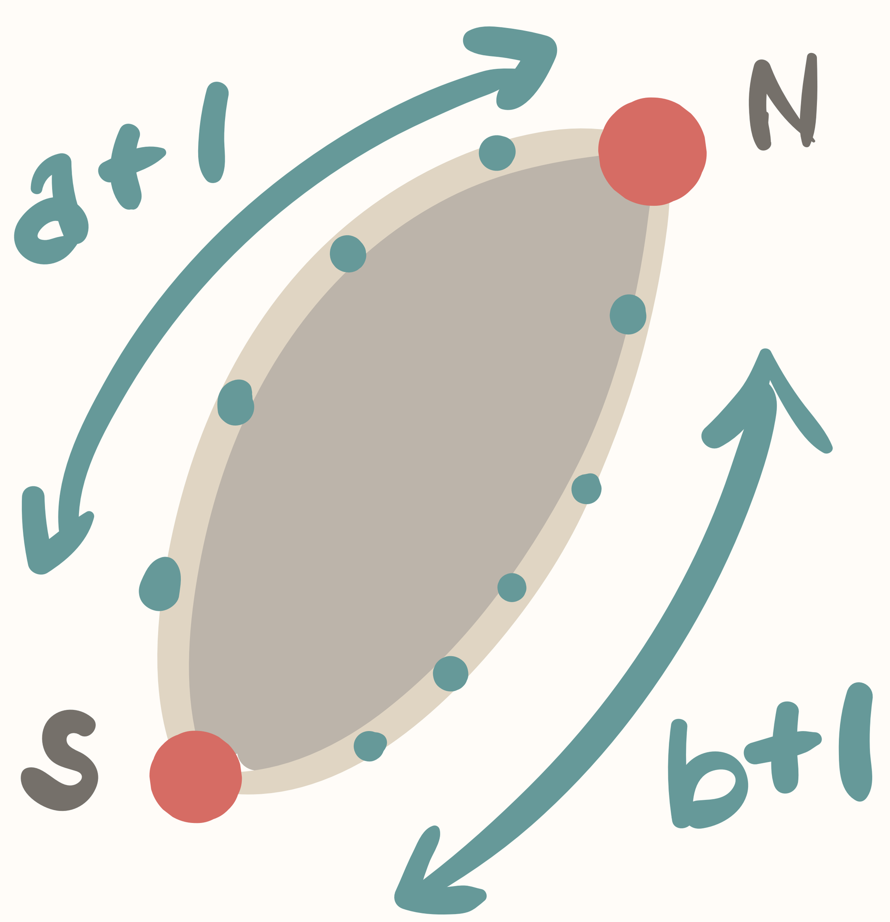

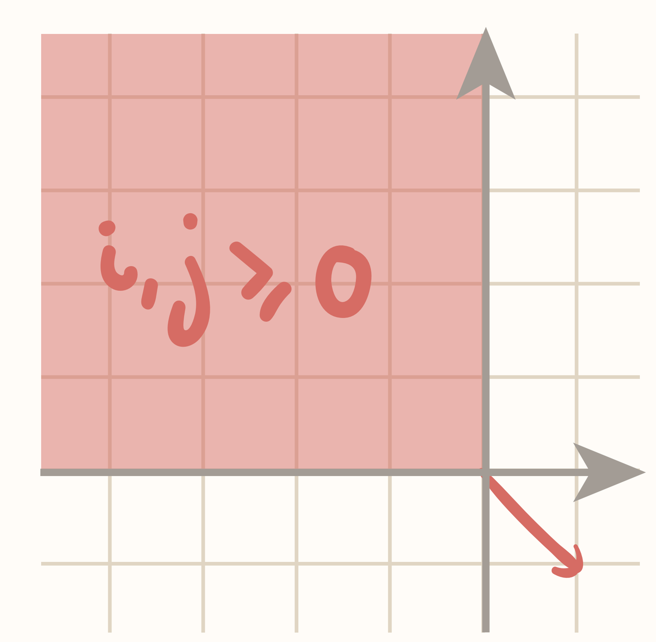

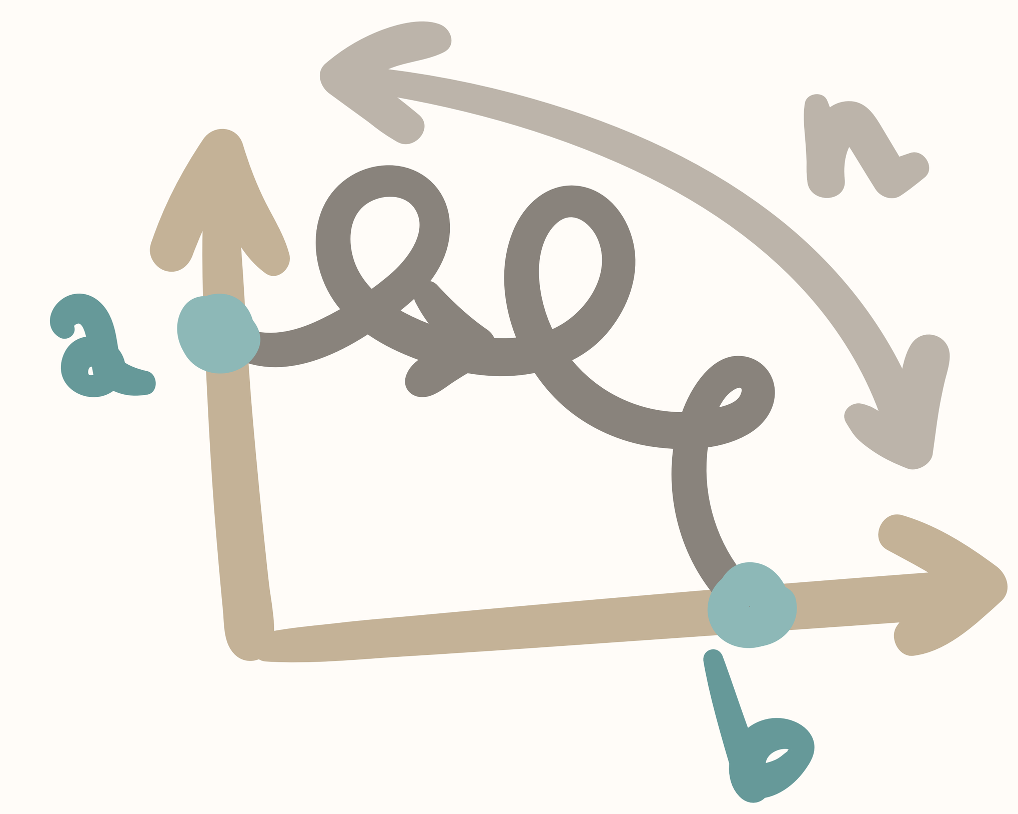

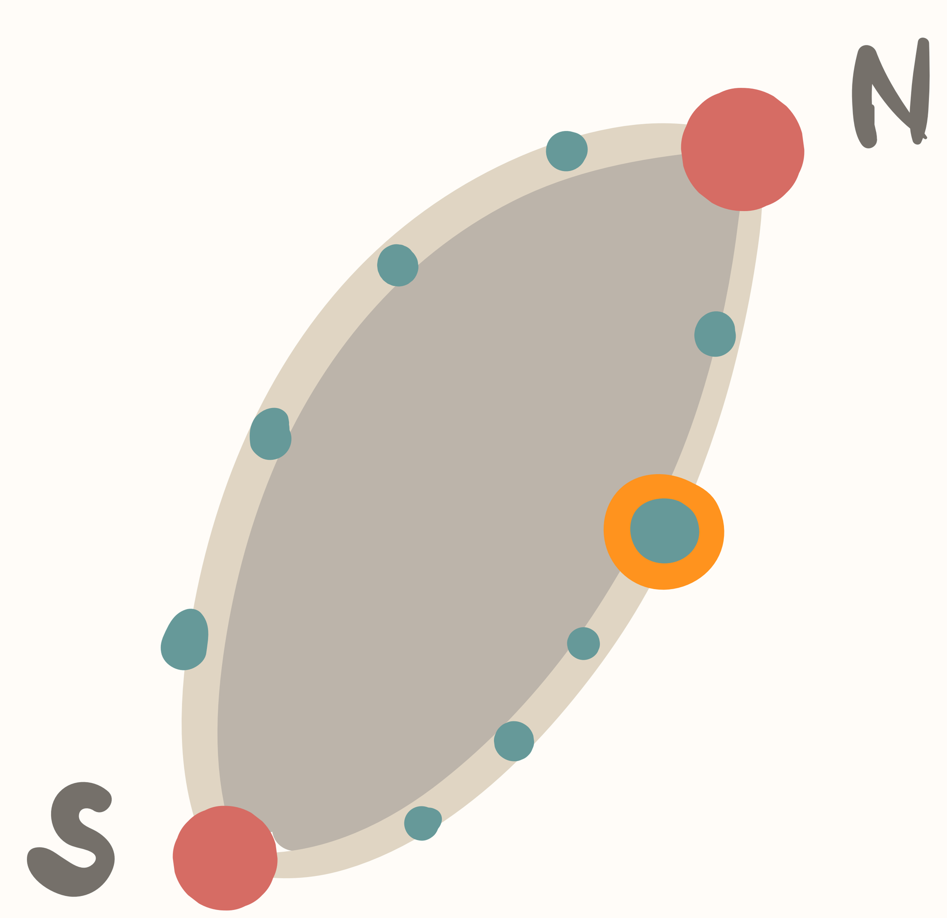

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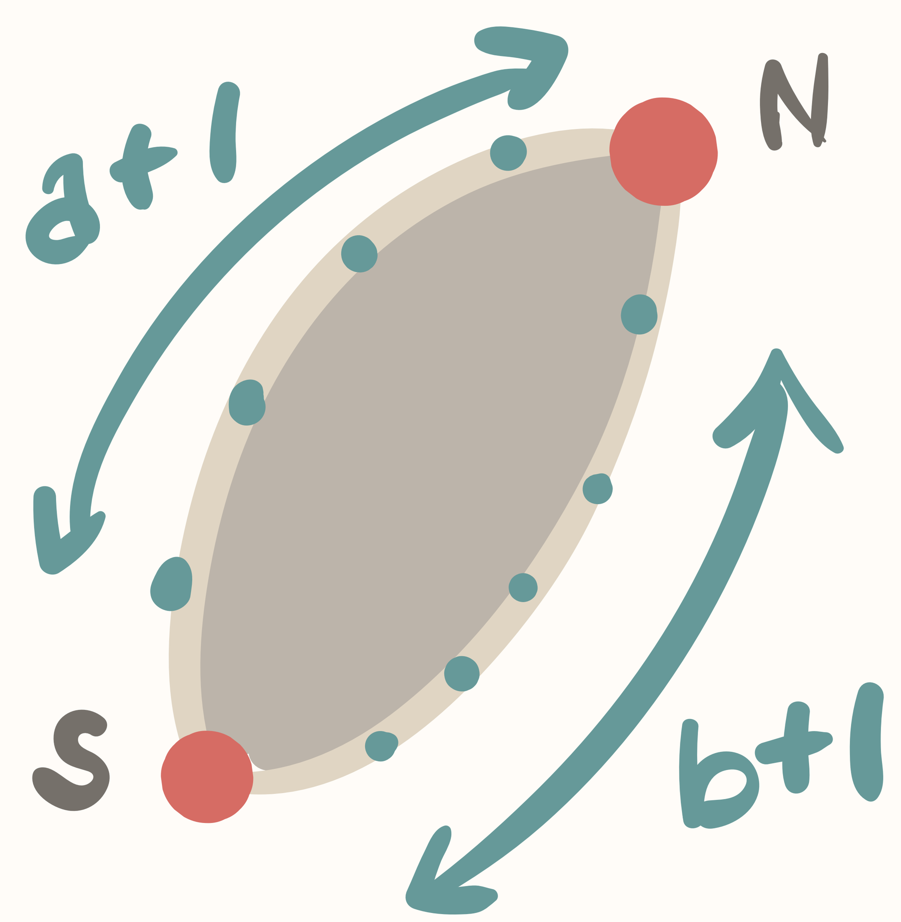

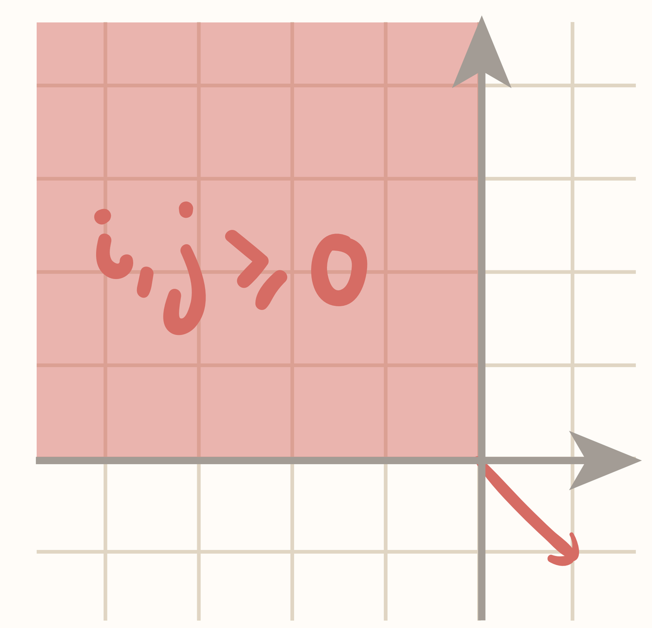

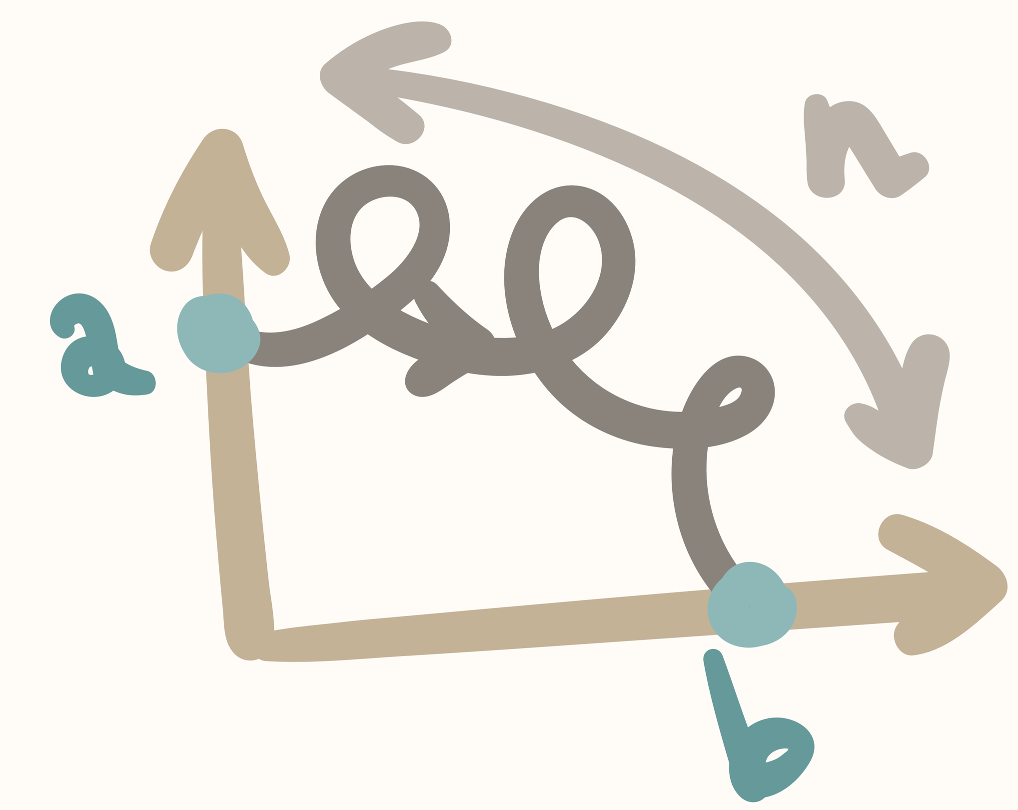

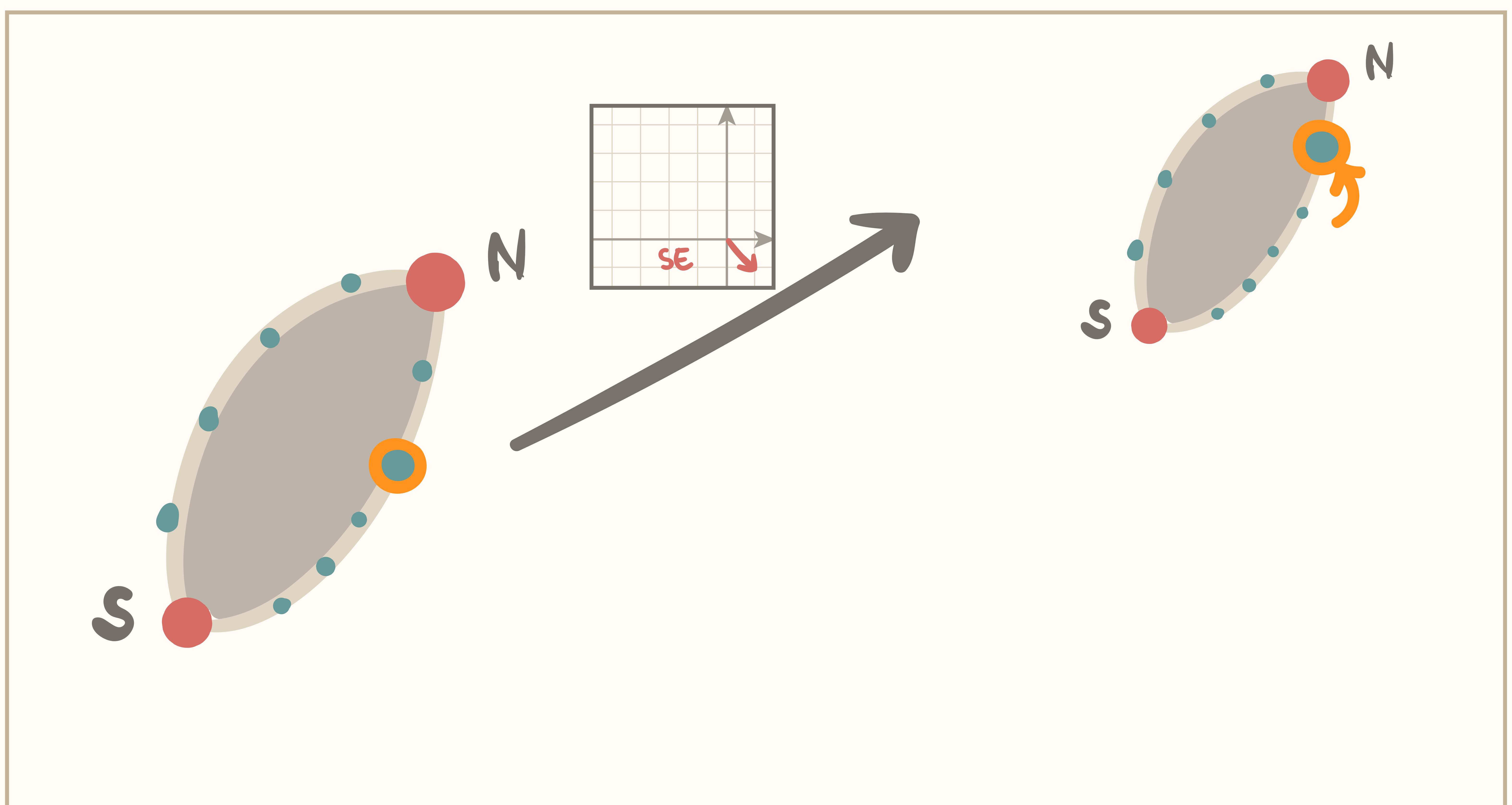

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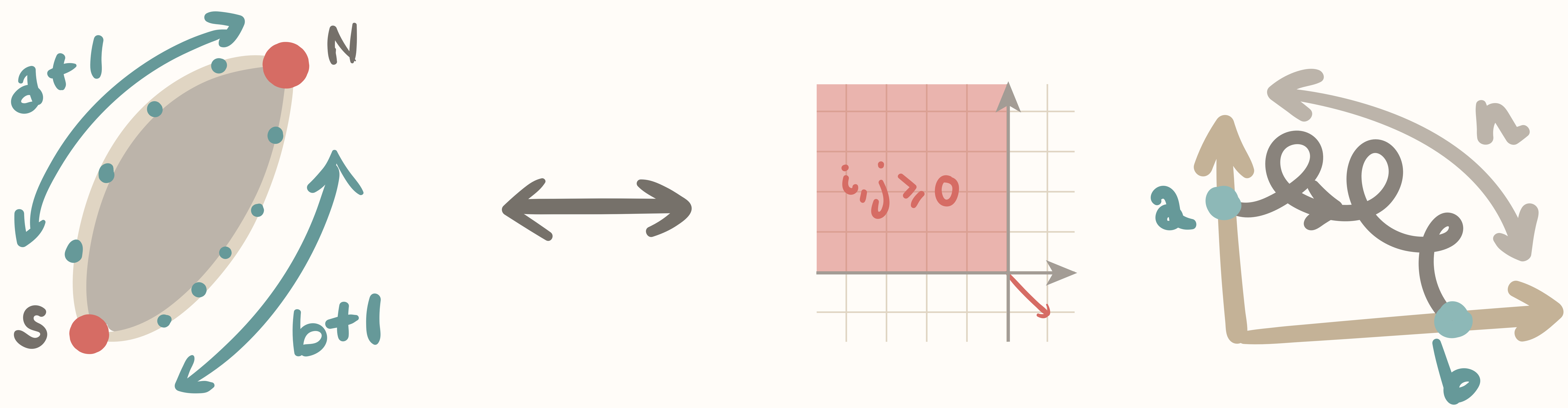

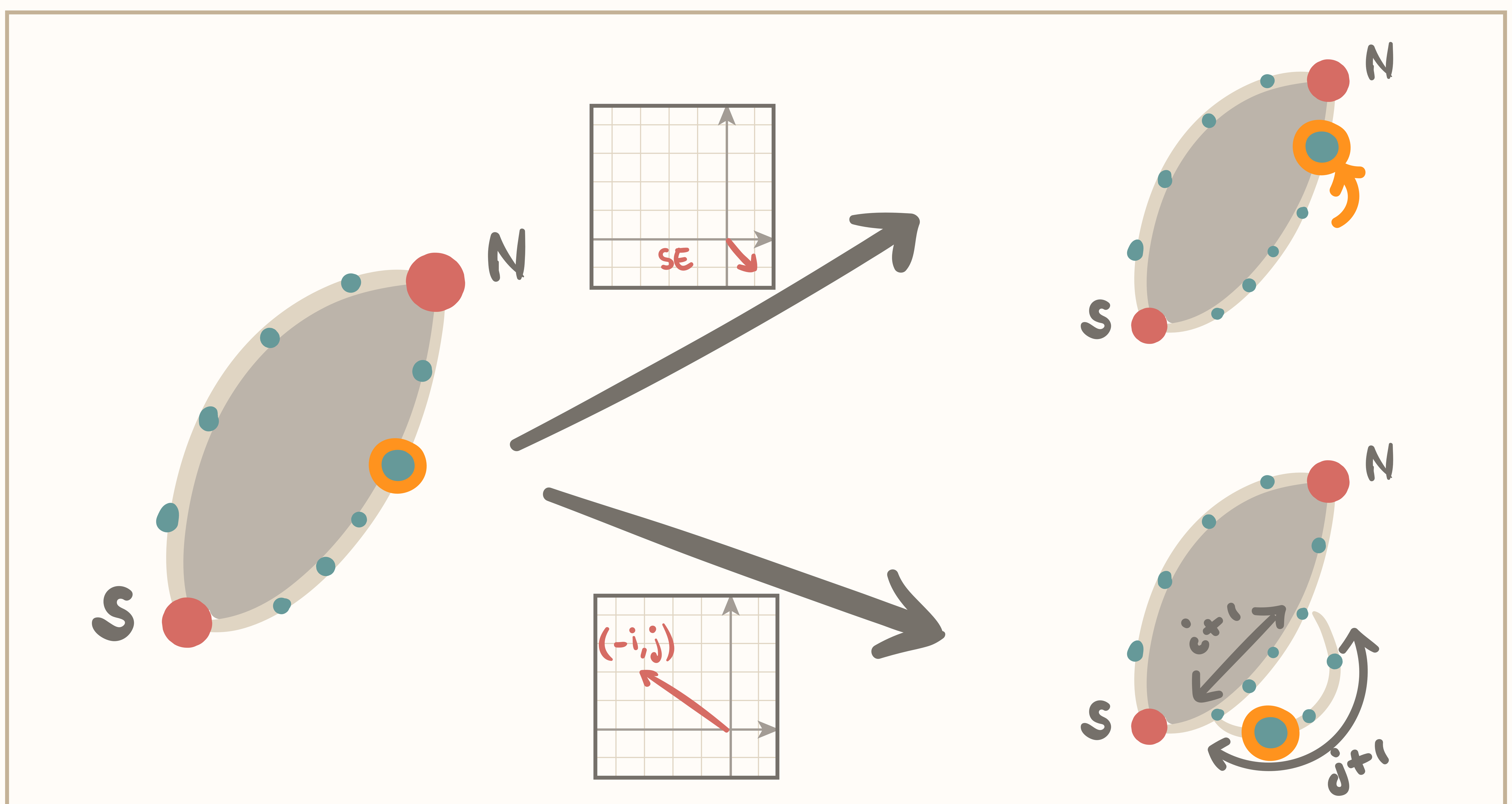

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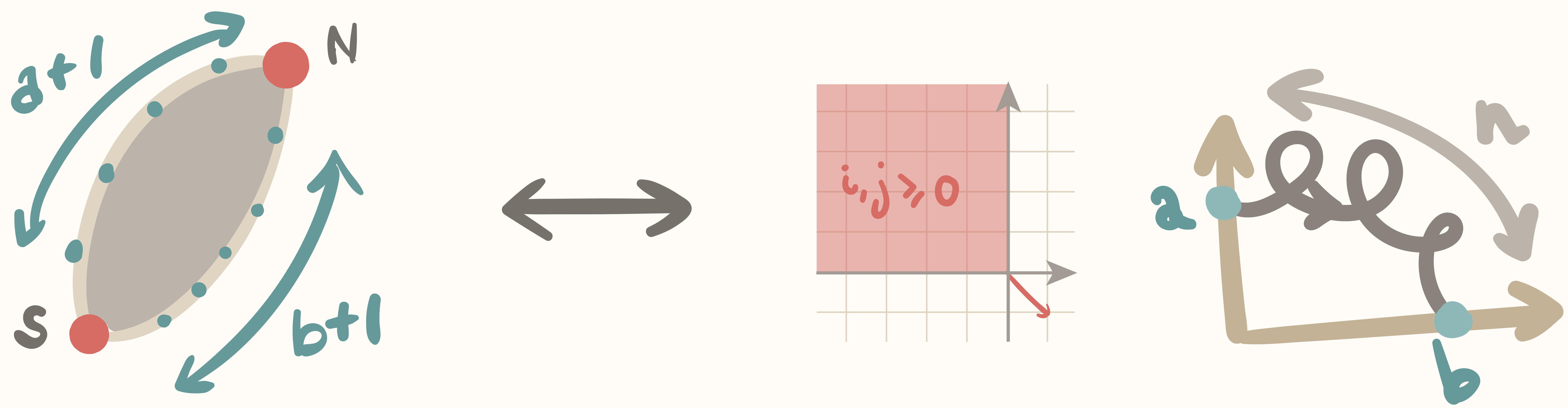
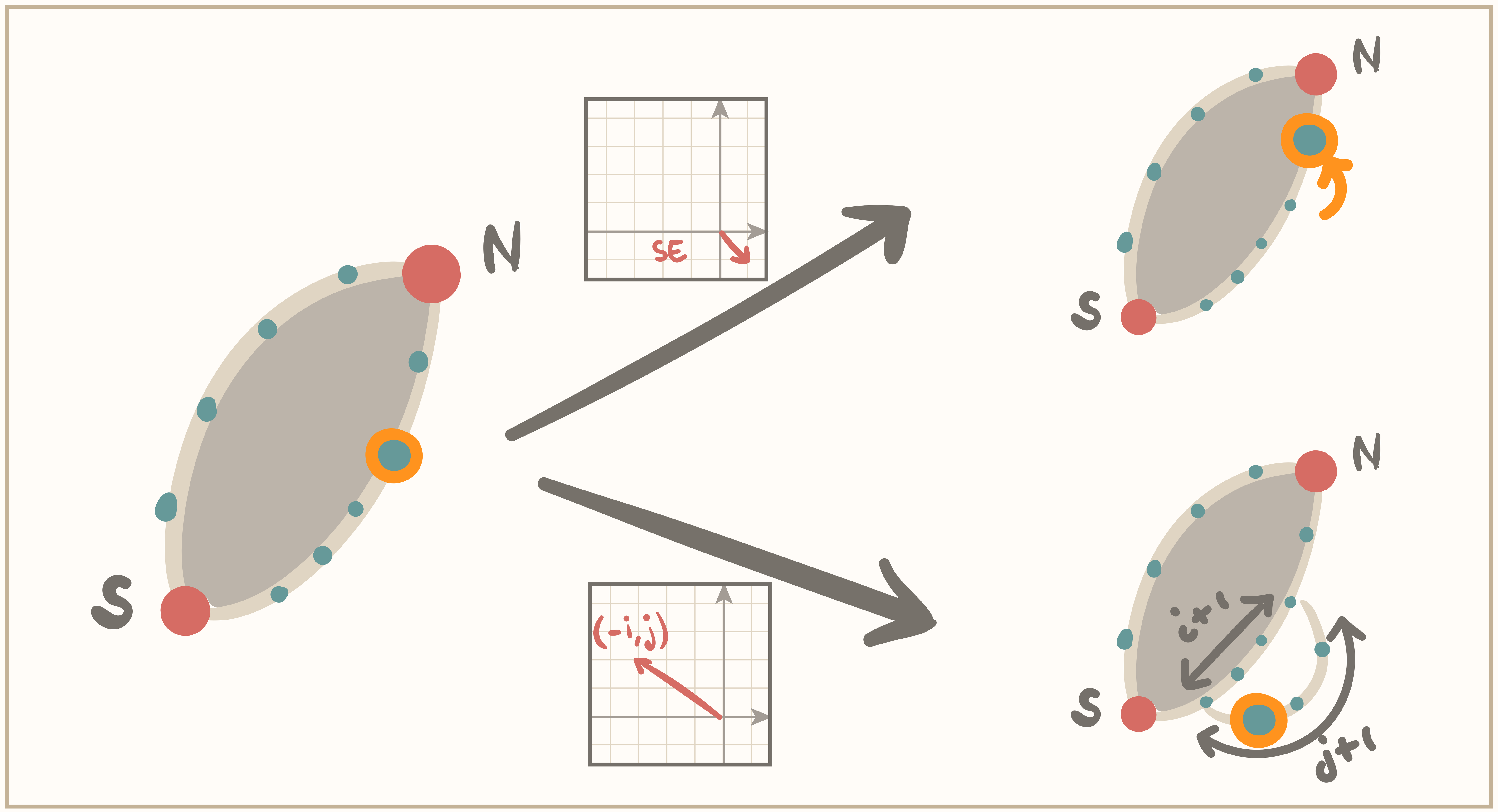
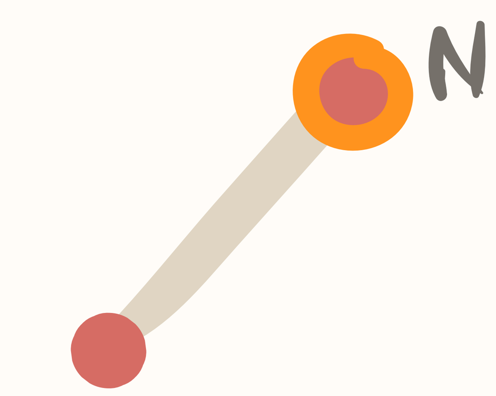




start :



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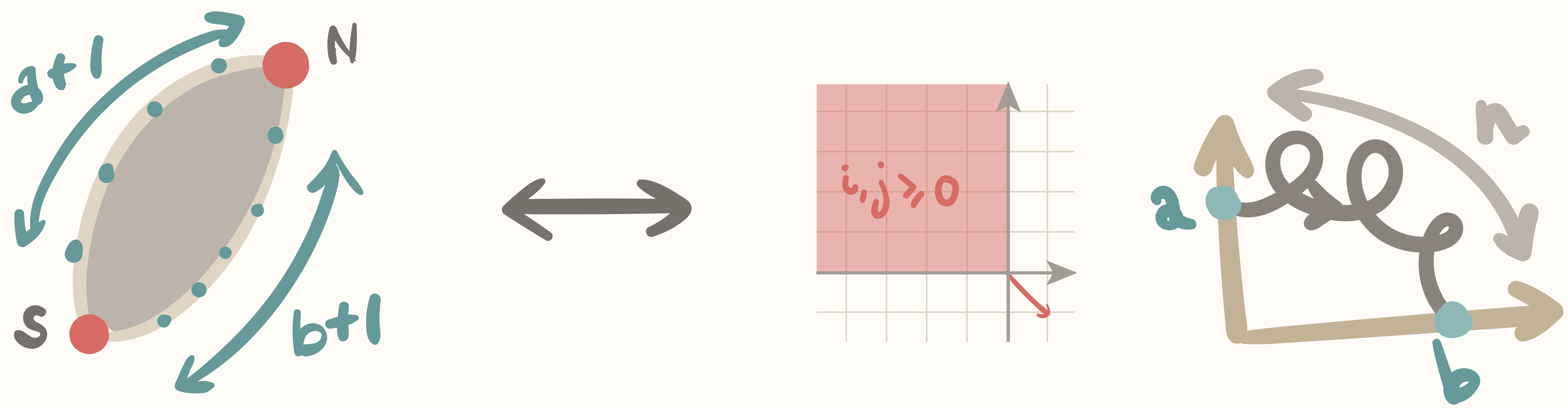
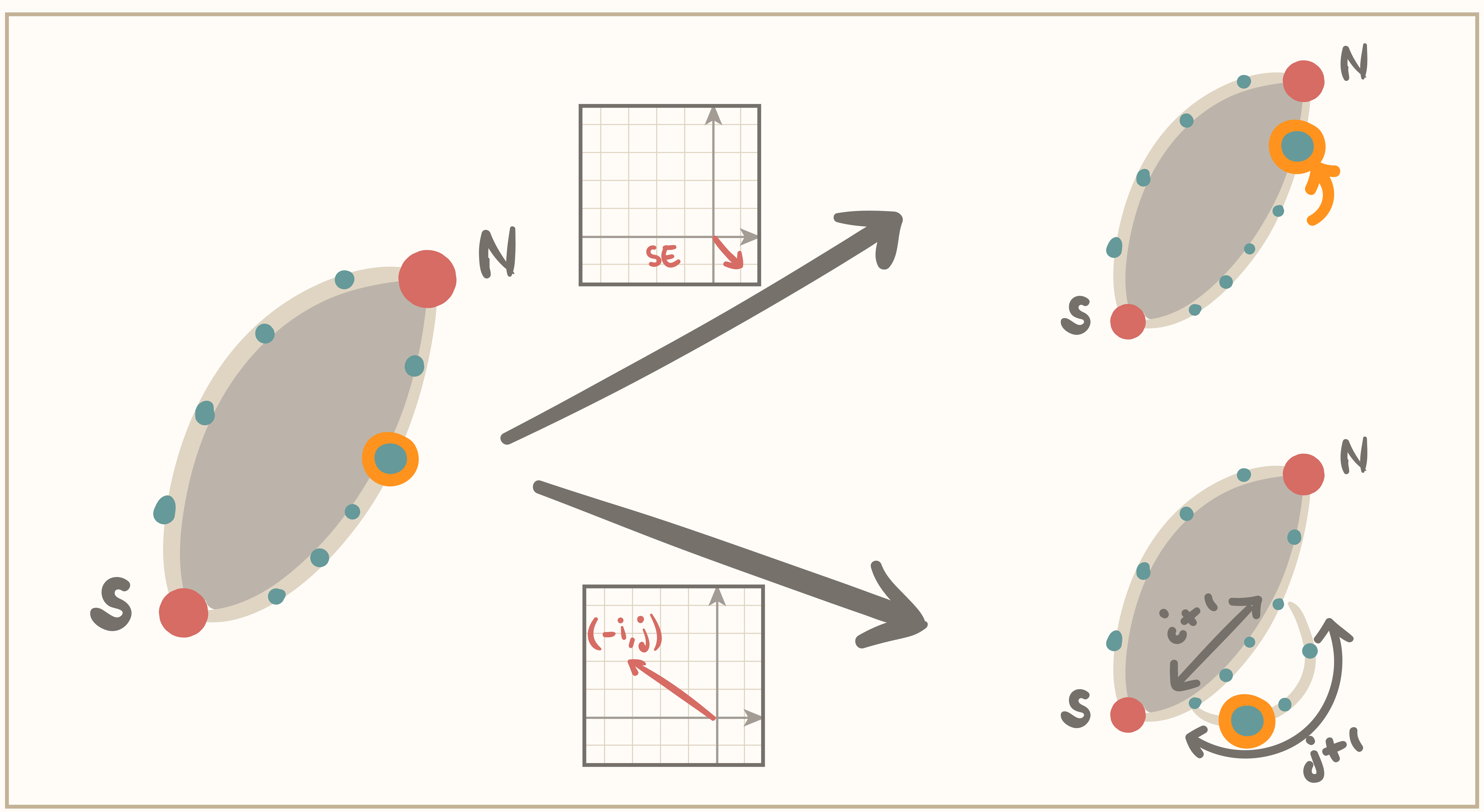
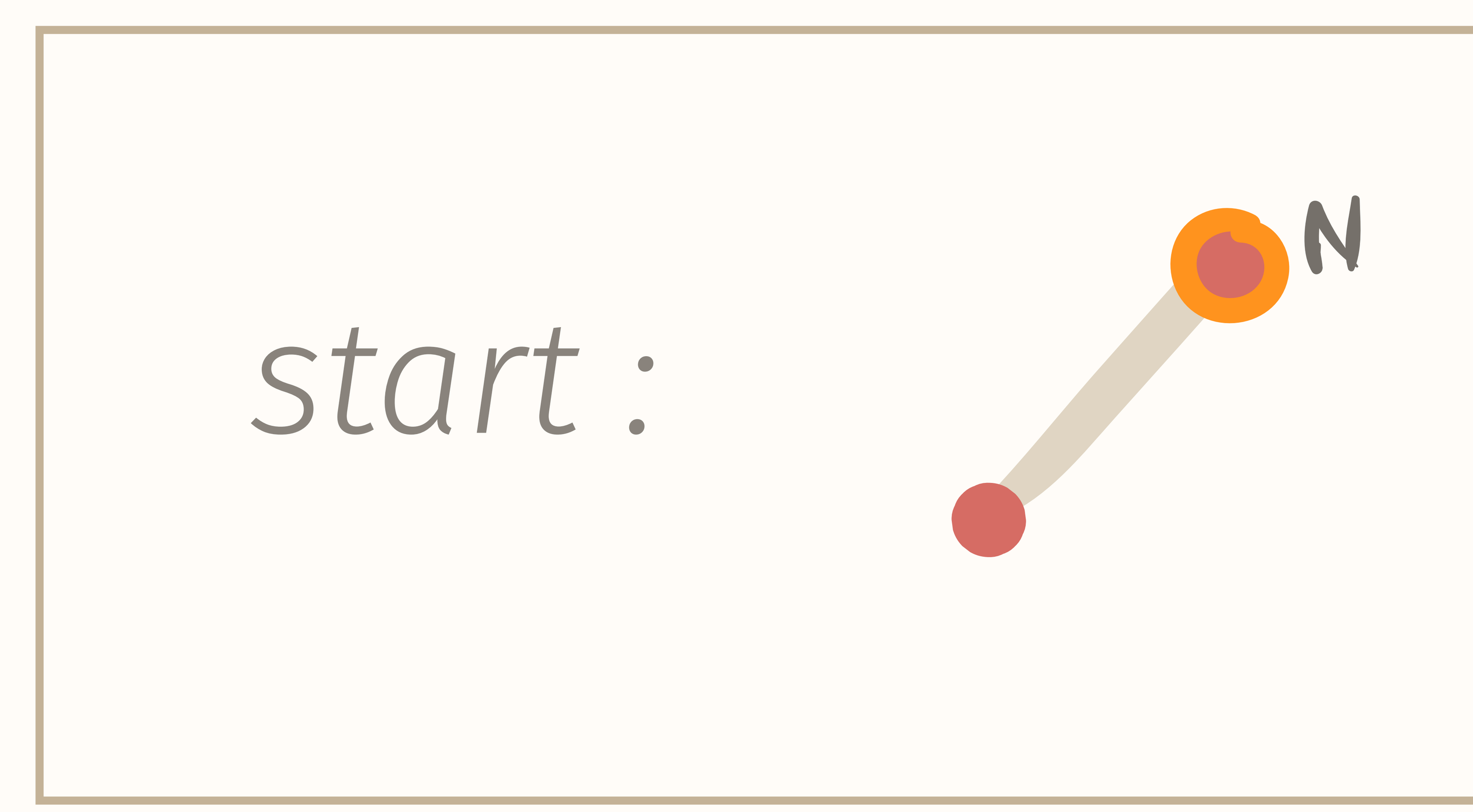
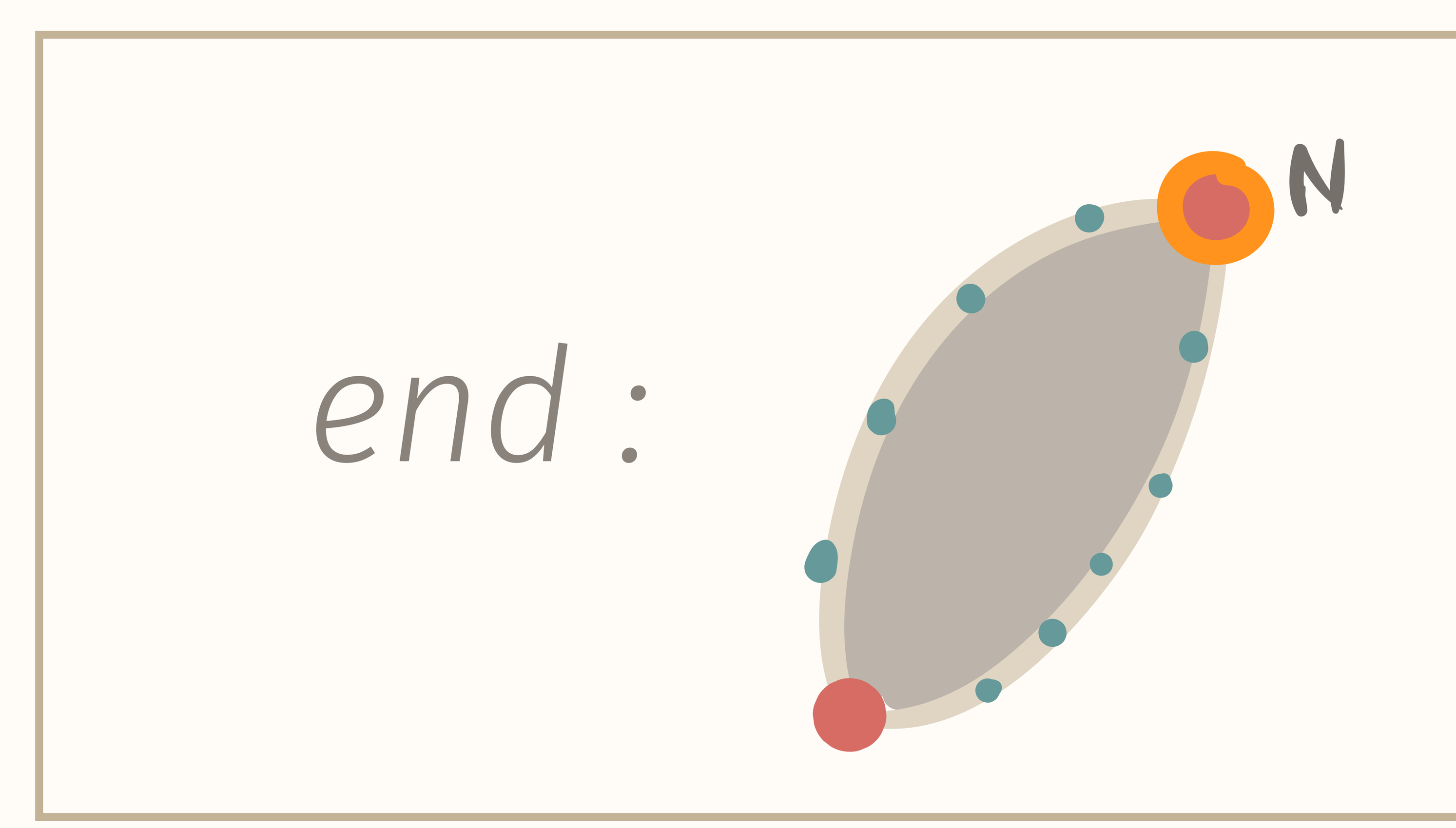
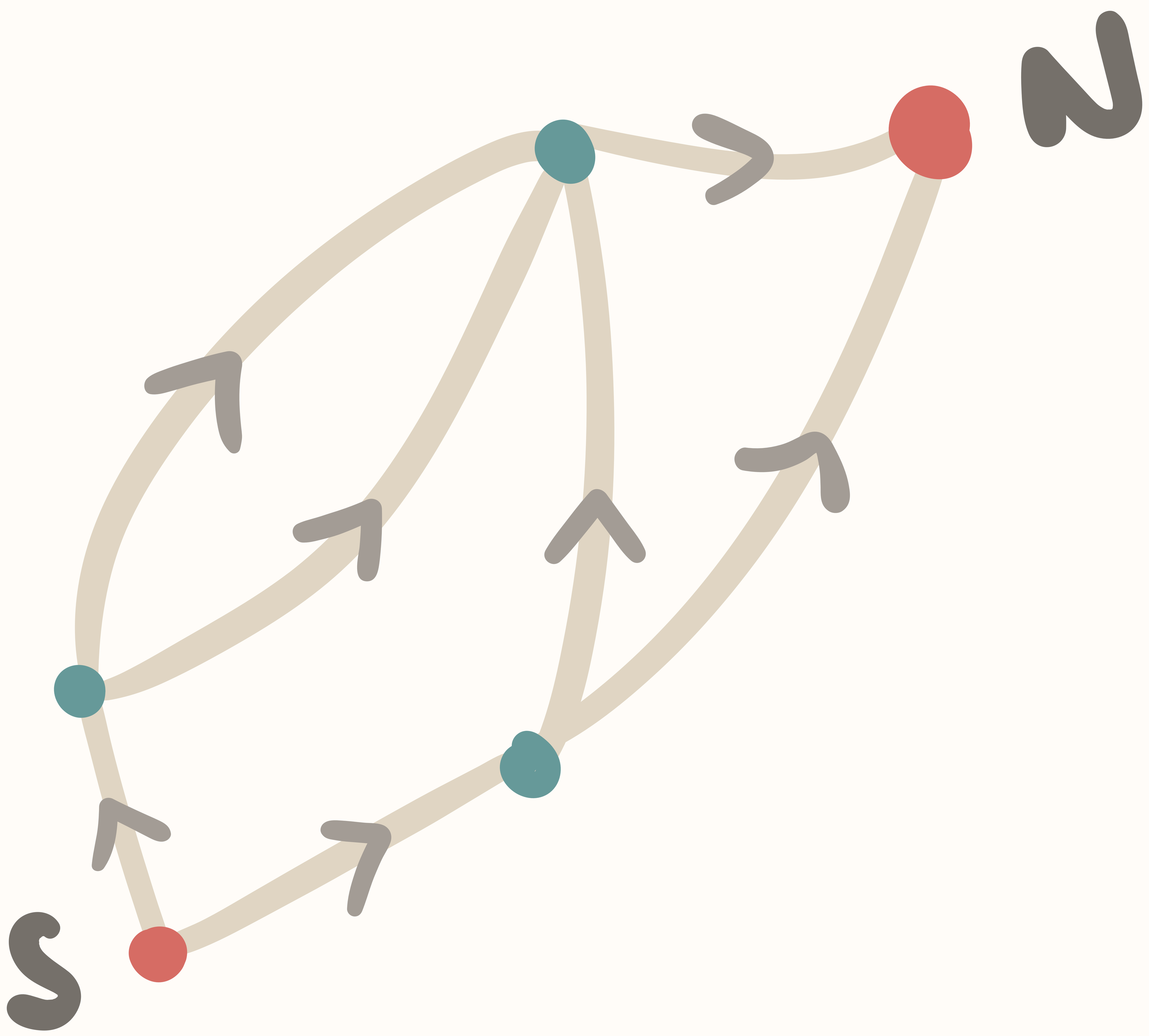


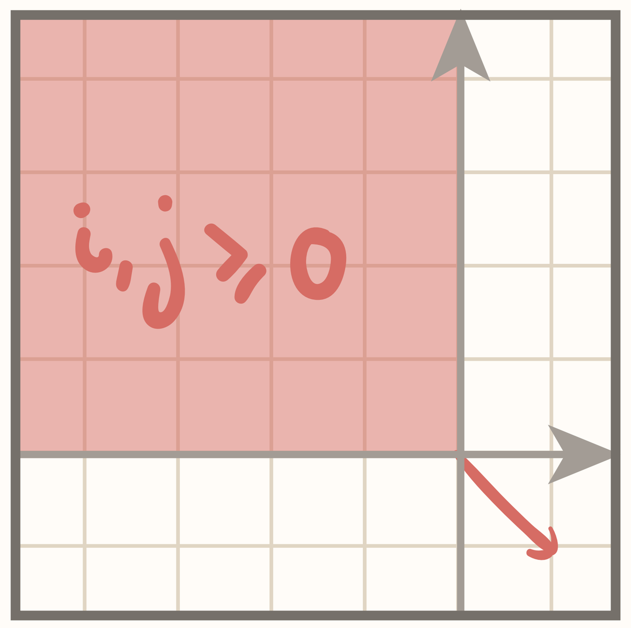


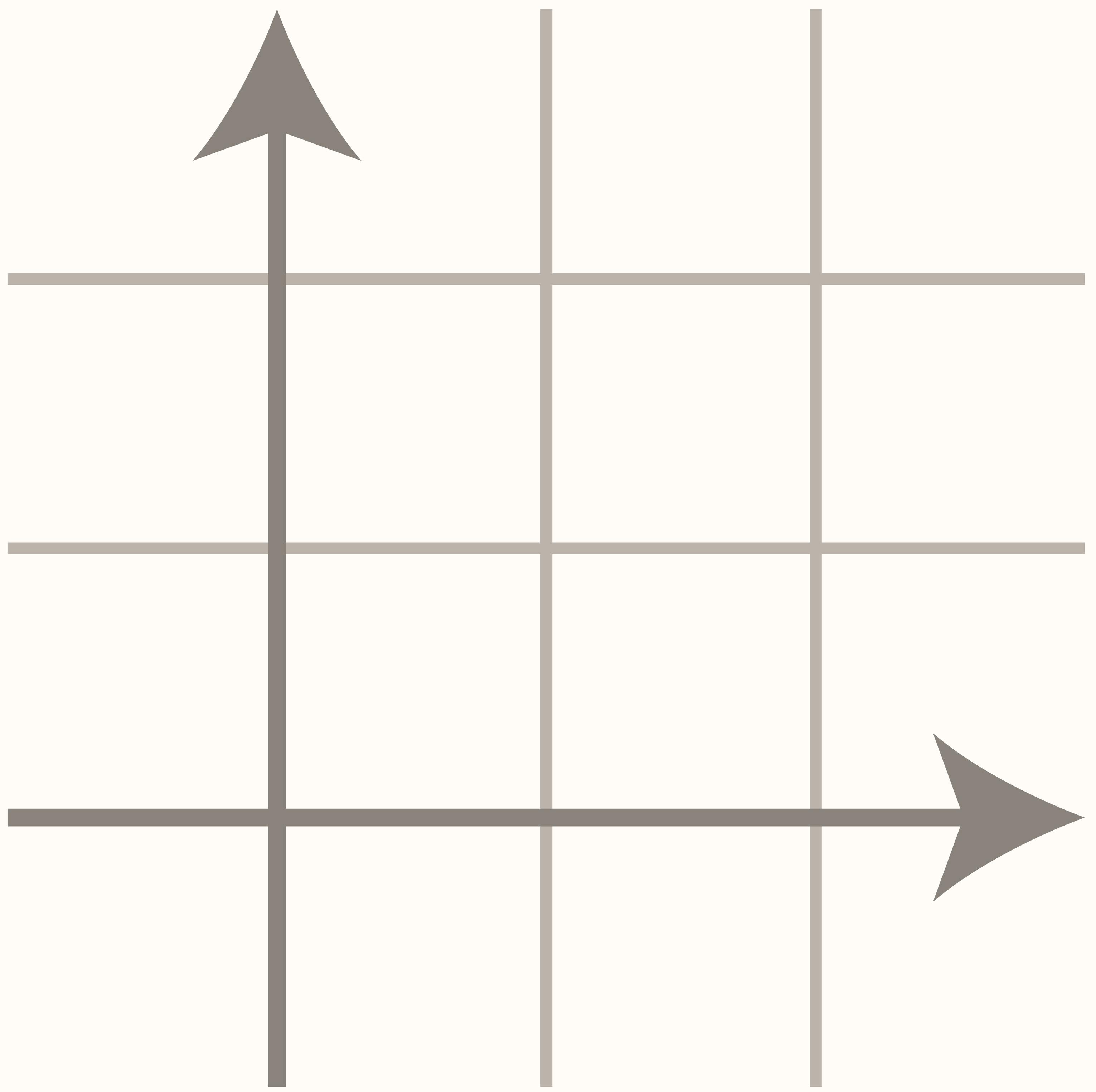




KMSW bijection example







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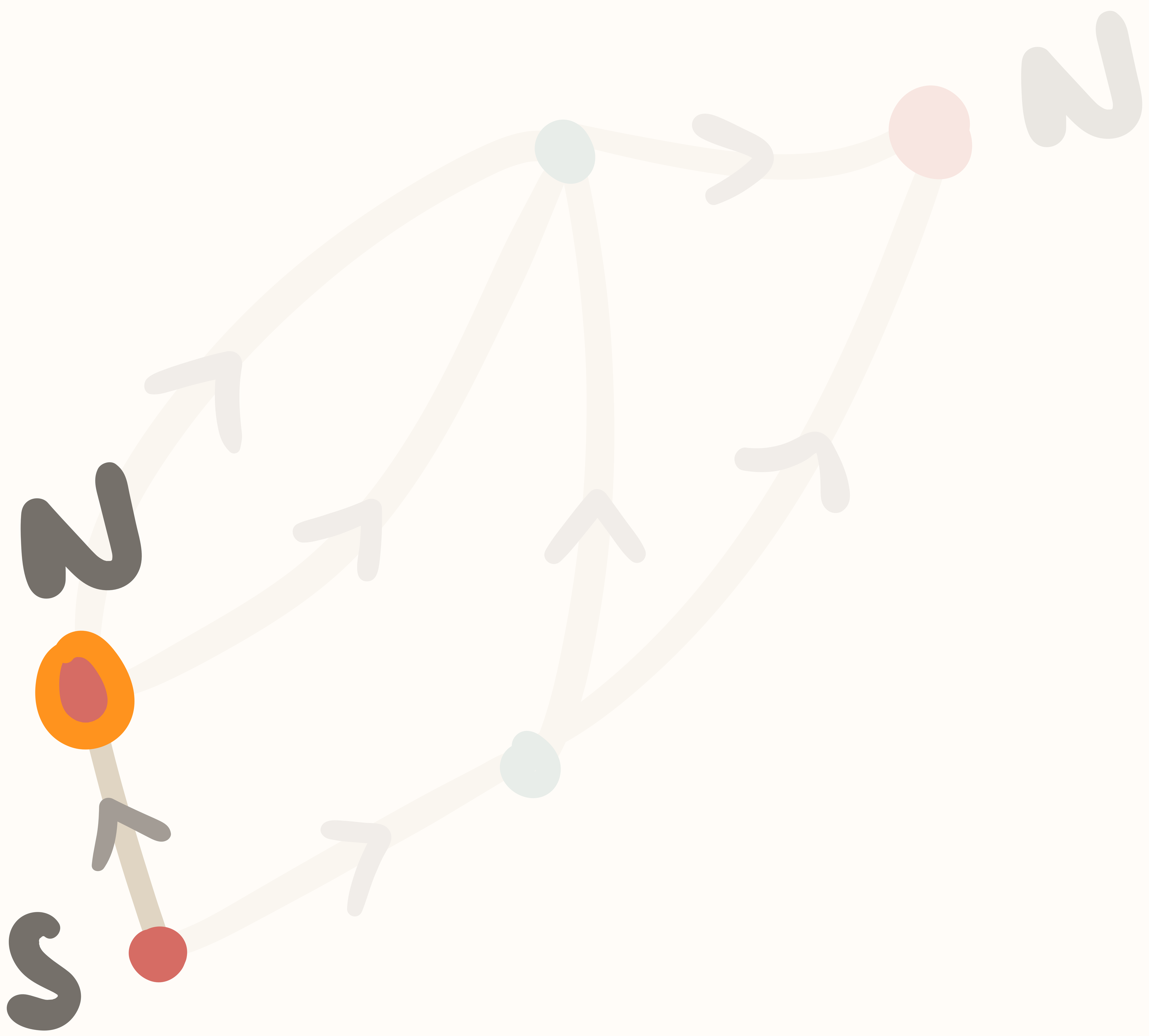

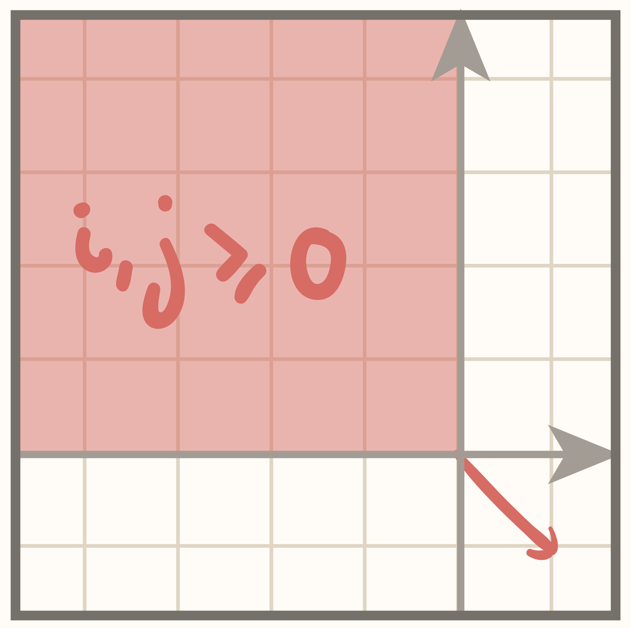

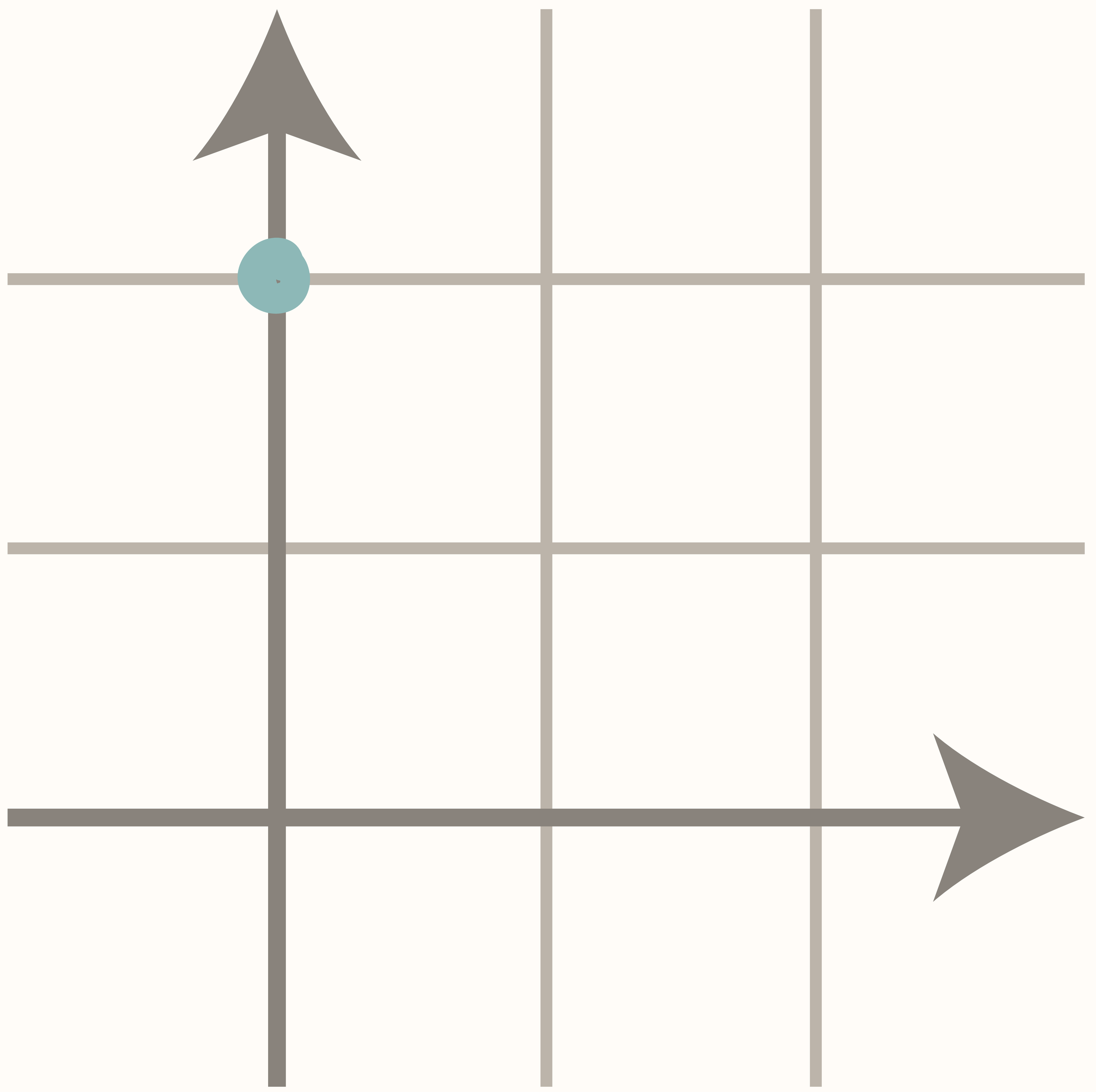

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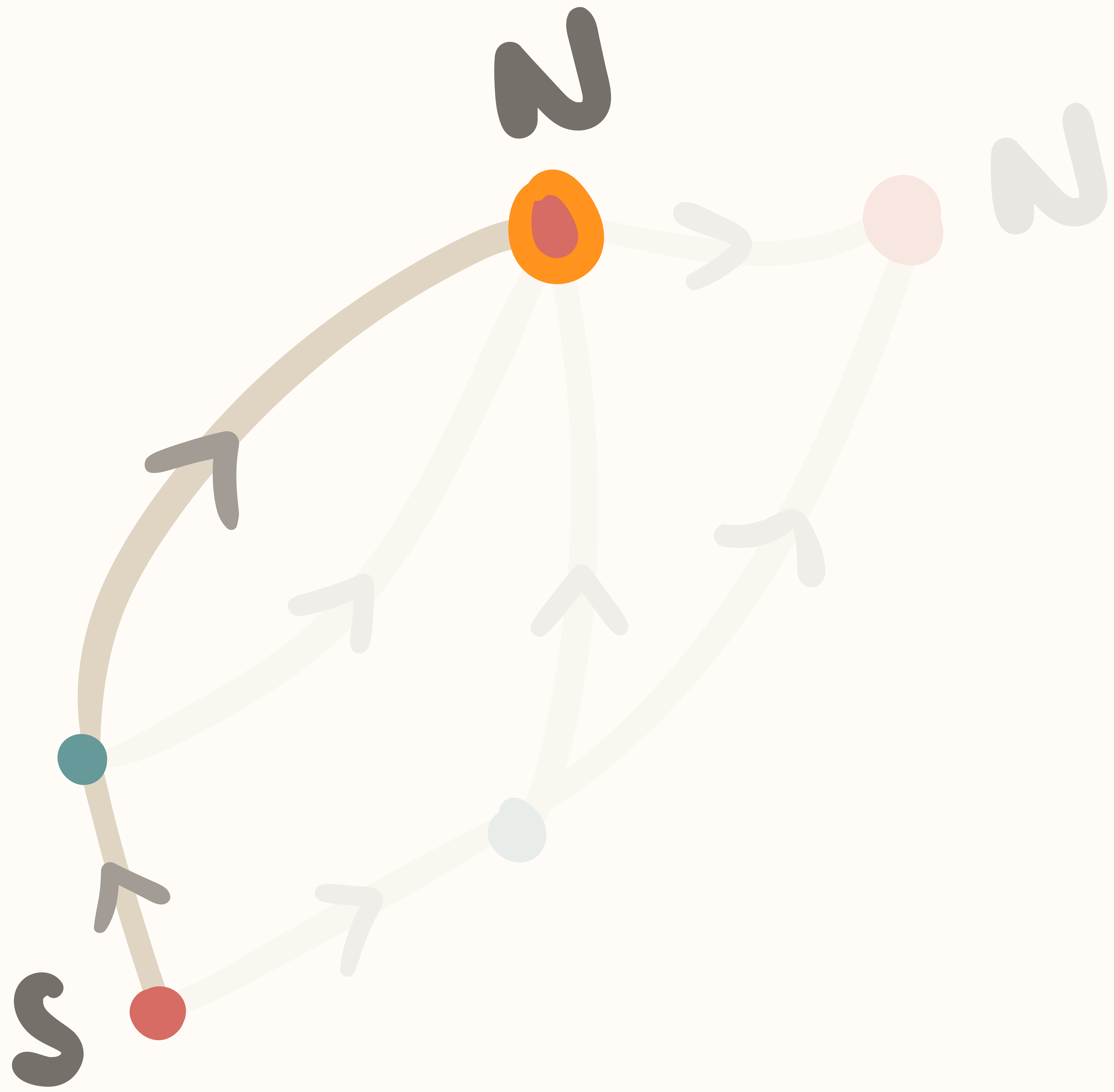

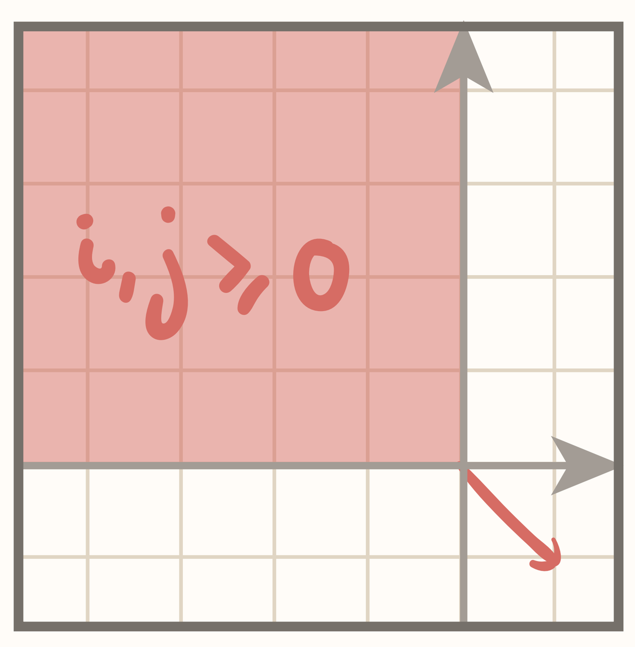

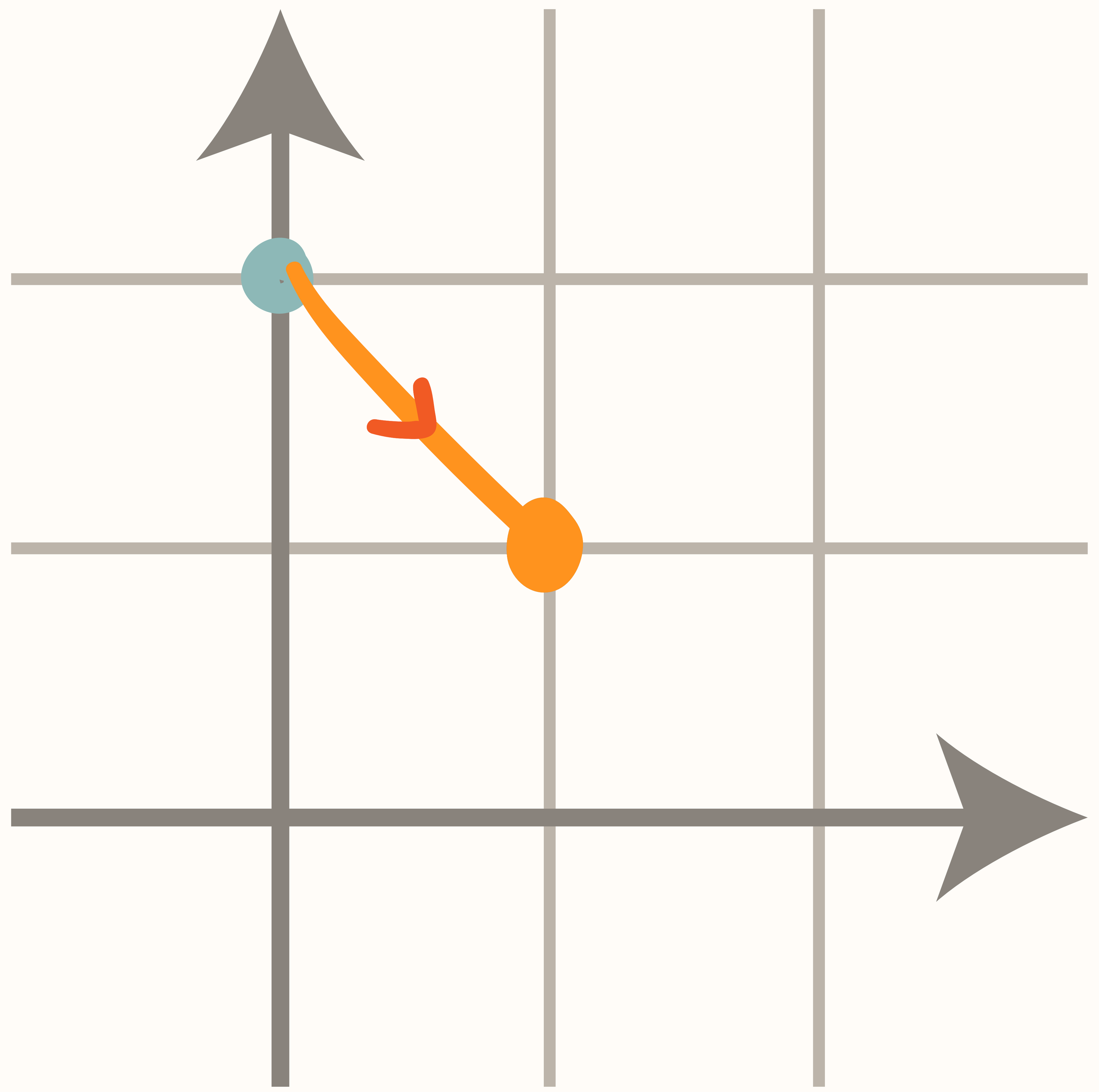

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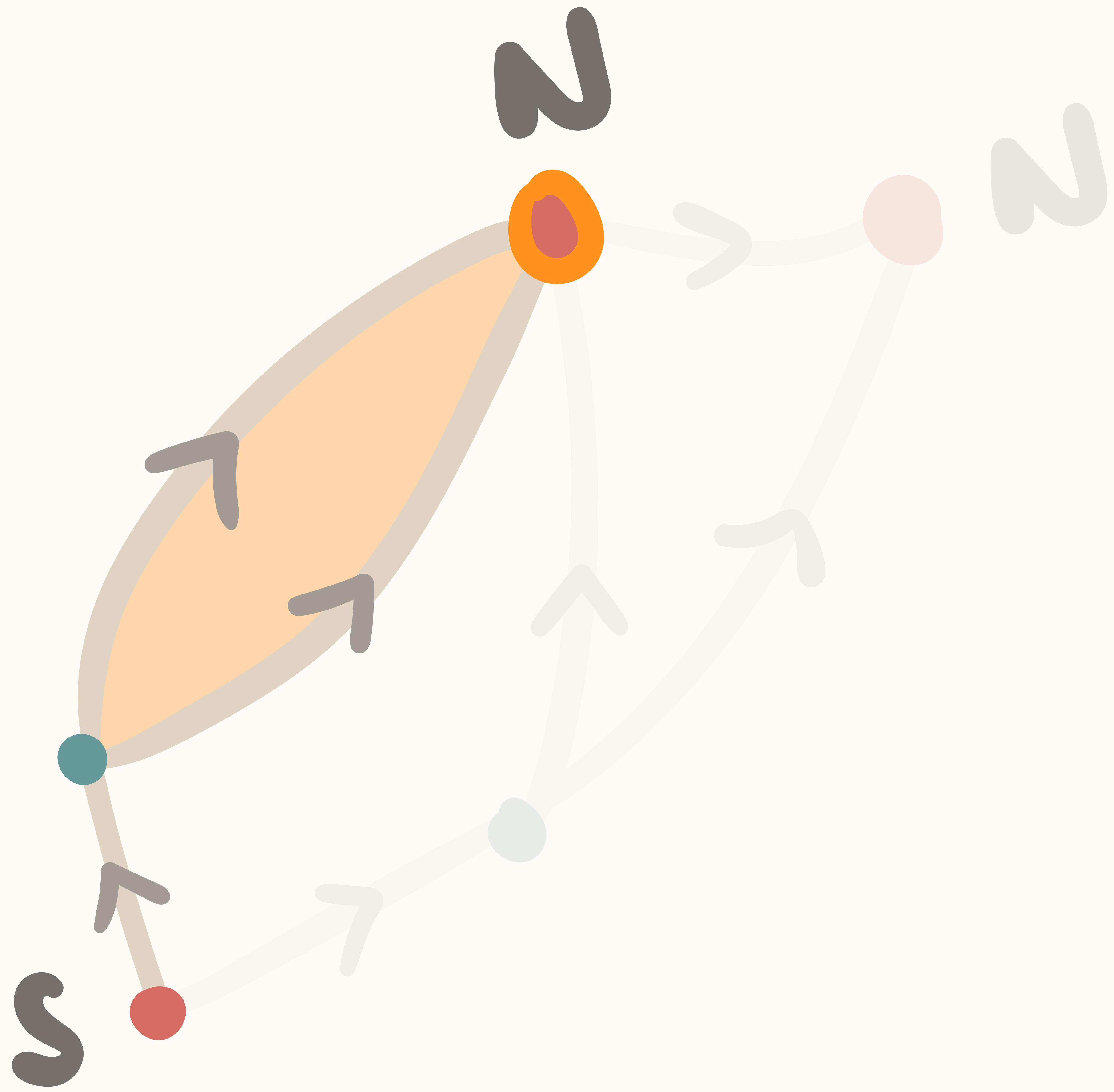

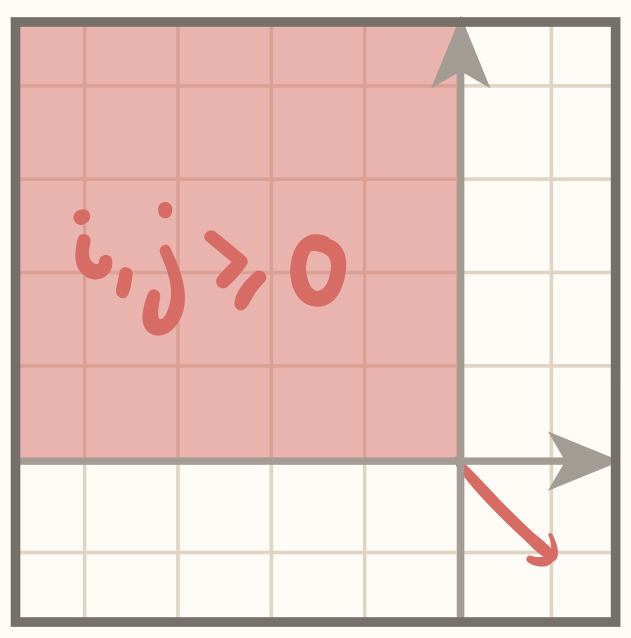

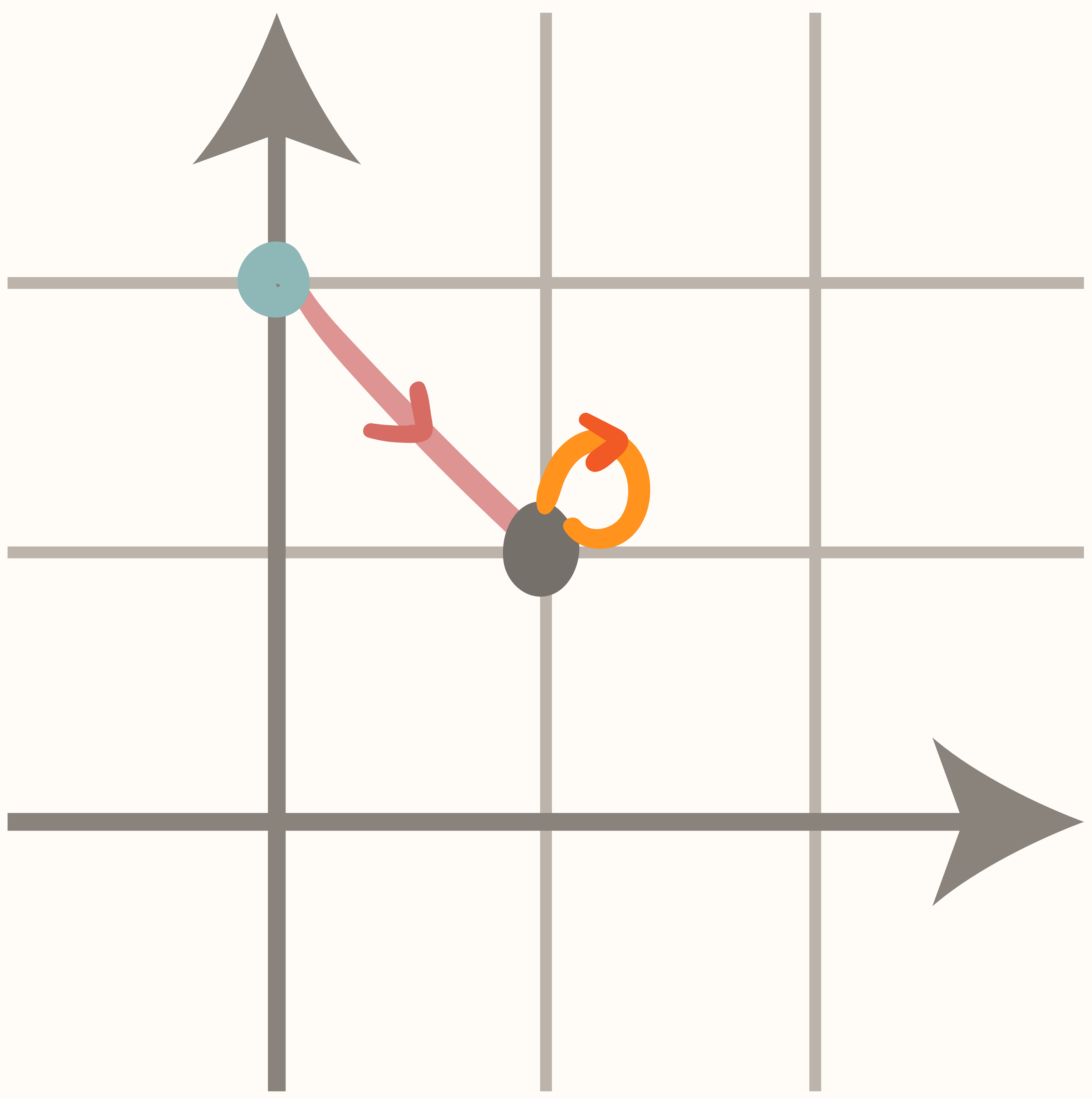

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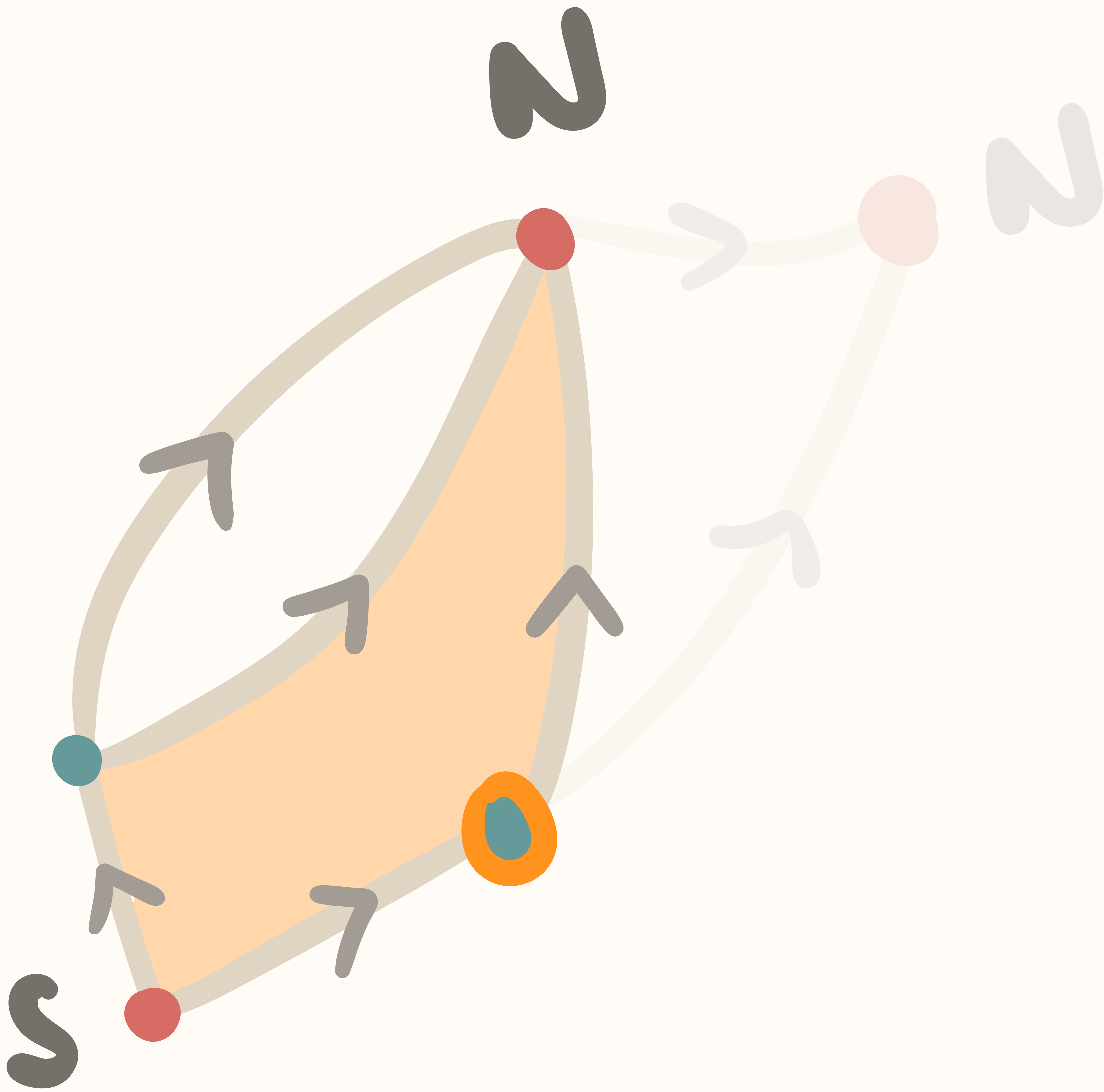

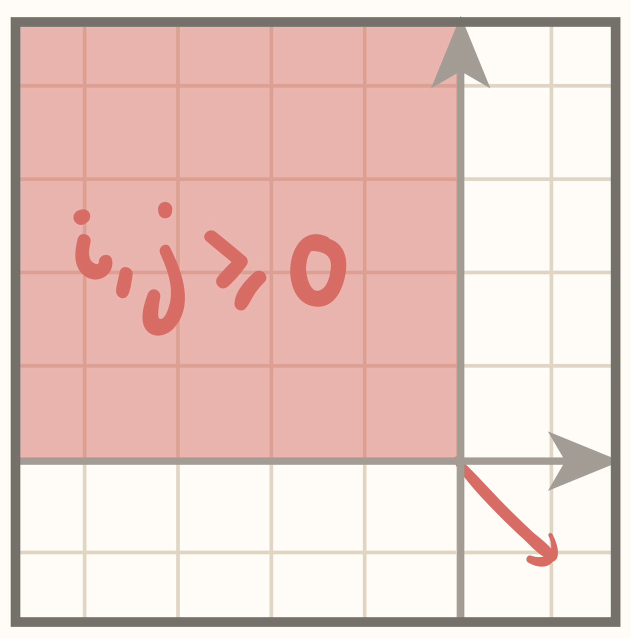

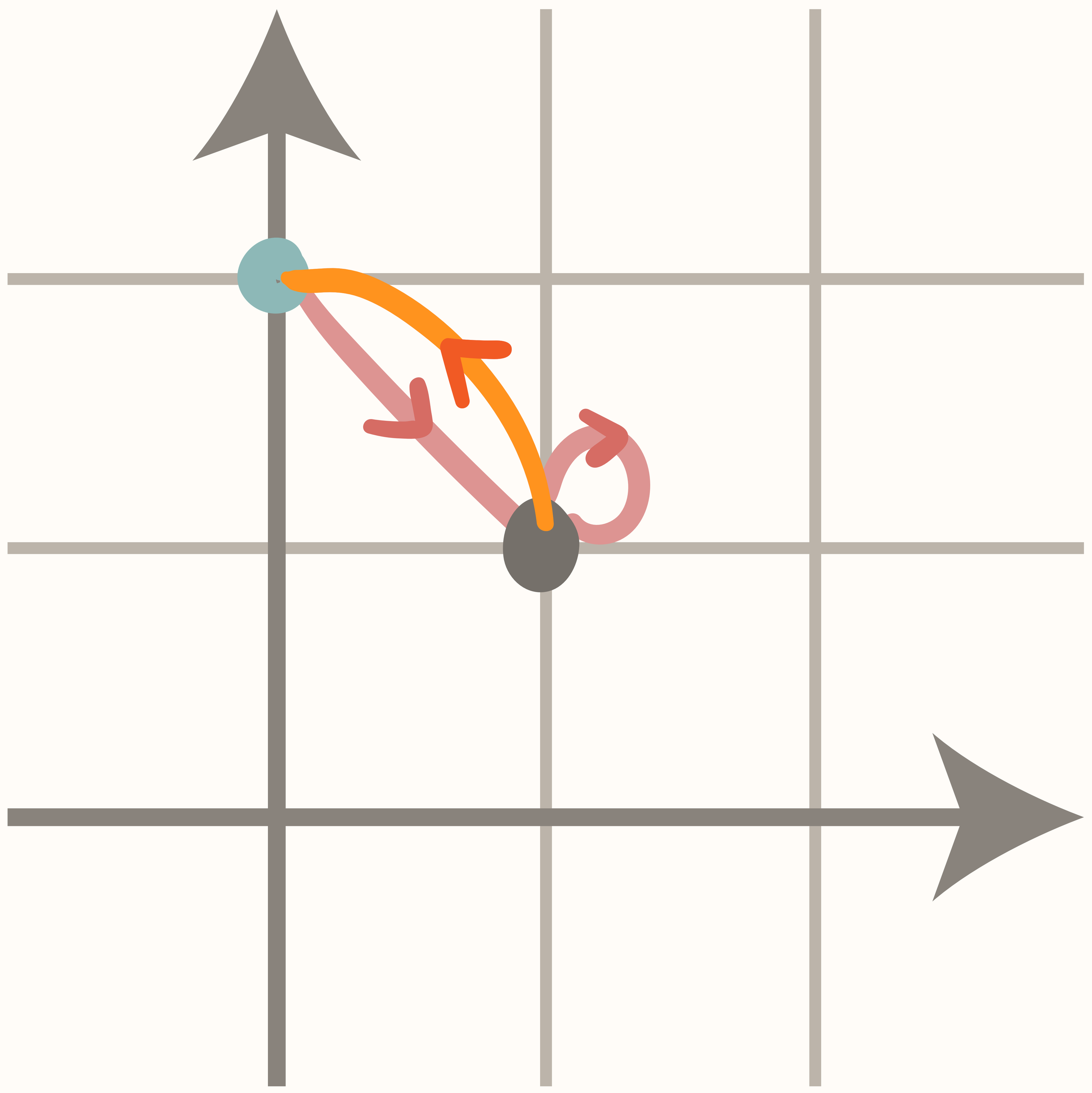

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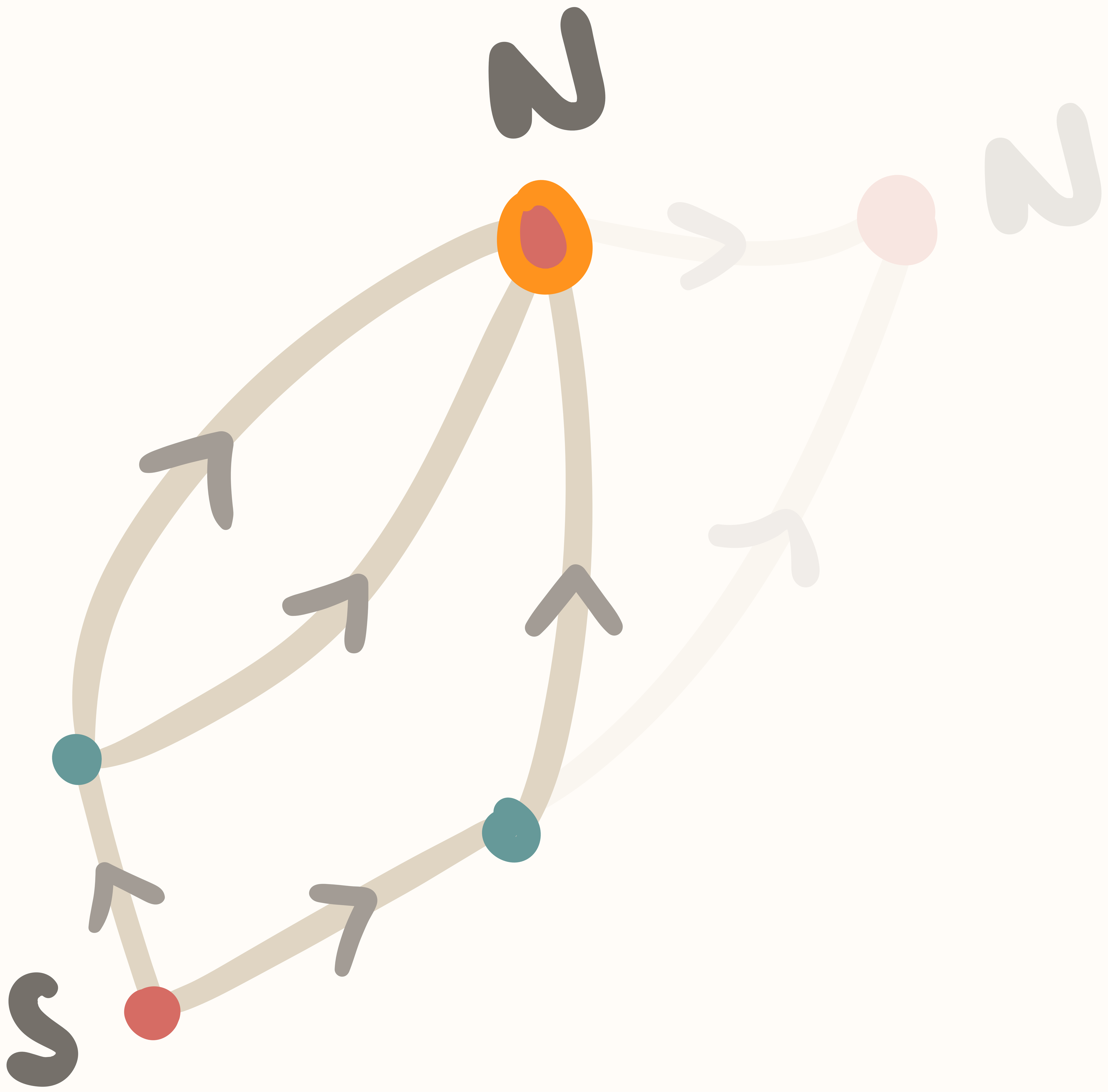

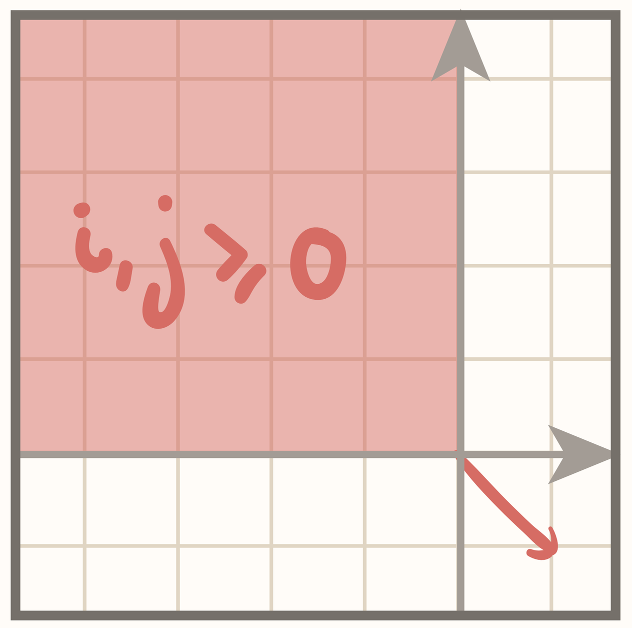

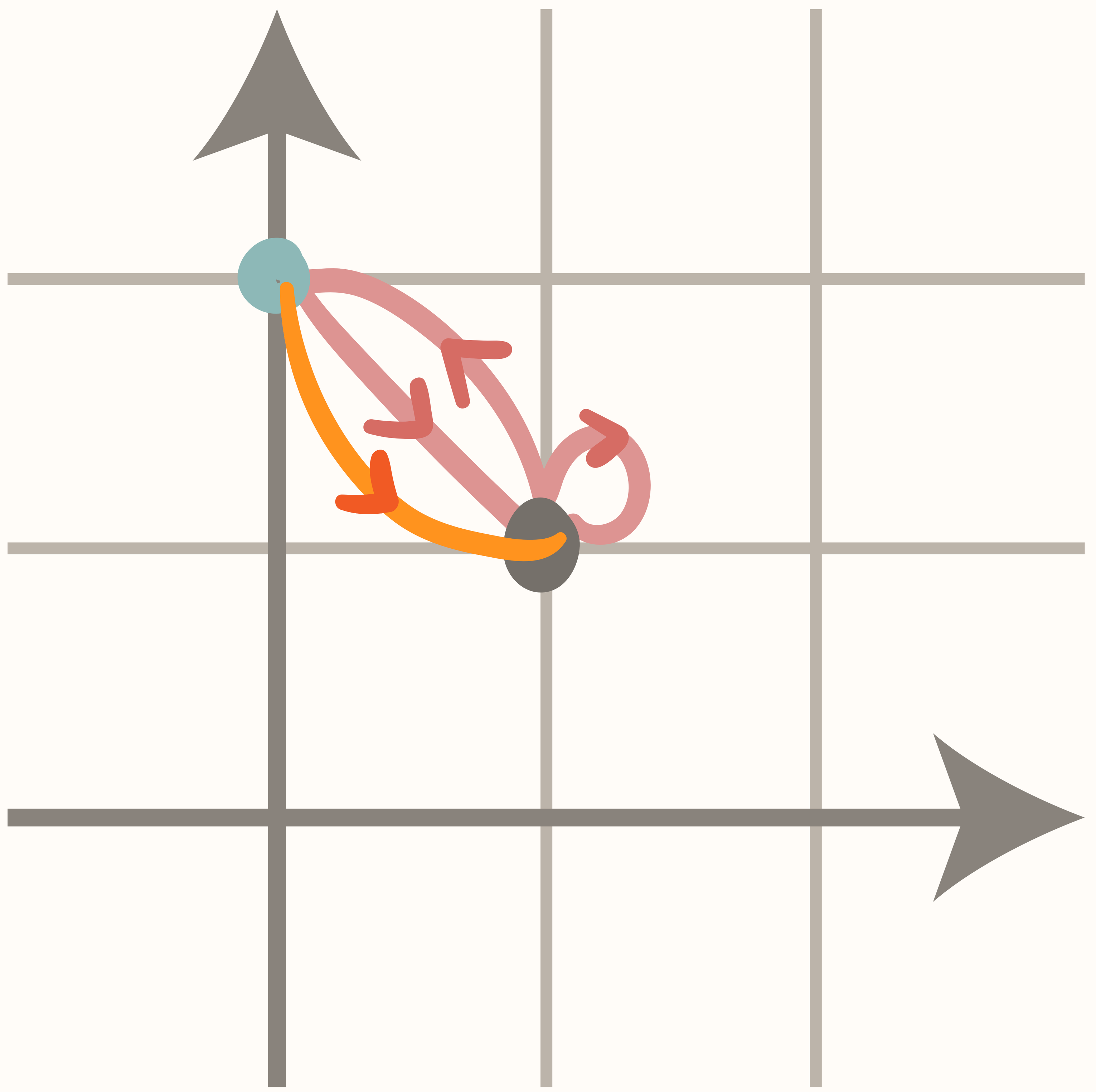

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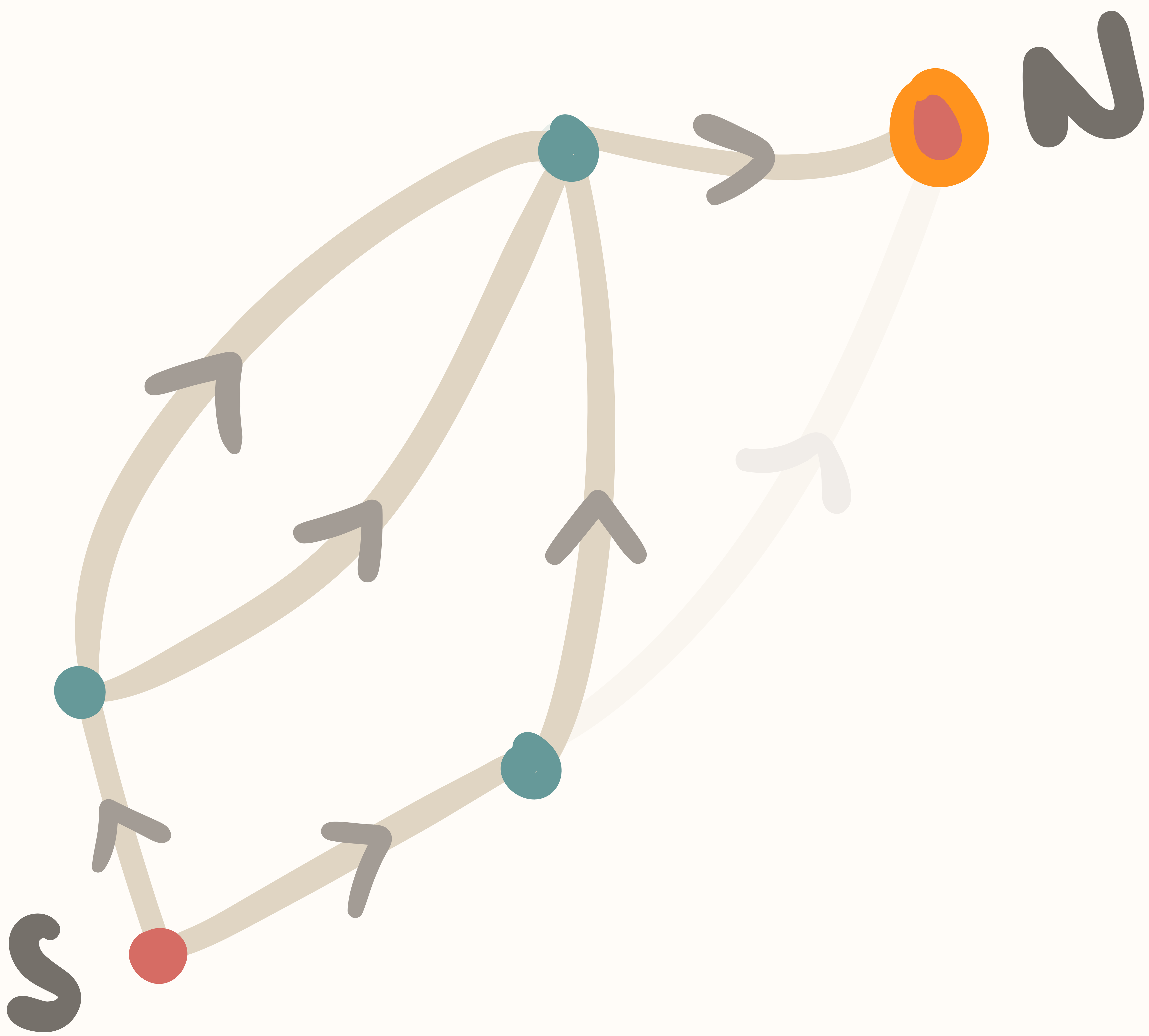

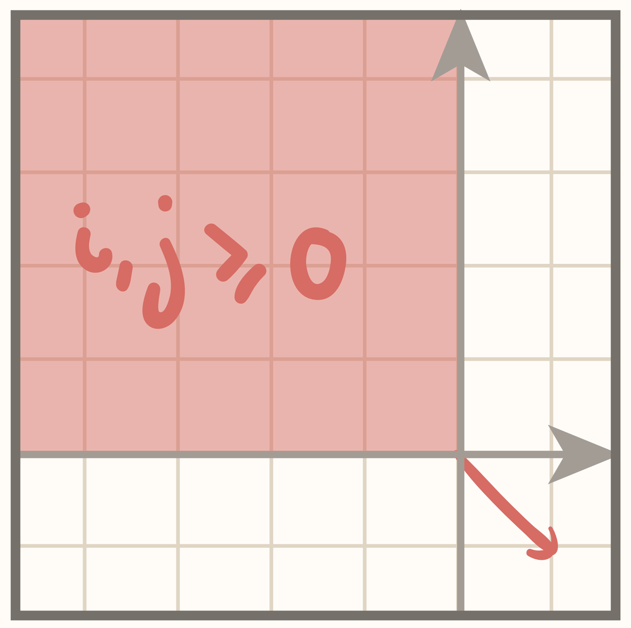

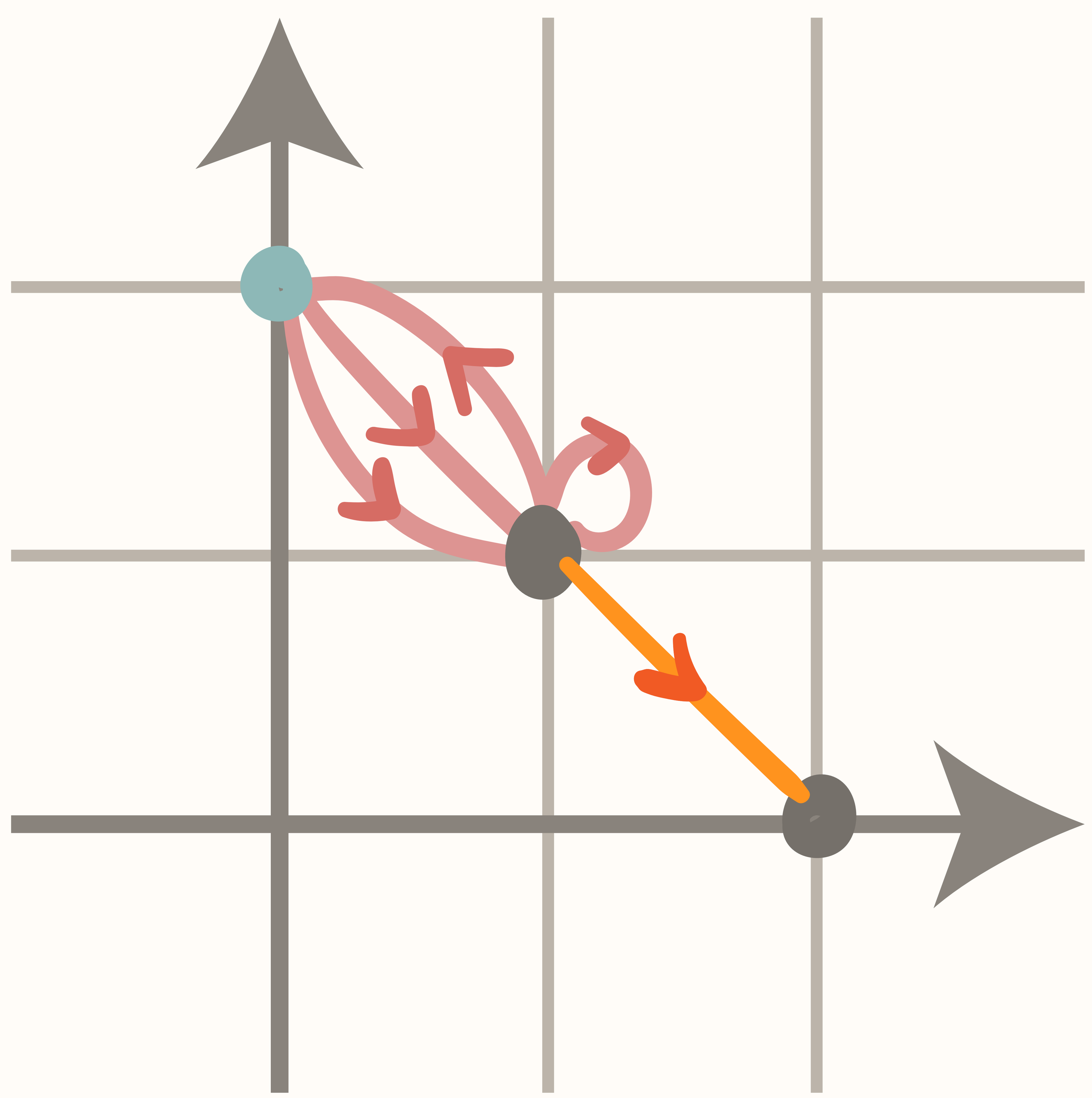

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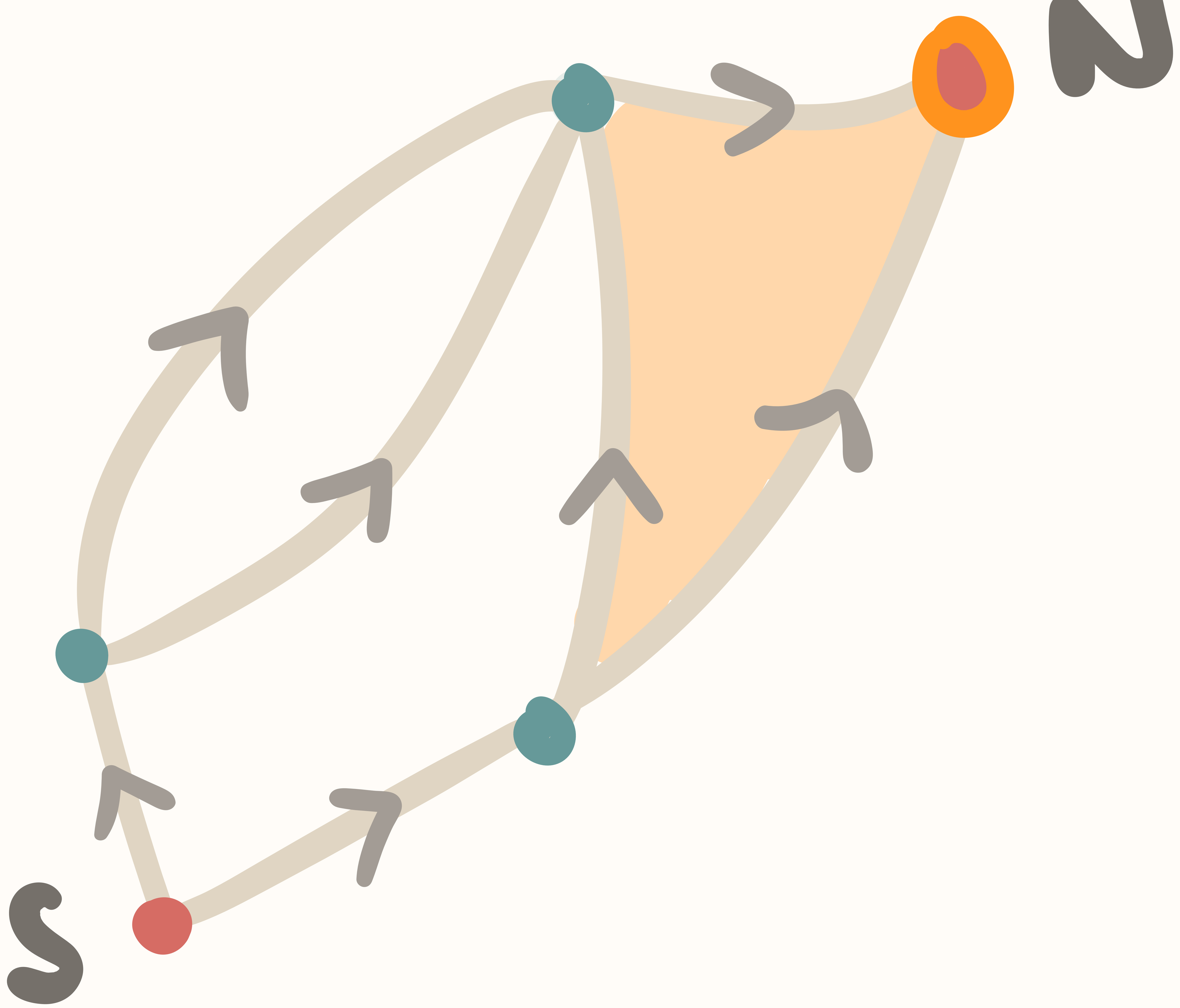
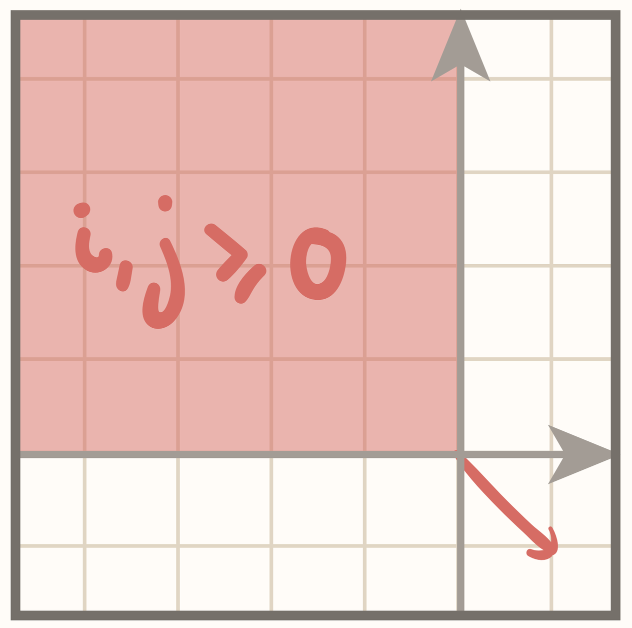
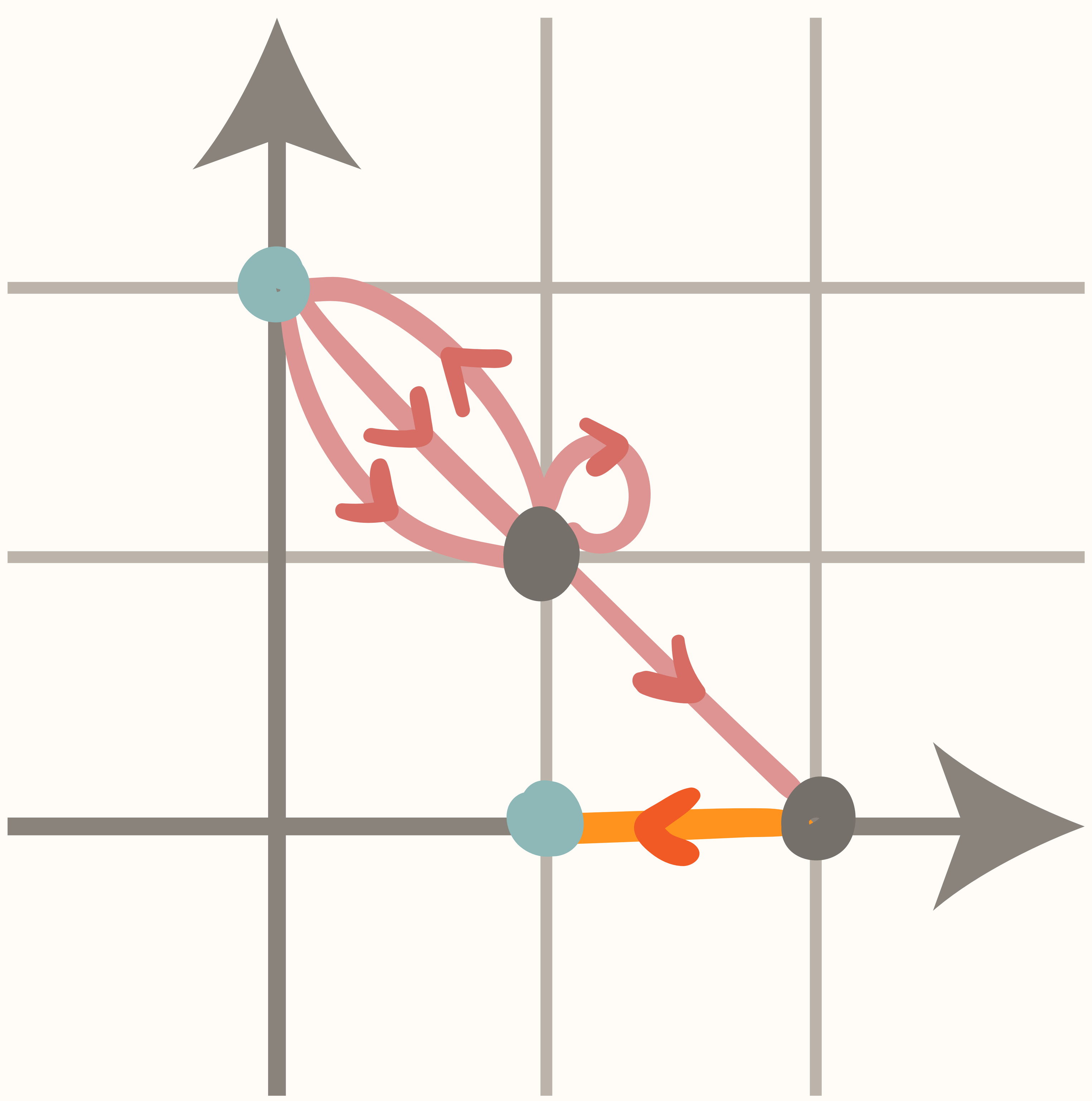
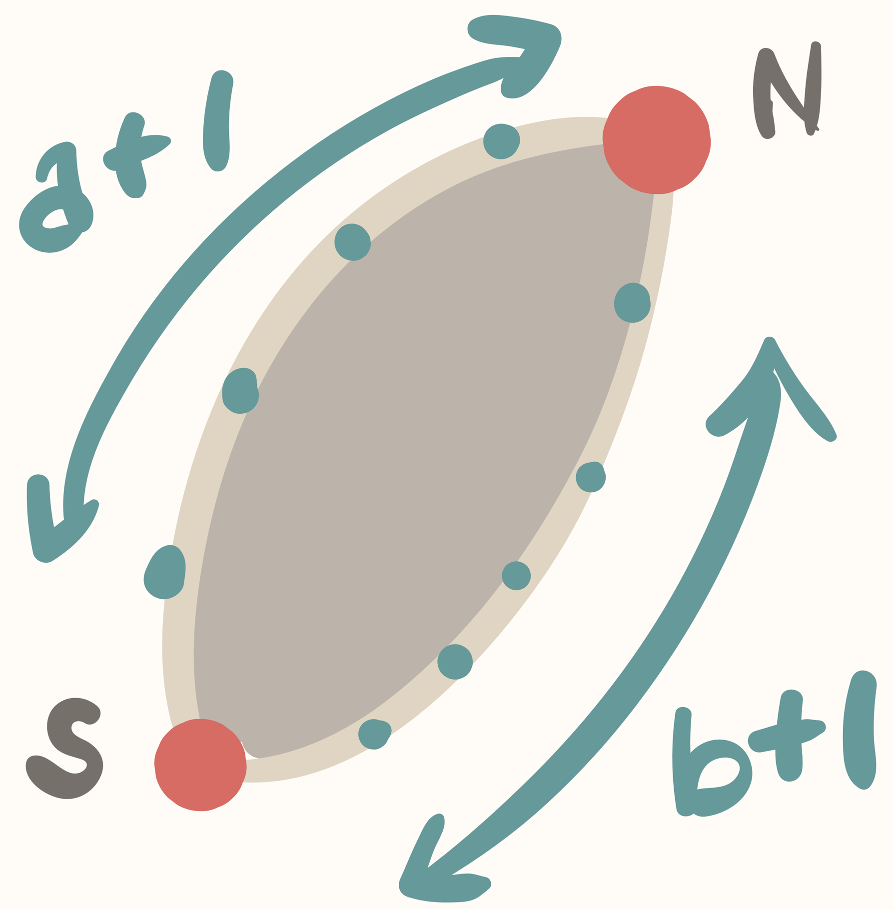




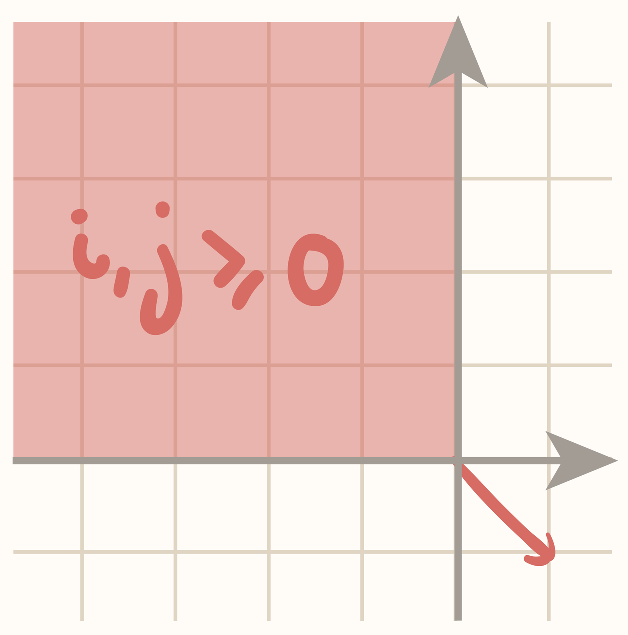


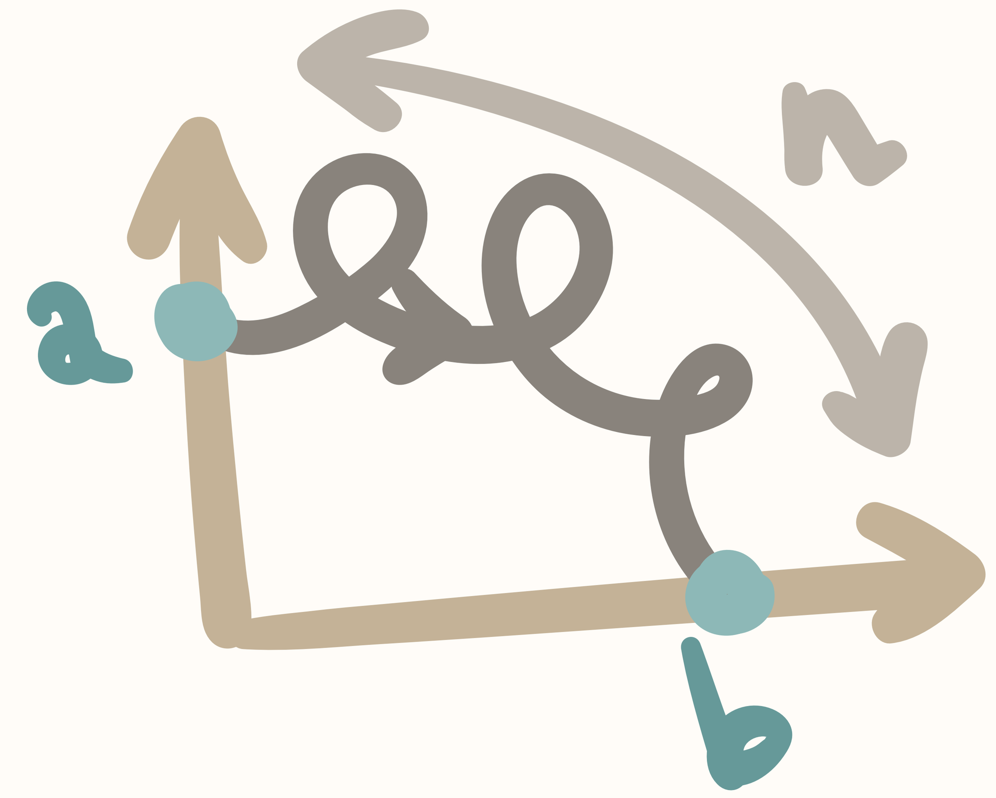
The KMSW bijection

bipolar orientations ⟷ *tandems walks in the quarter plane*



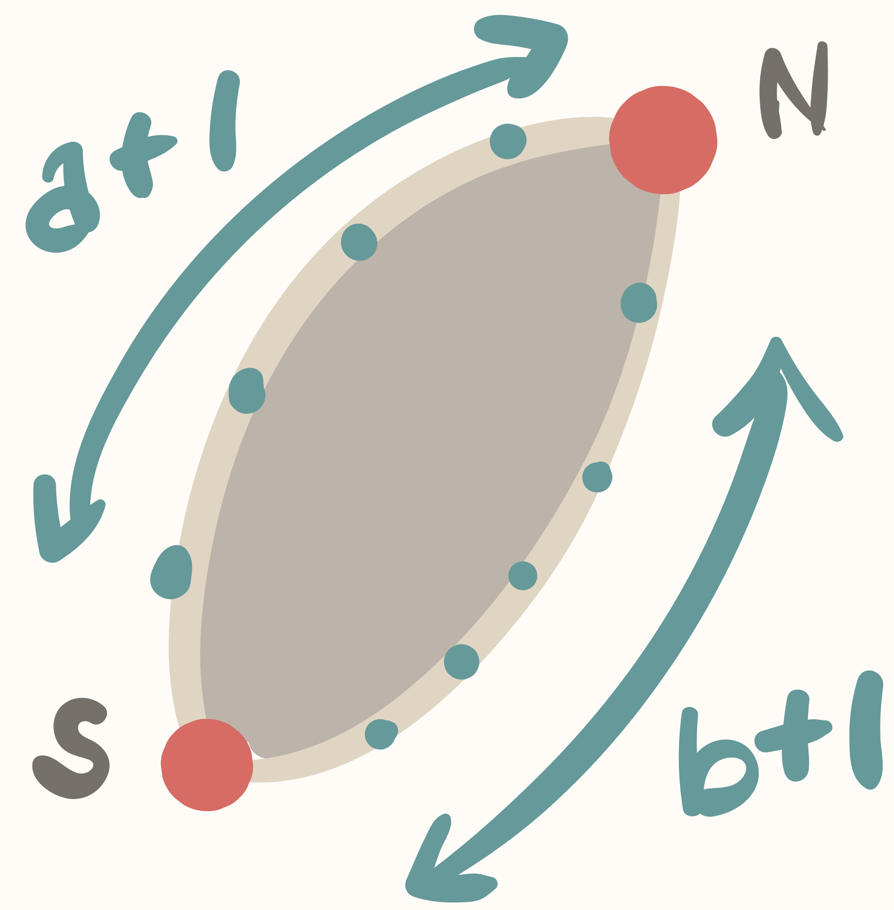
⟷





The KMSW bijection

bipolar orientations ⟷ **tandems walks in the quarter plane**



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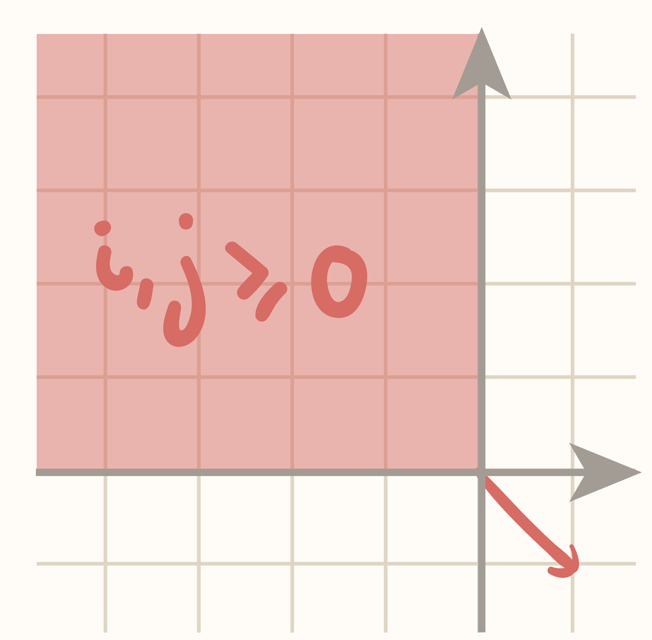
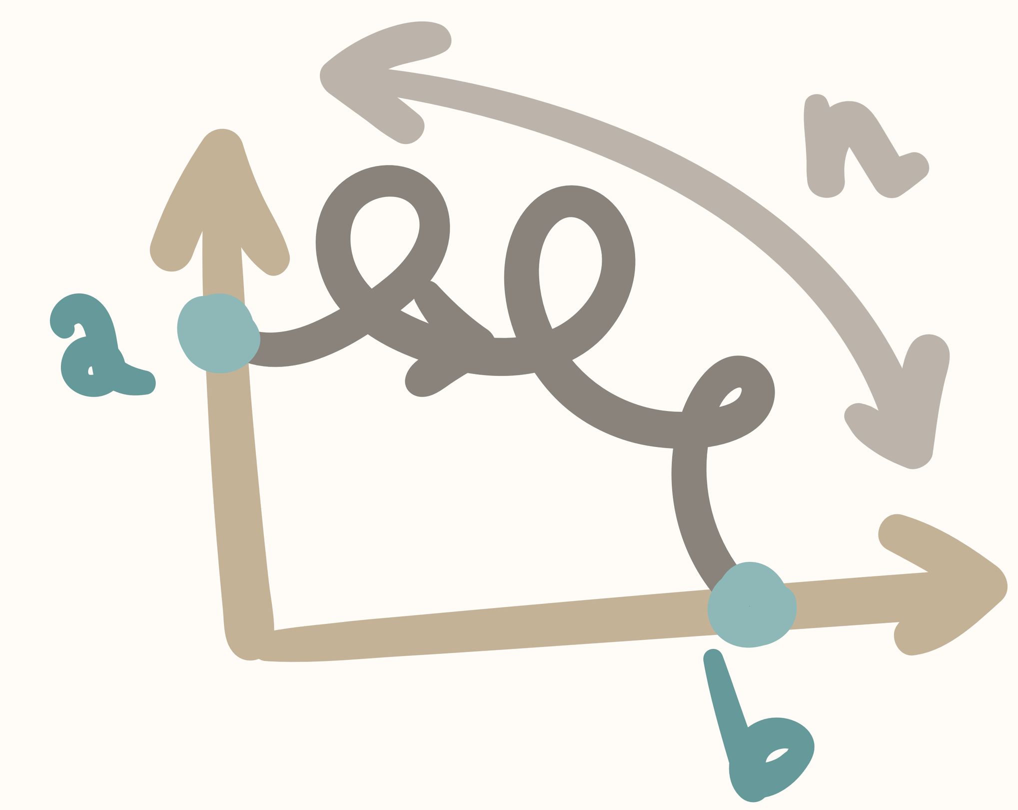
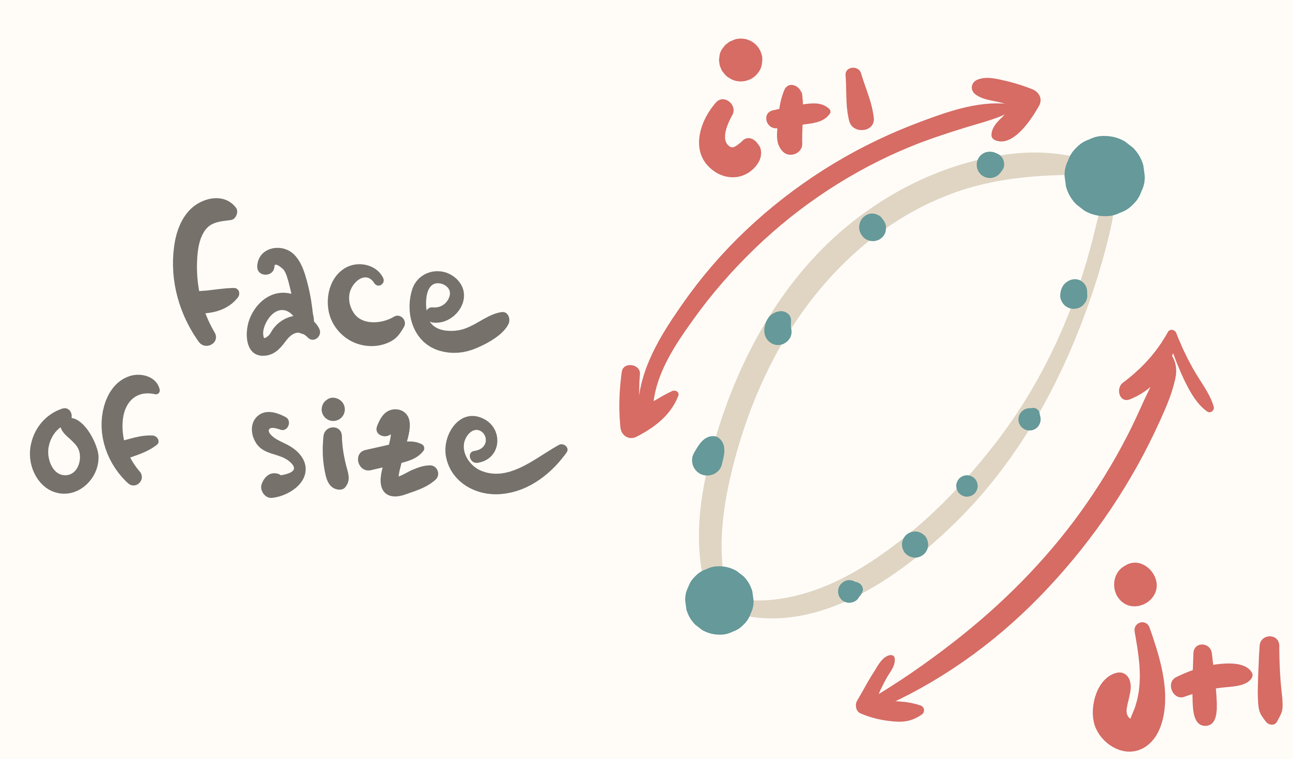




↠ *Bipolar orientations on planar maps and* SLE_{12}, R. Kenyon, J. Miller, S. Sheffield and D. Wison (2015)



⟷

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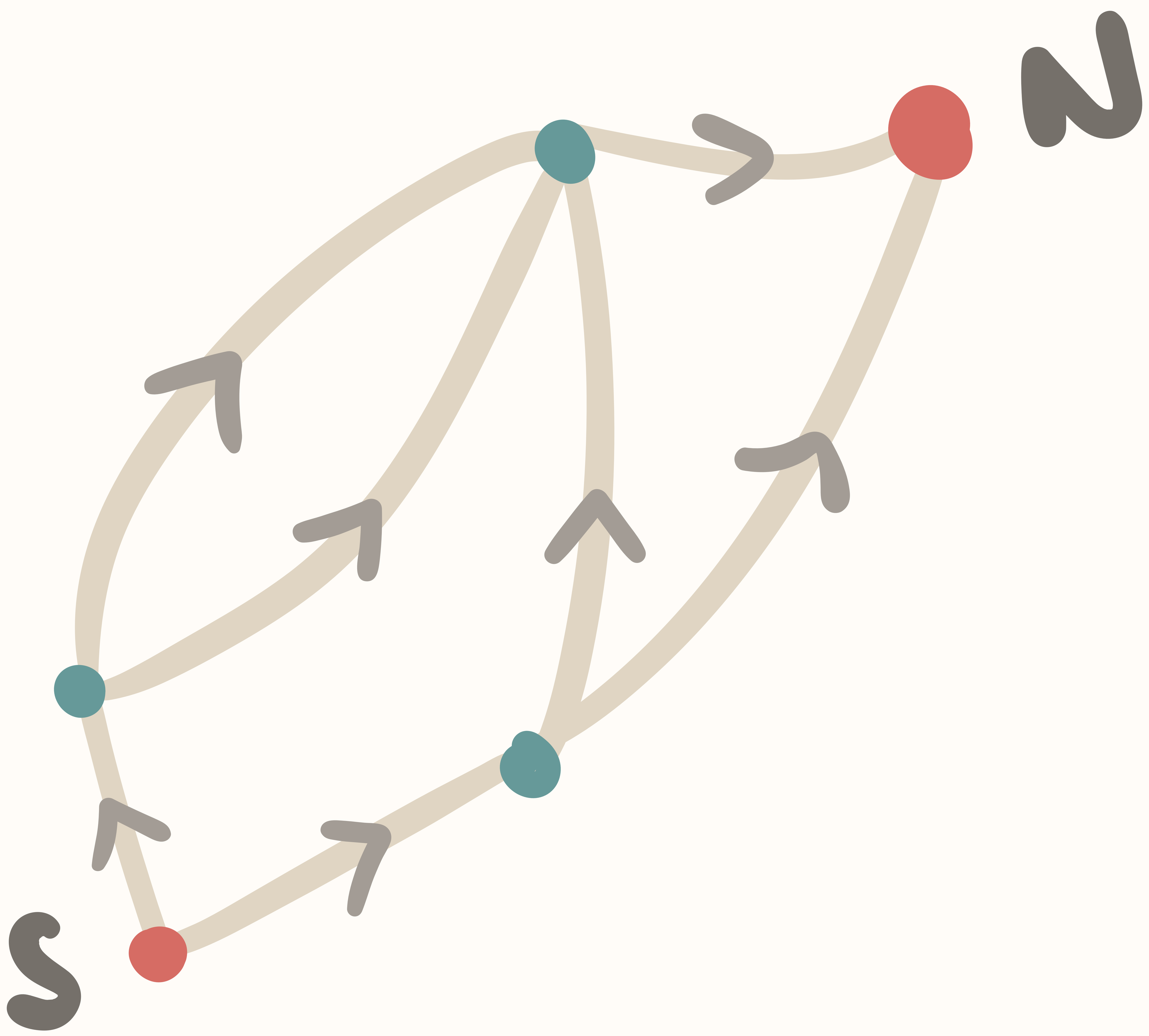
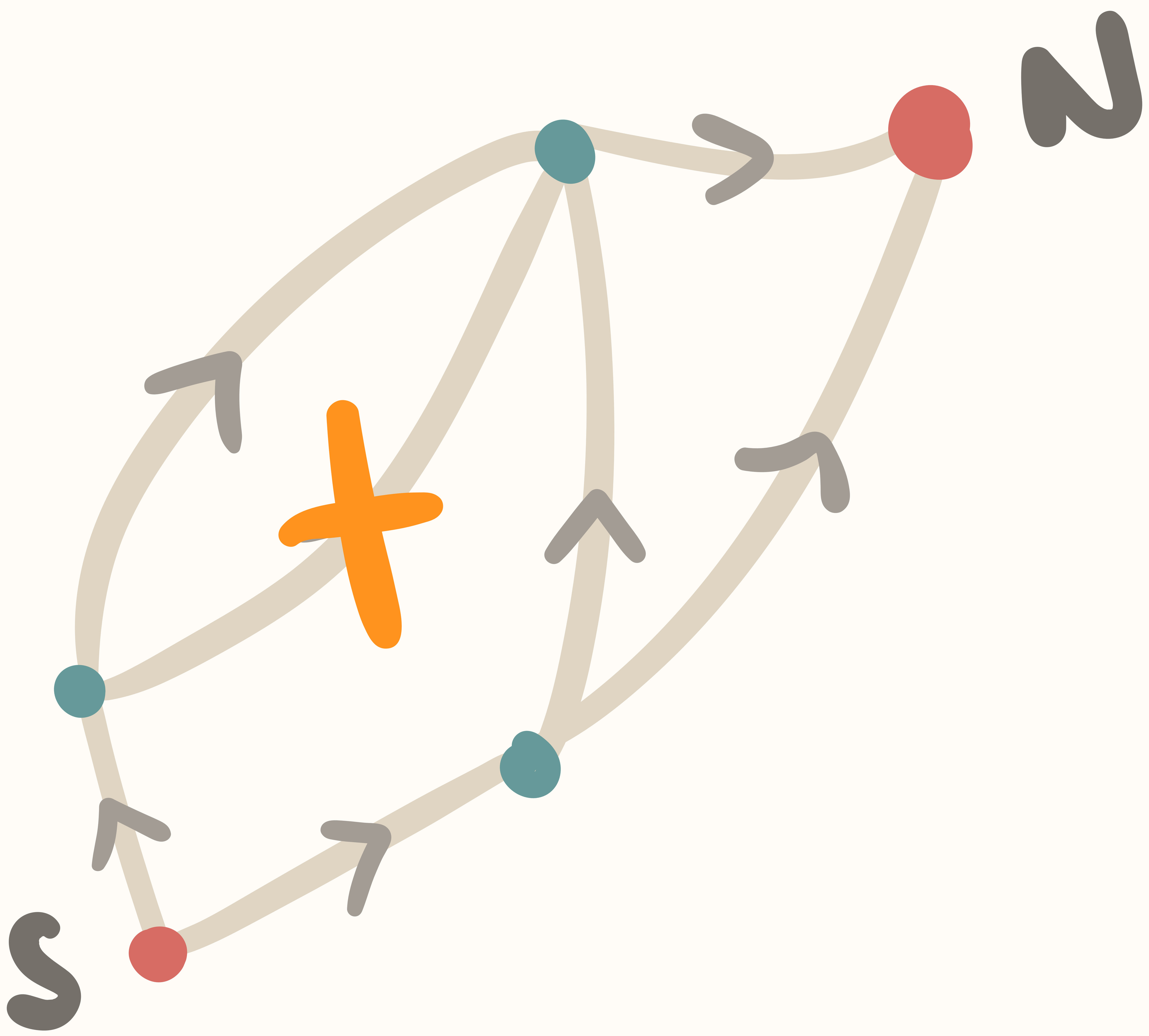
Plane bipolar poset



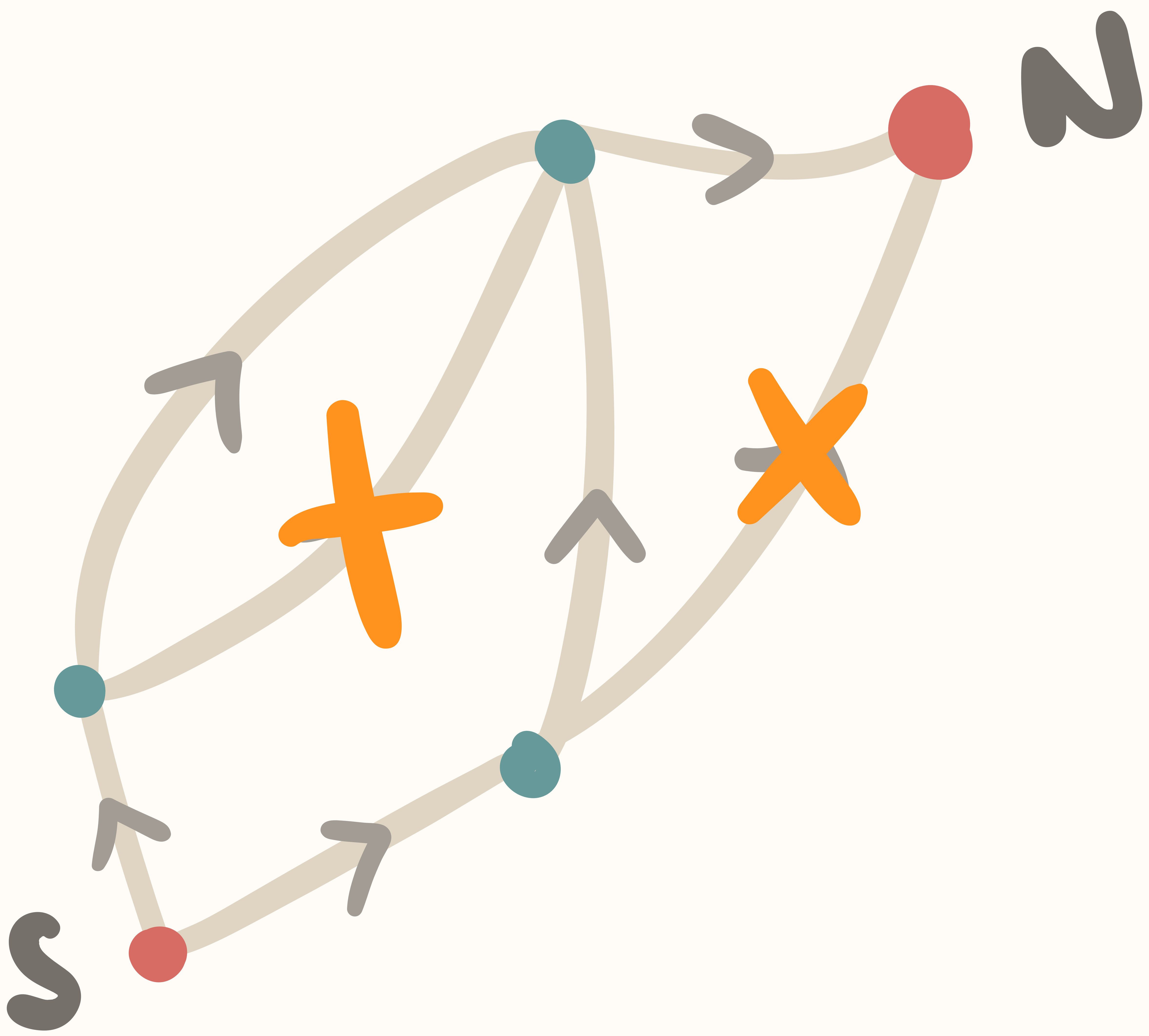
Plane bipolar poset



Poset
(plane bipolar poset)

= ***Bipolar orientation***
No multiple edge

Plane bipolar poset

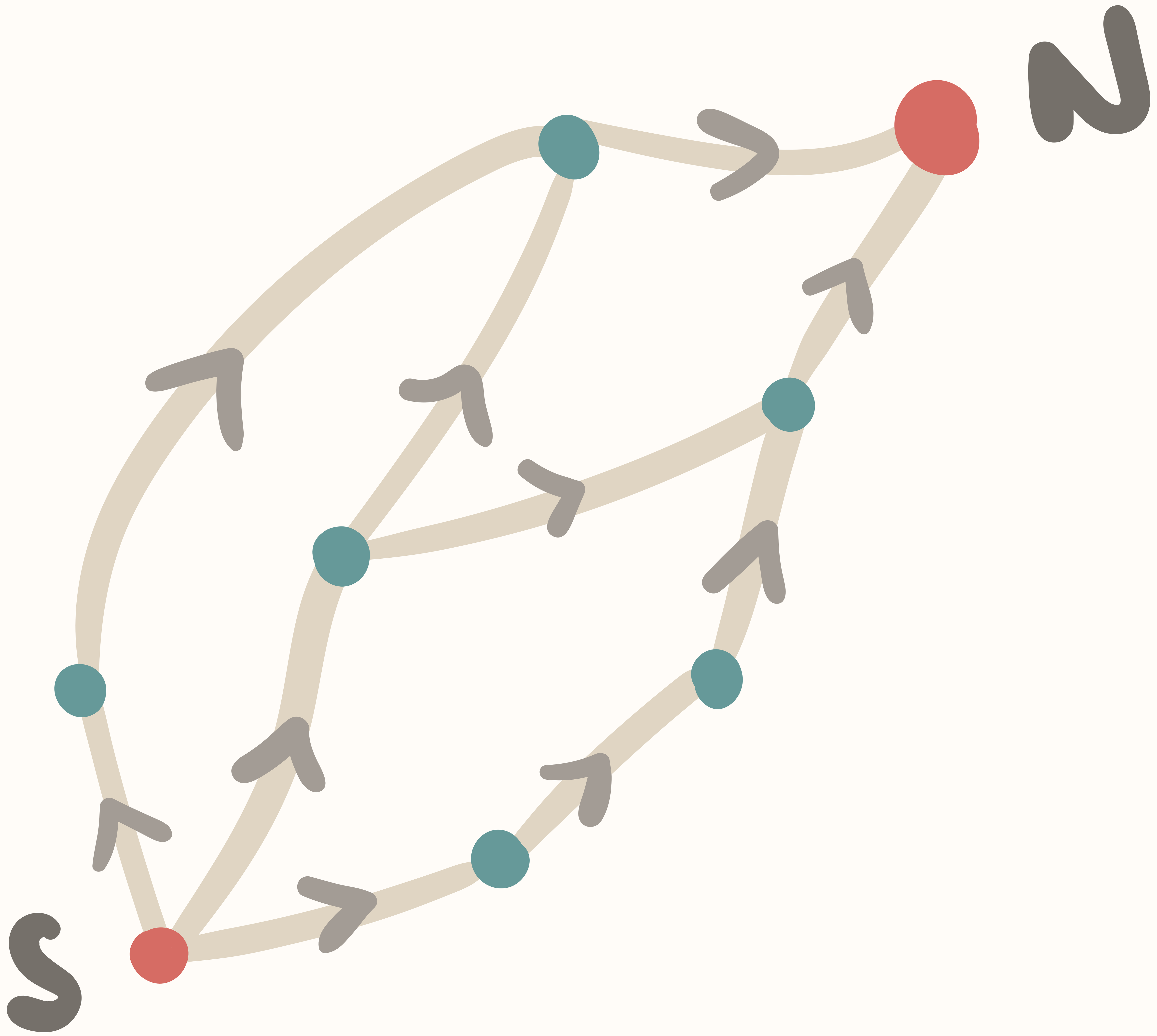


Poset
(plane bipolar poset)

= ***Bipolar orientation***

No multiple edge
No transitive edge

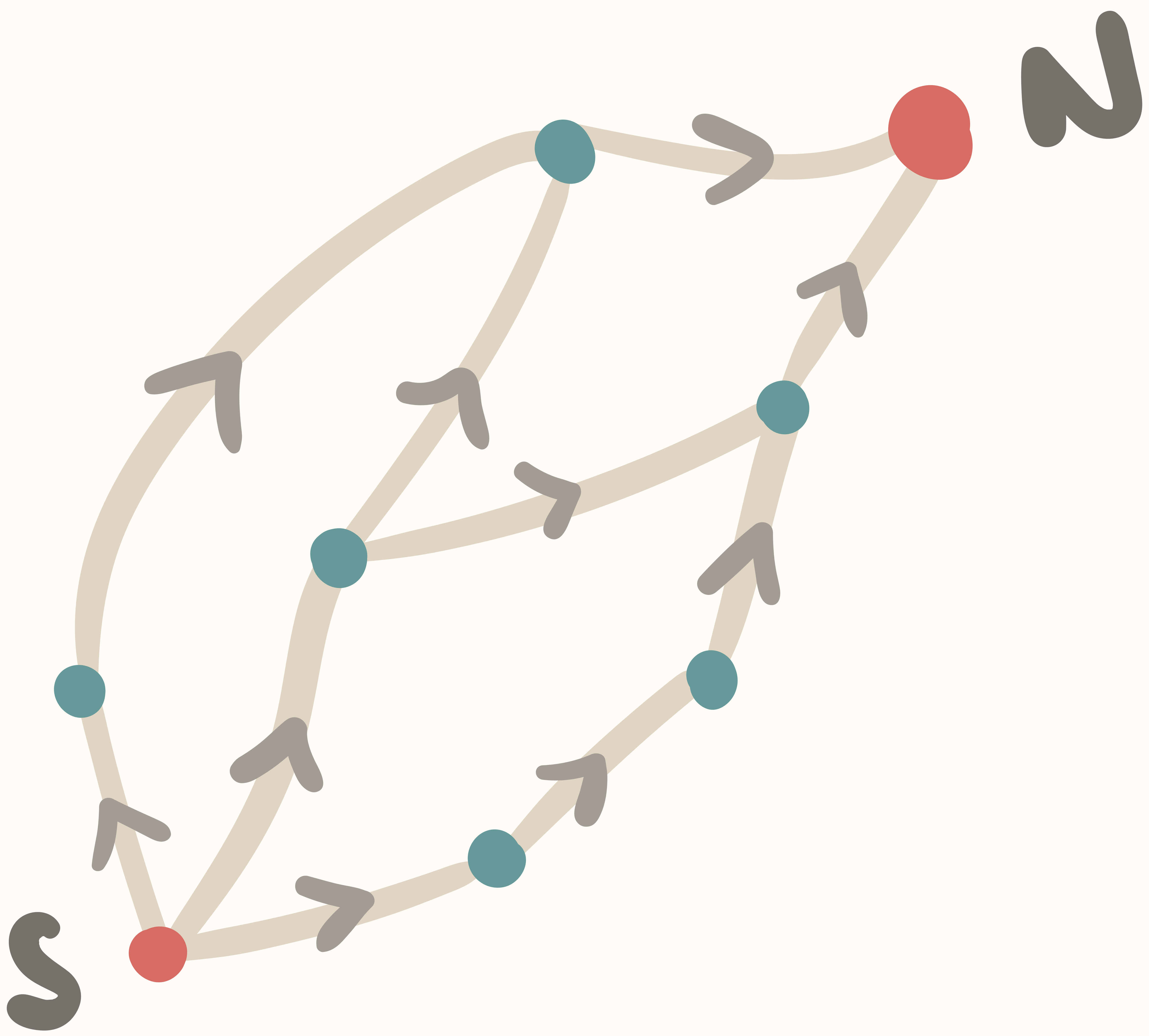
Plane bipolar poset



Poset
(plane bipolar poset)

= **Bipolar orientation**
No multiple edge
No transitive edge

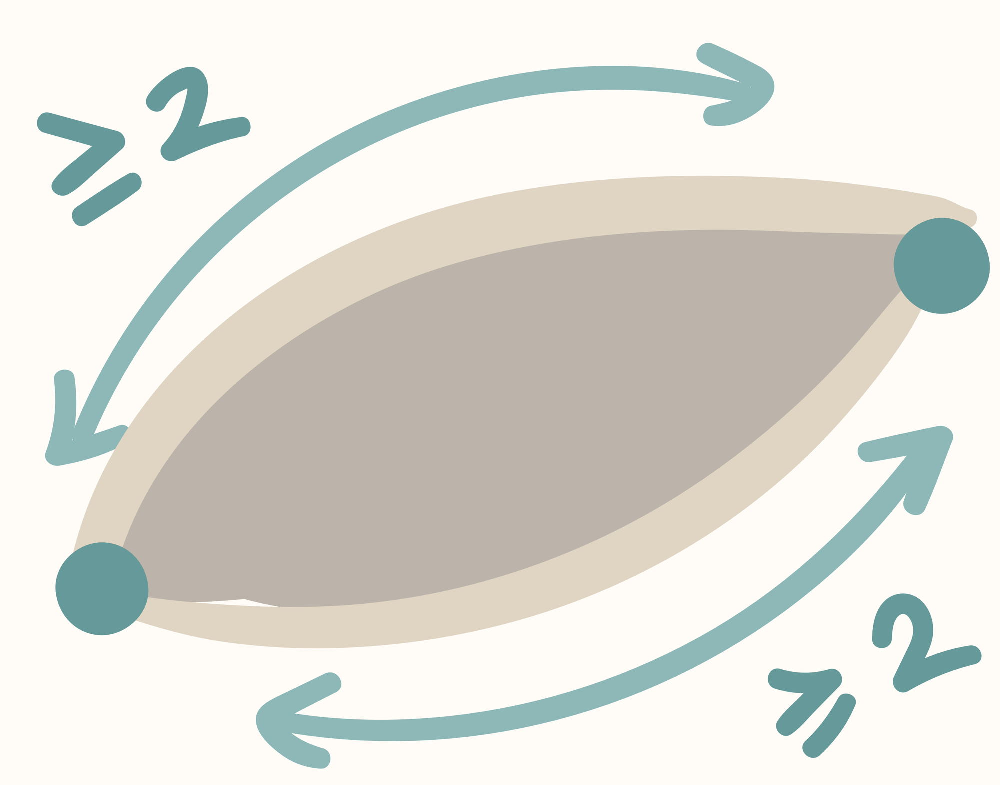
Plane bipolar poset



Poset
(plane bipolar poset)

= **Bipolar orientation**
No multiple edge
No transitive edge

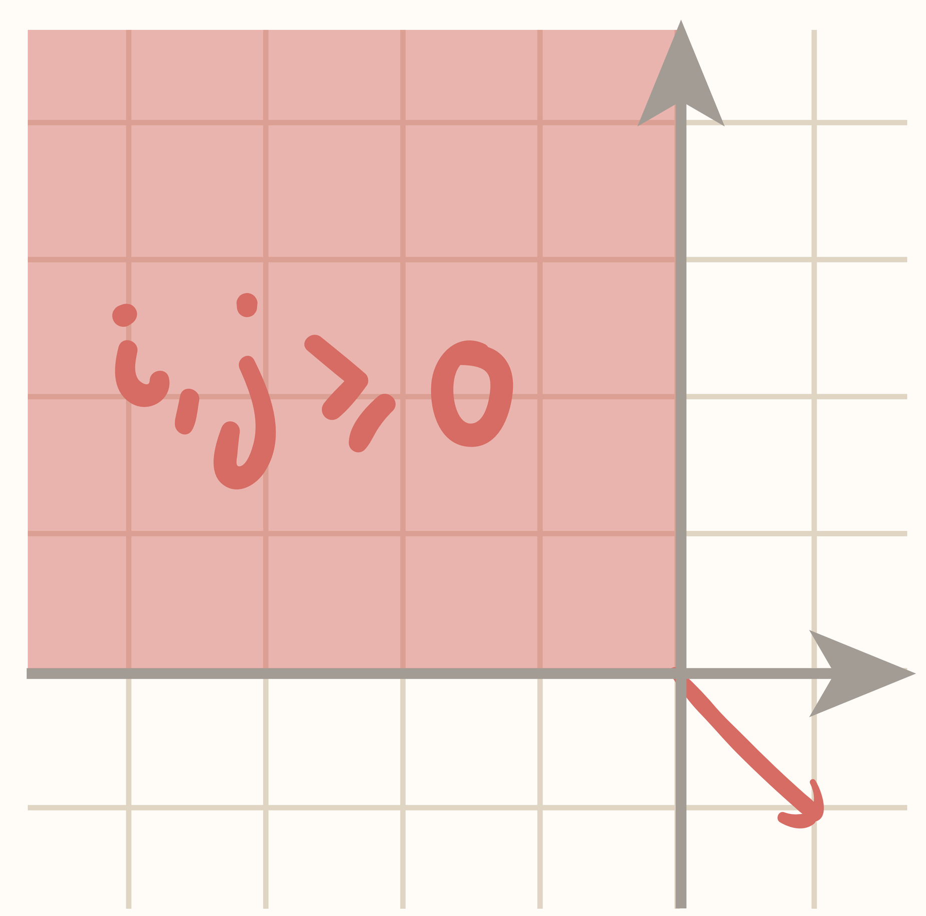
= **Bipolar orientation**



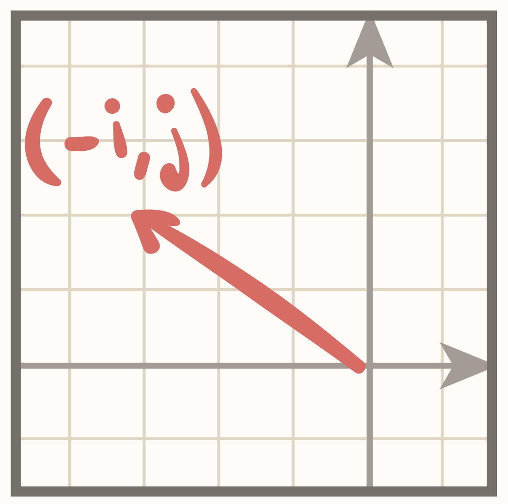
Specialization to Posets

Specialization to Posets

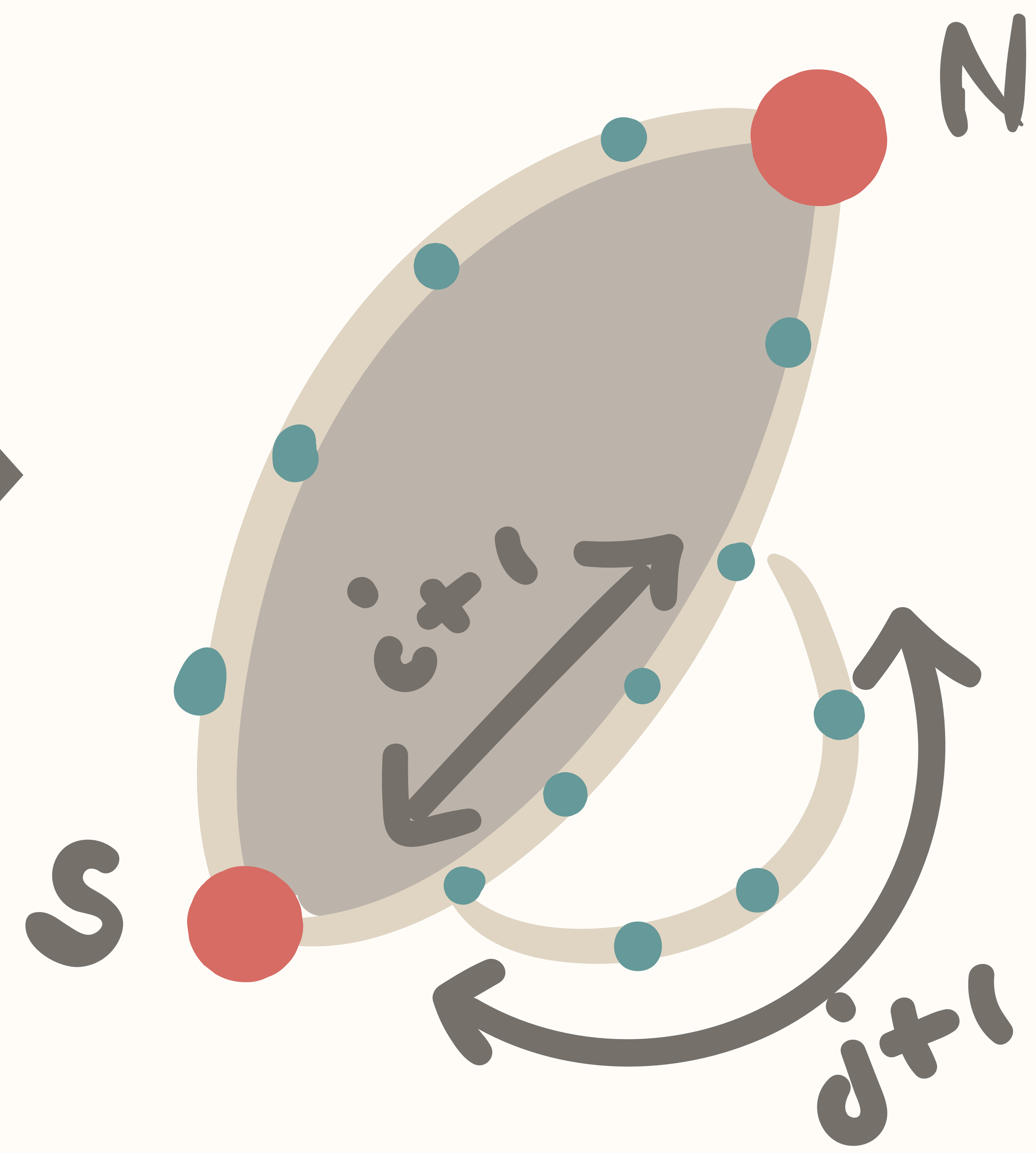
Bipolar orientation =



where

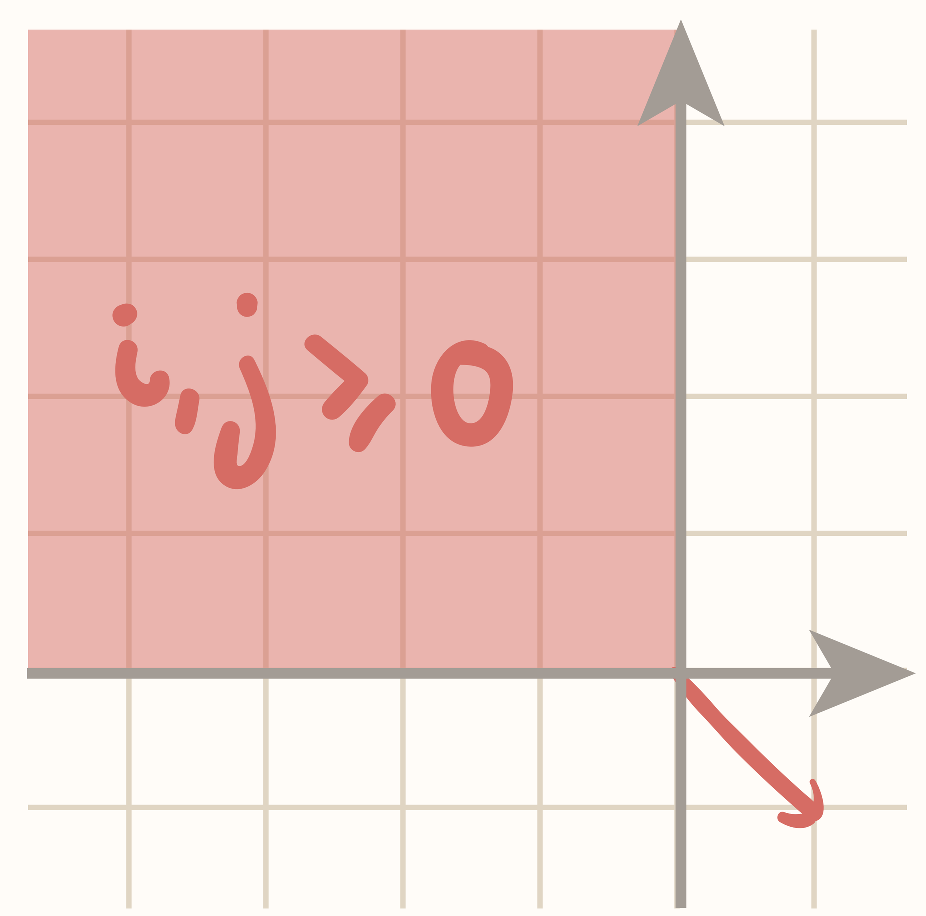


→

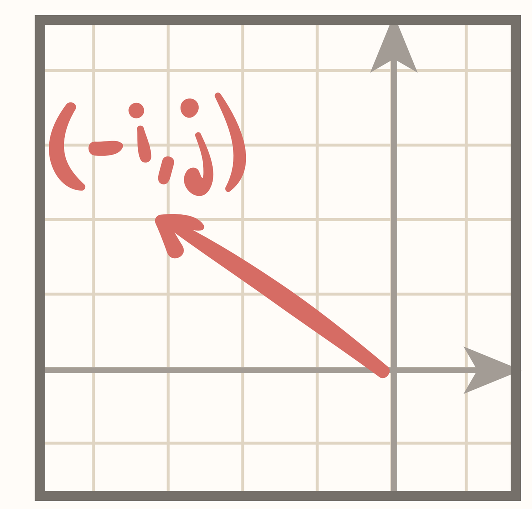


Specialization to Posets

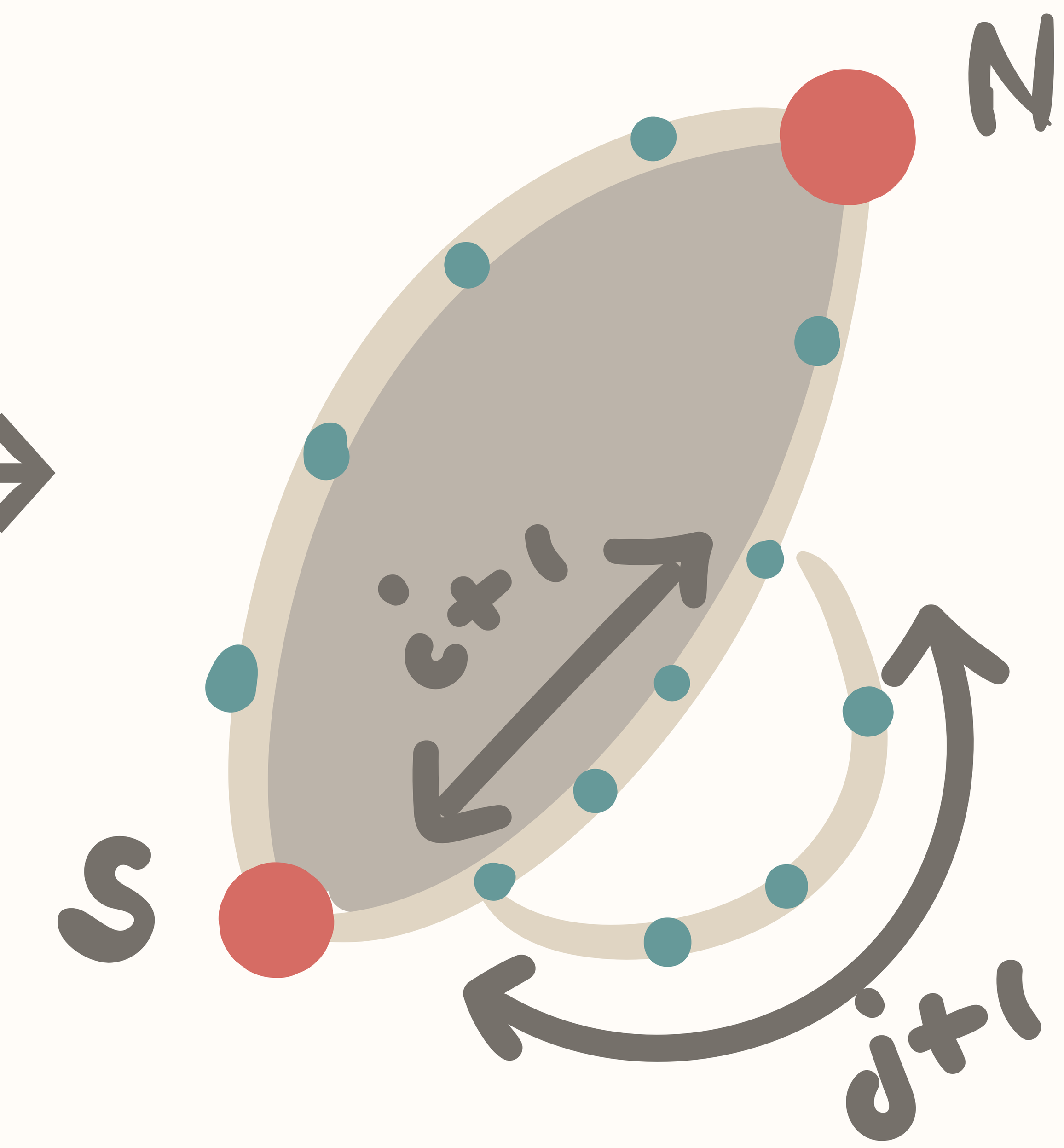
Bipolar orientation =



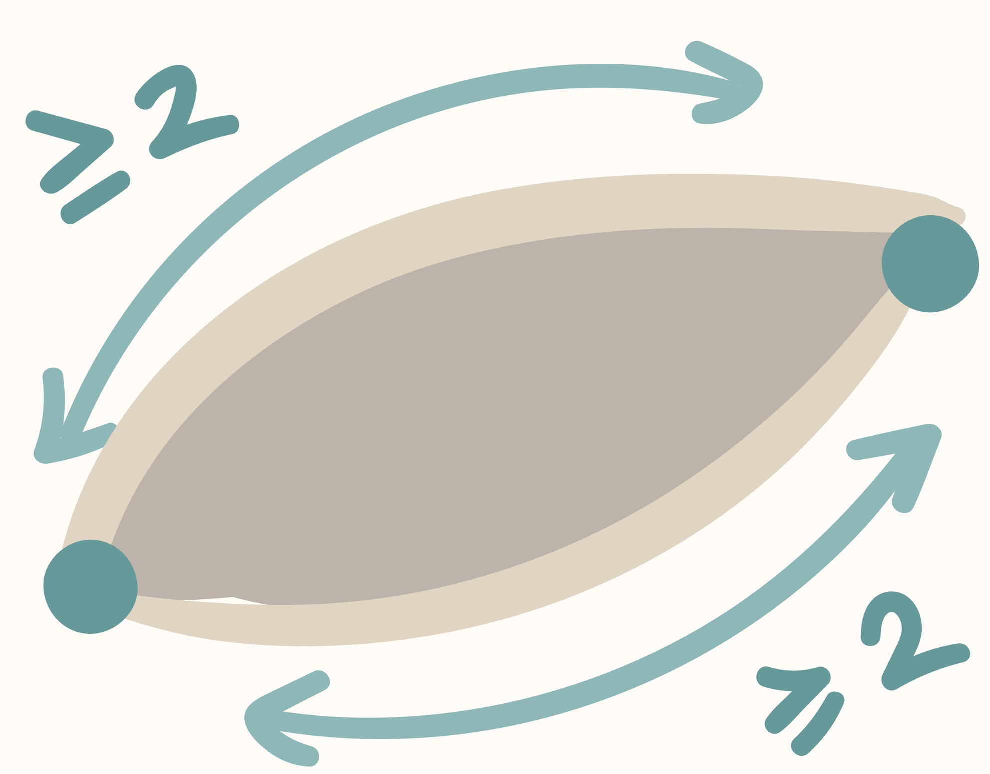
where



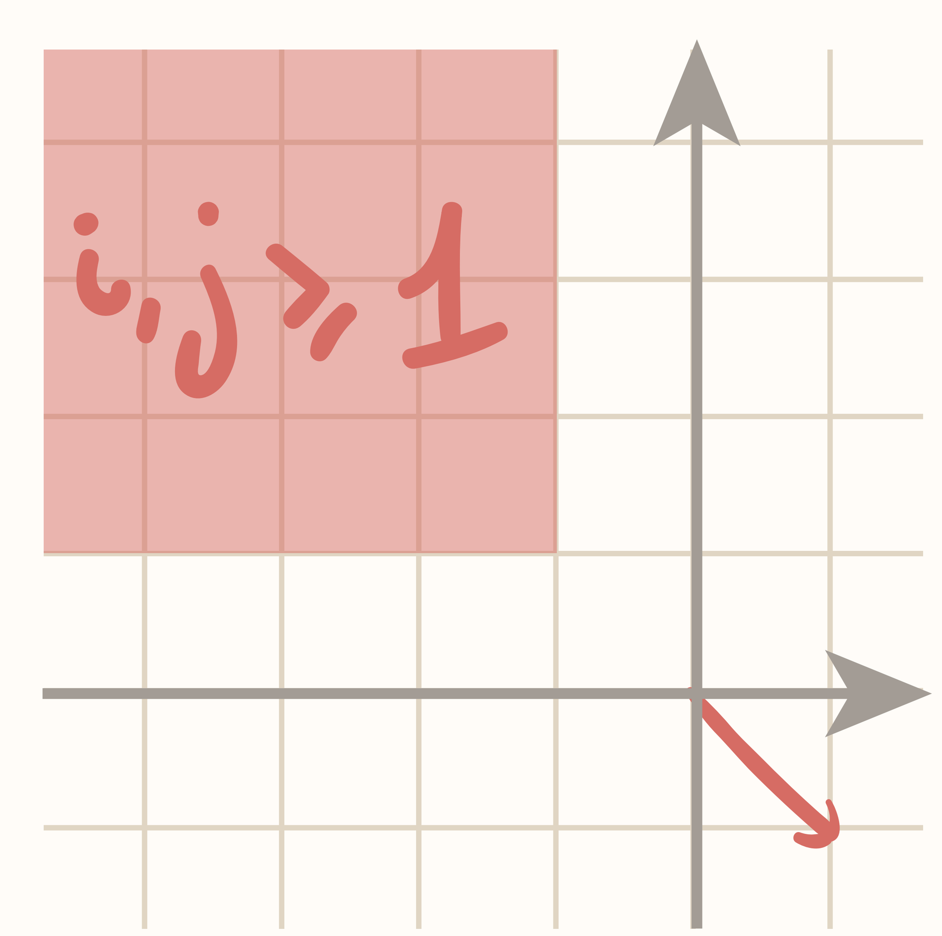
→



Poset =



=



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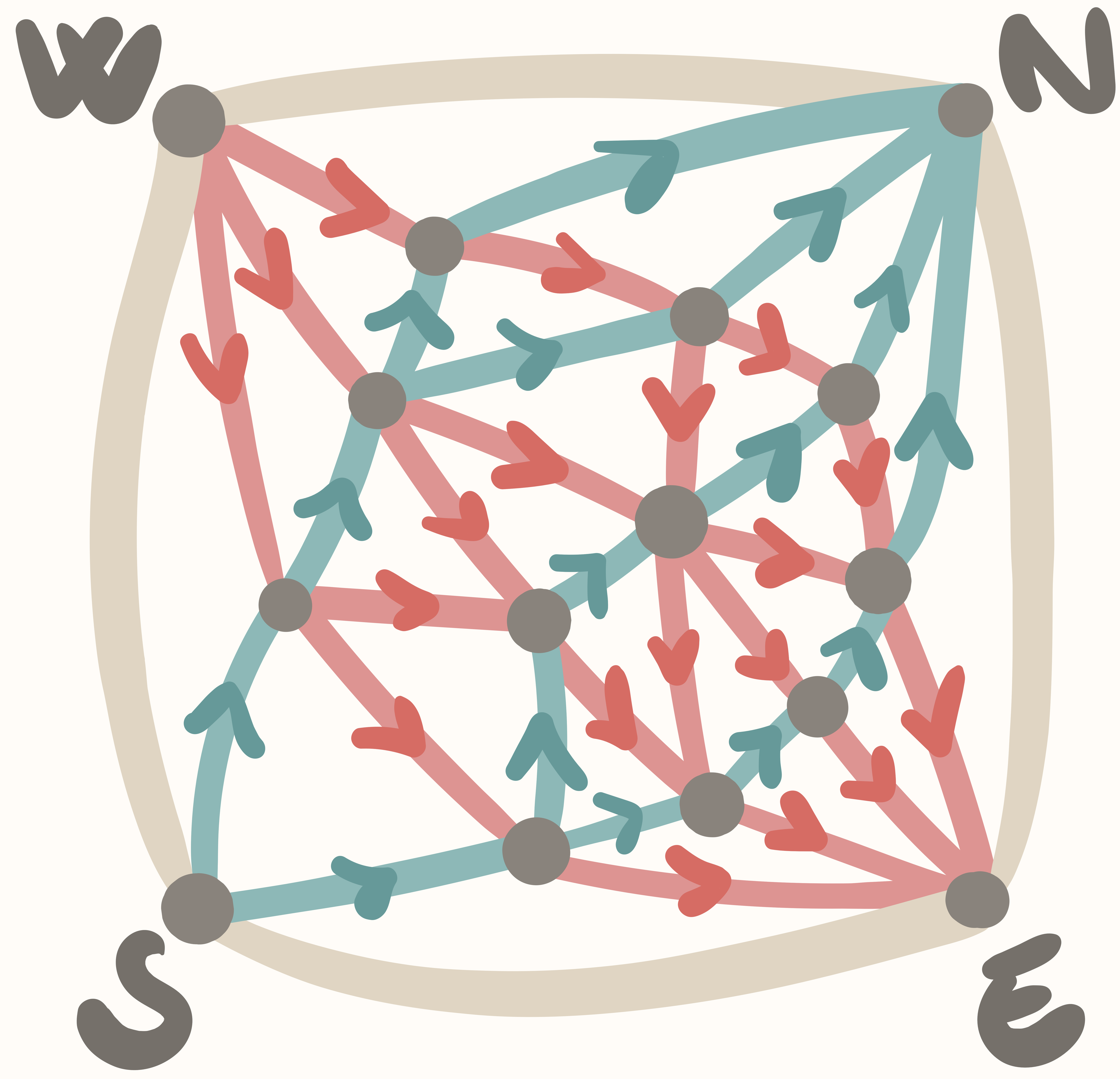
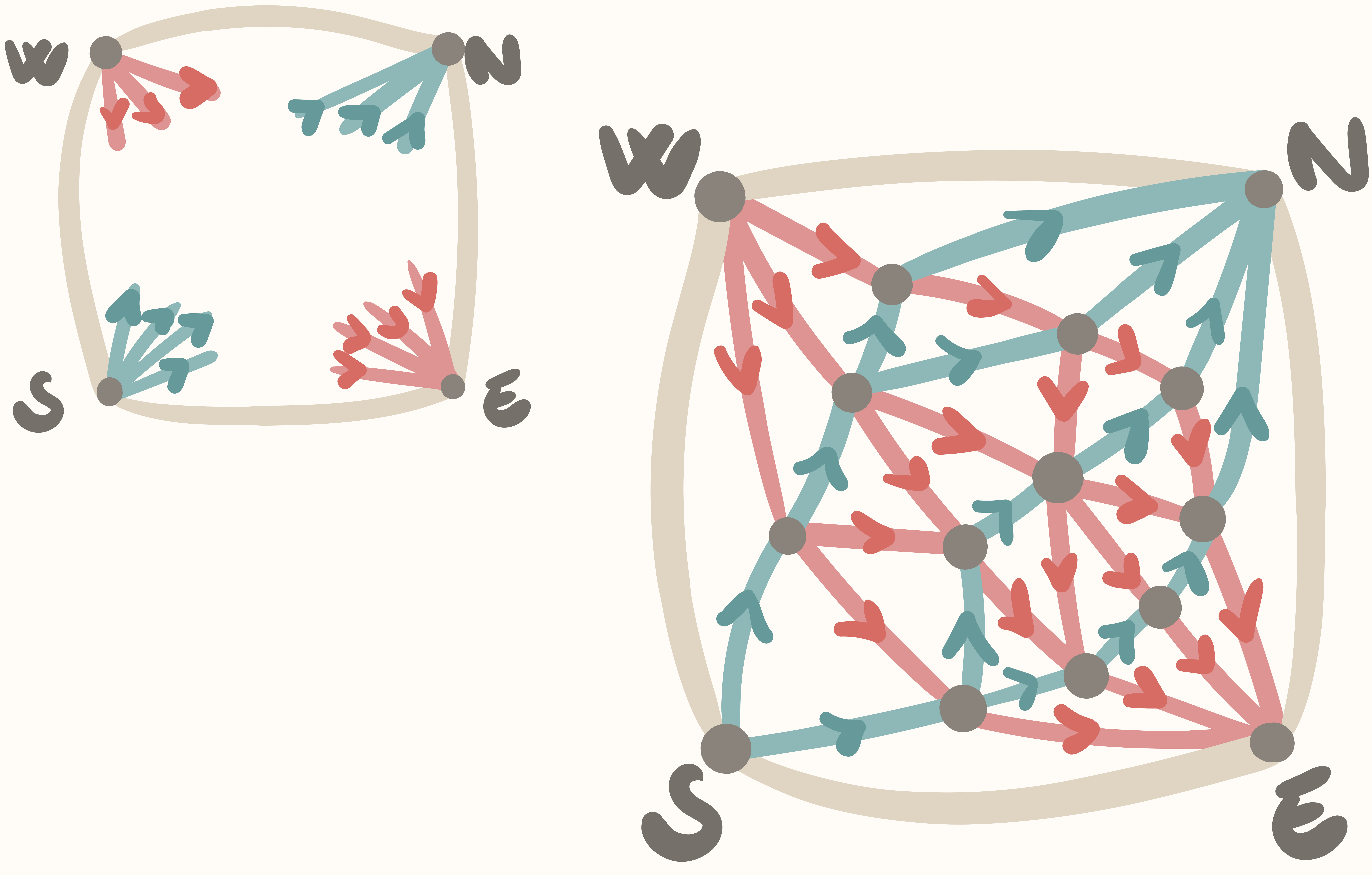
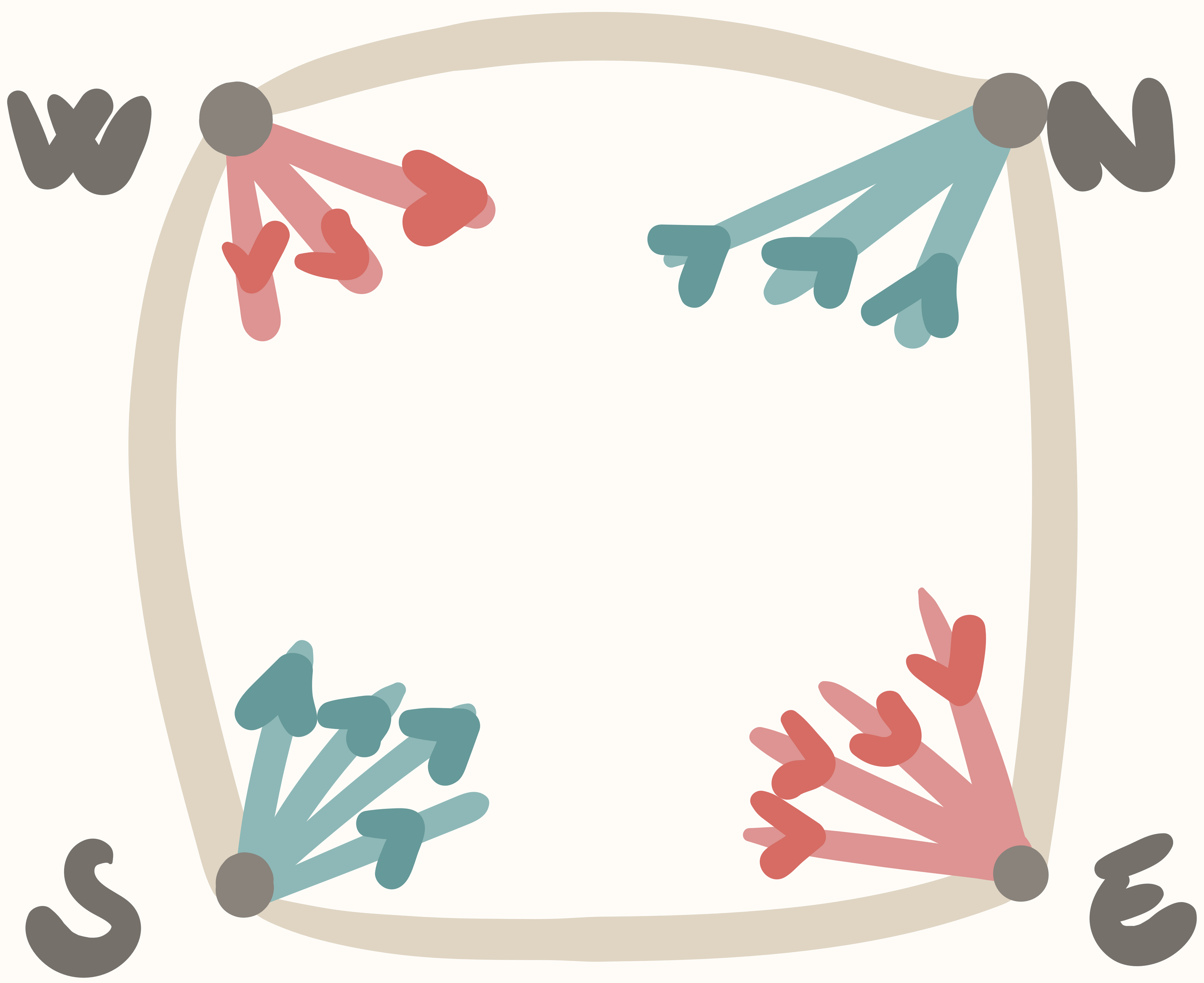
Transversal structures

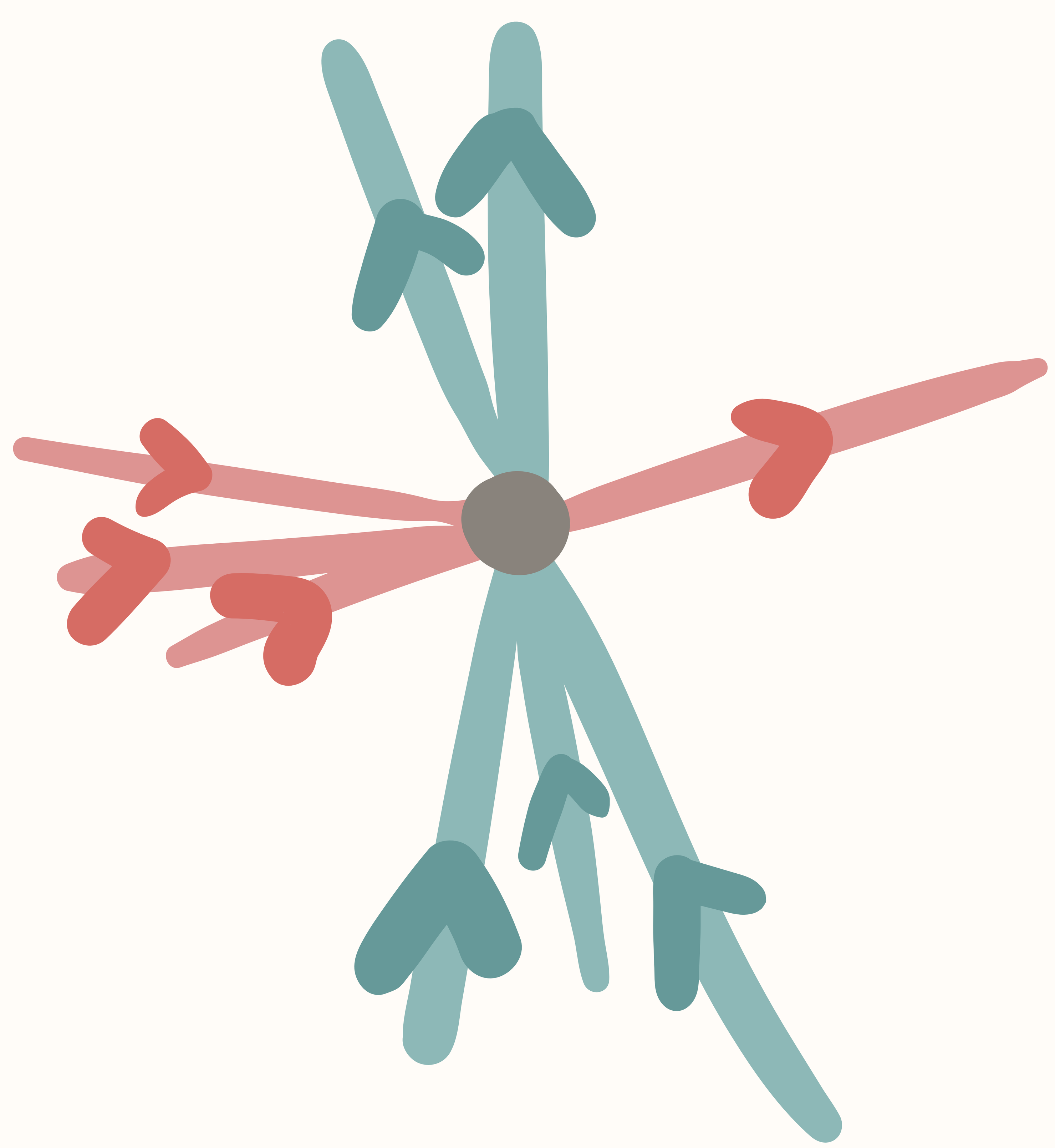


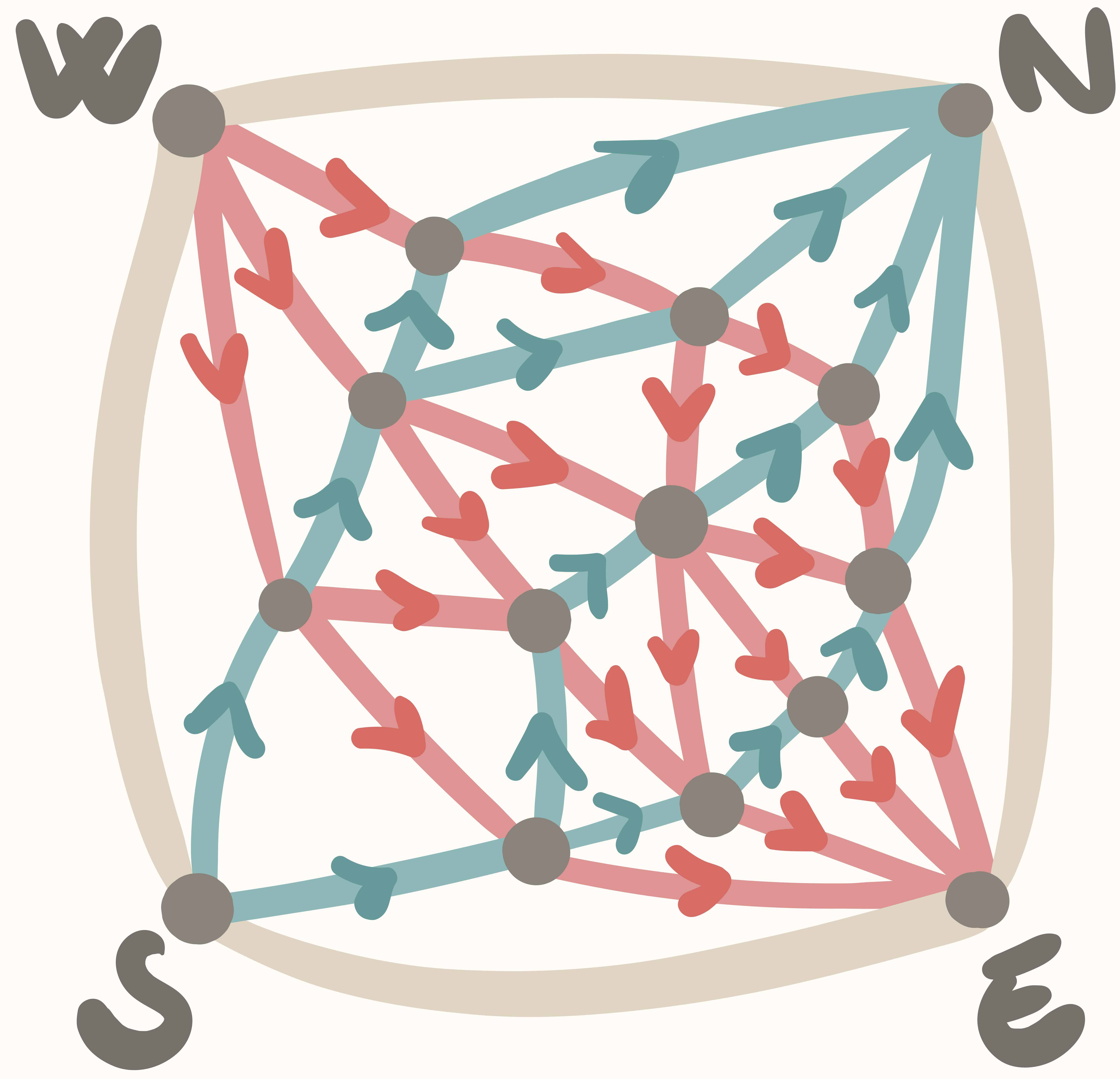
Transversal structures



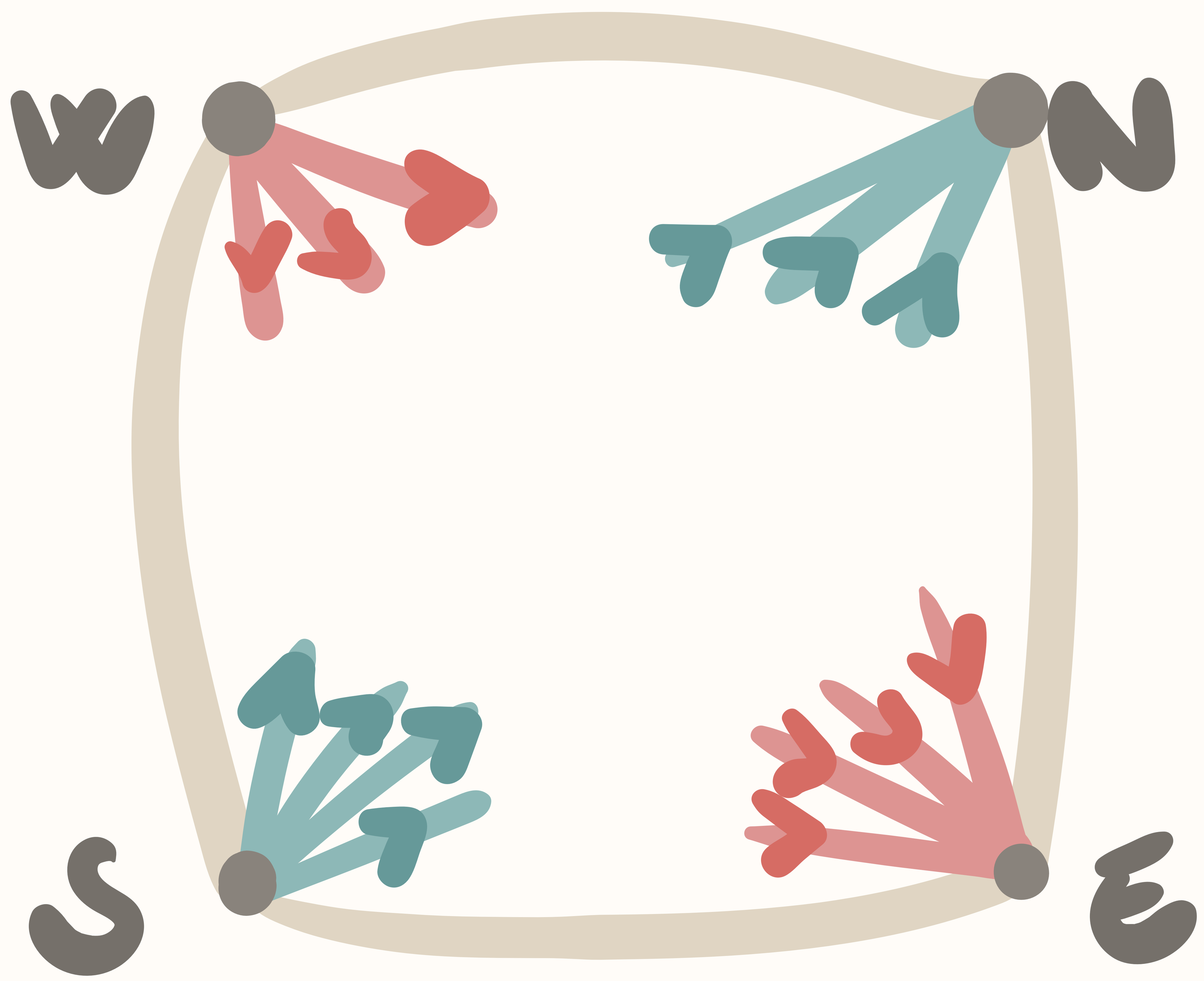
Transversal structures

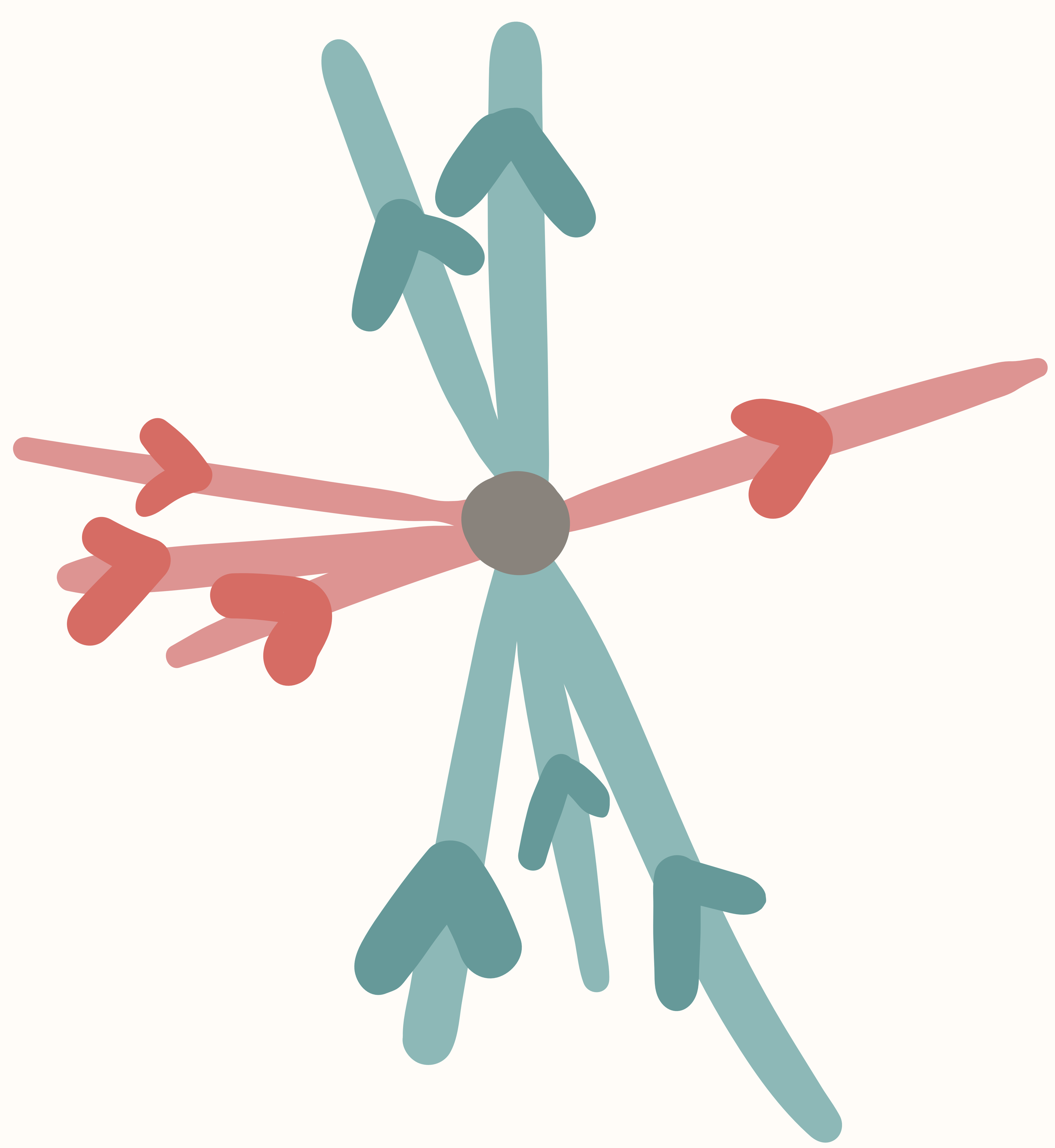


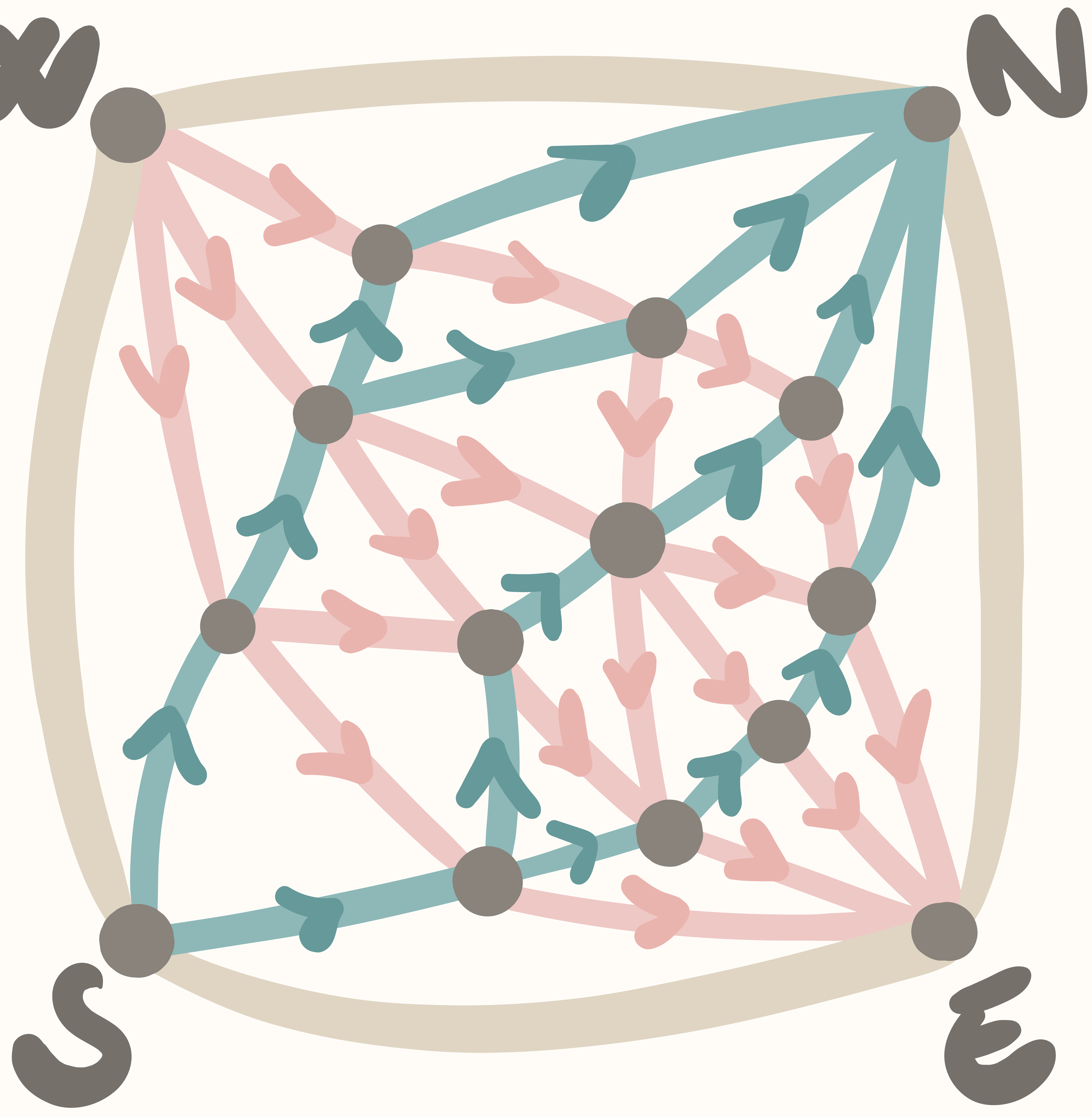




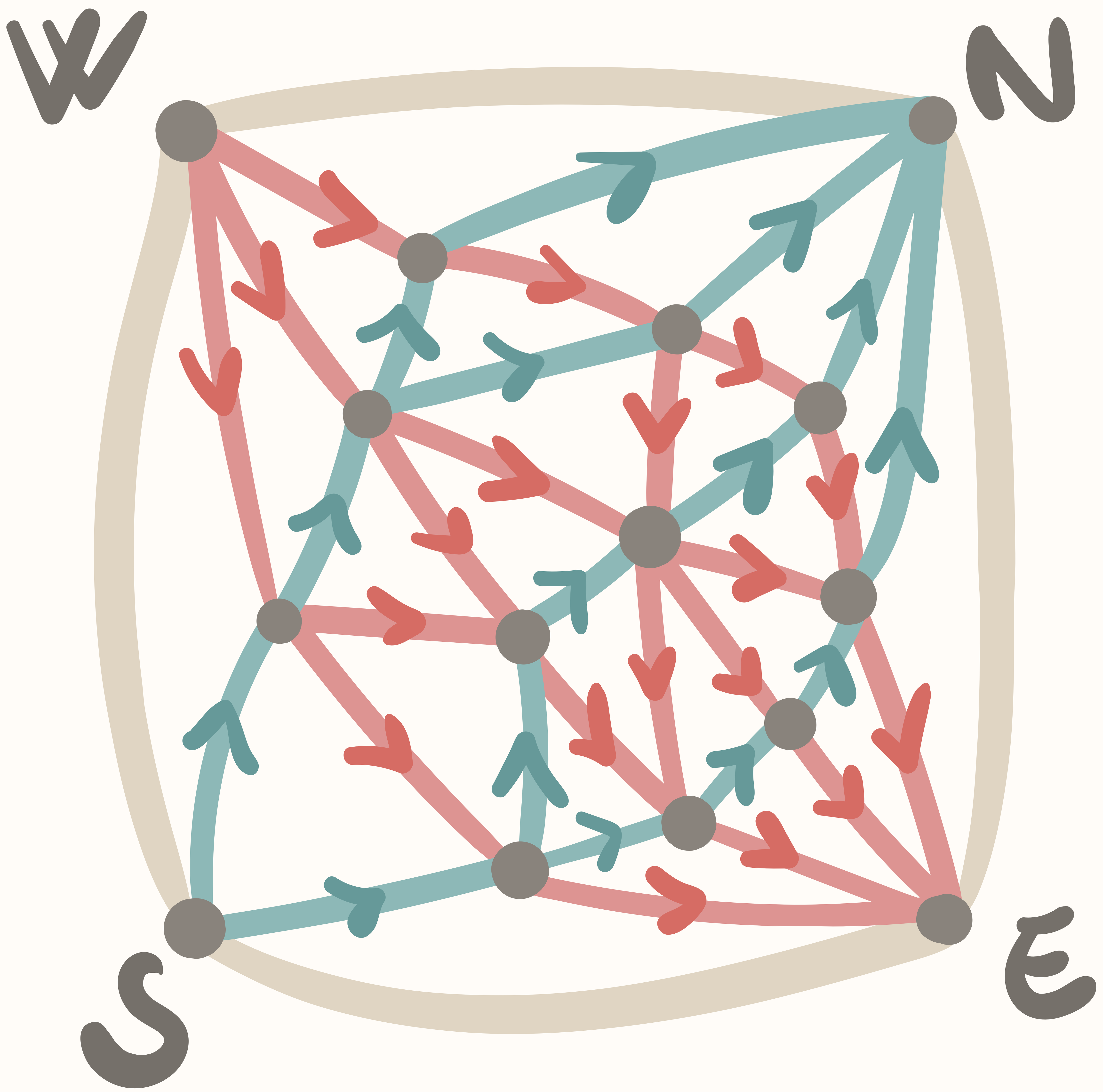
Transversal structures



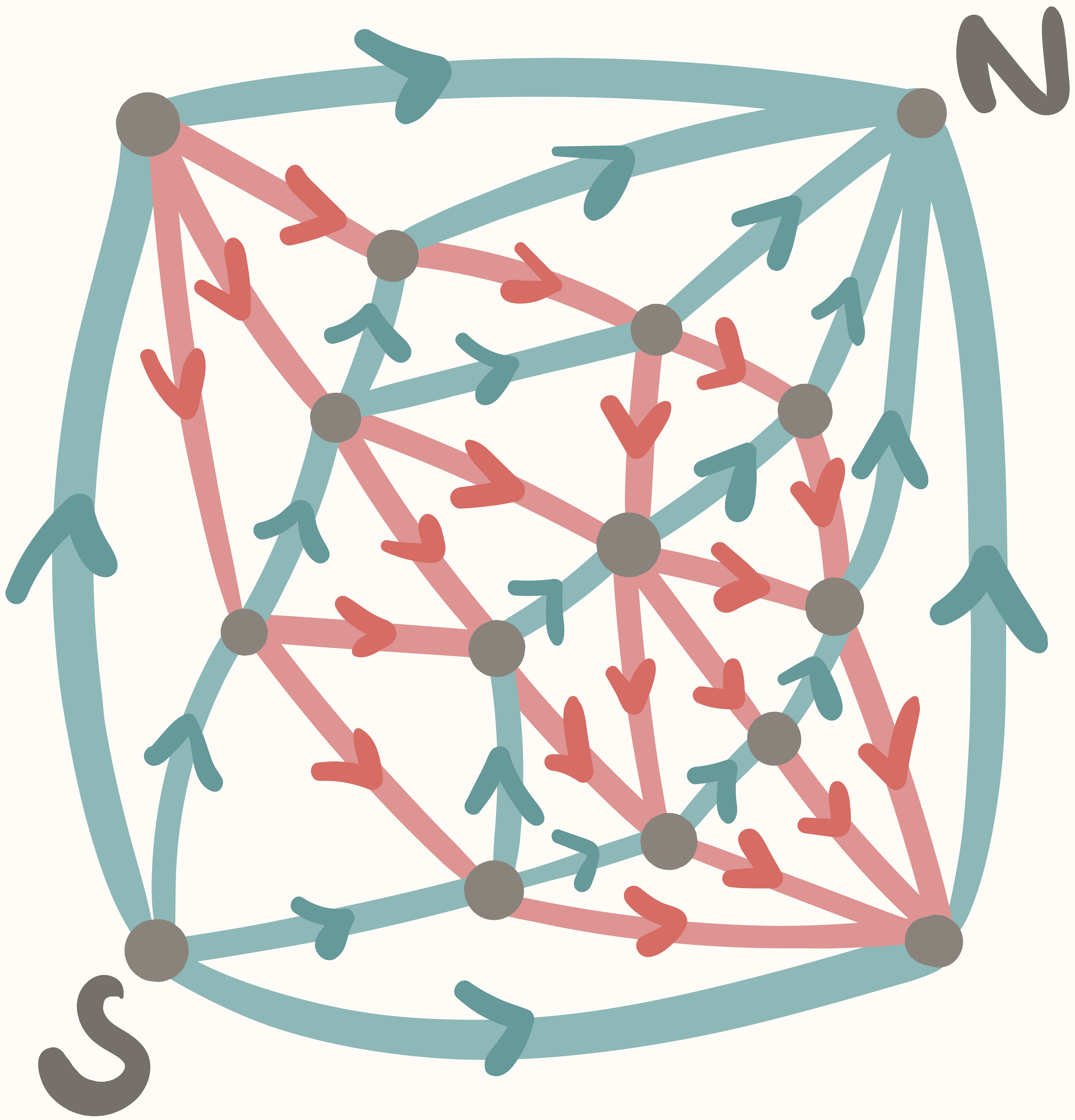




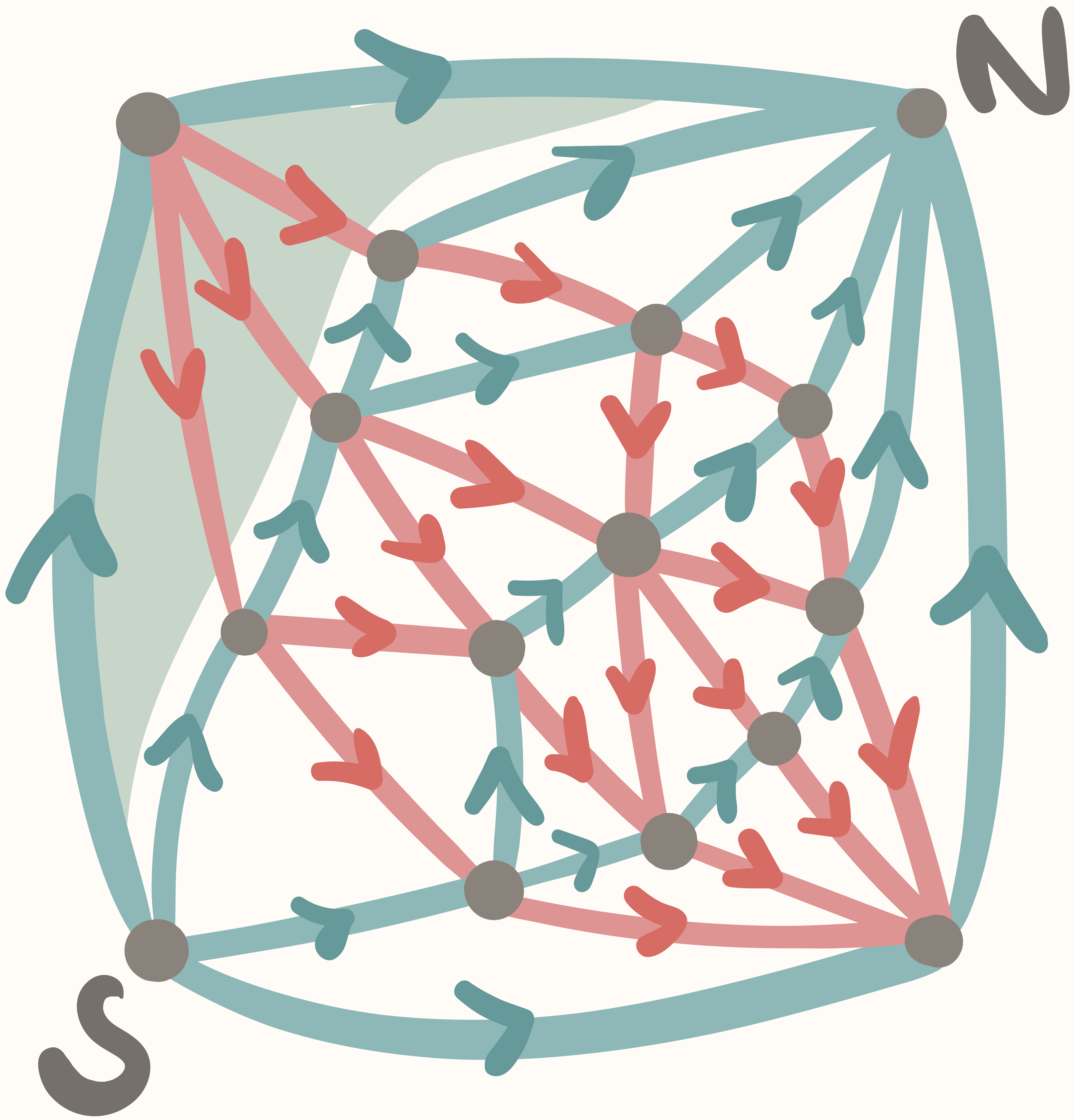
Specialization to transversal structures



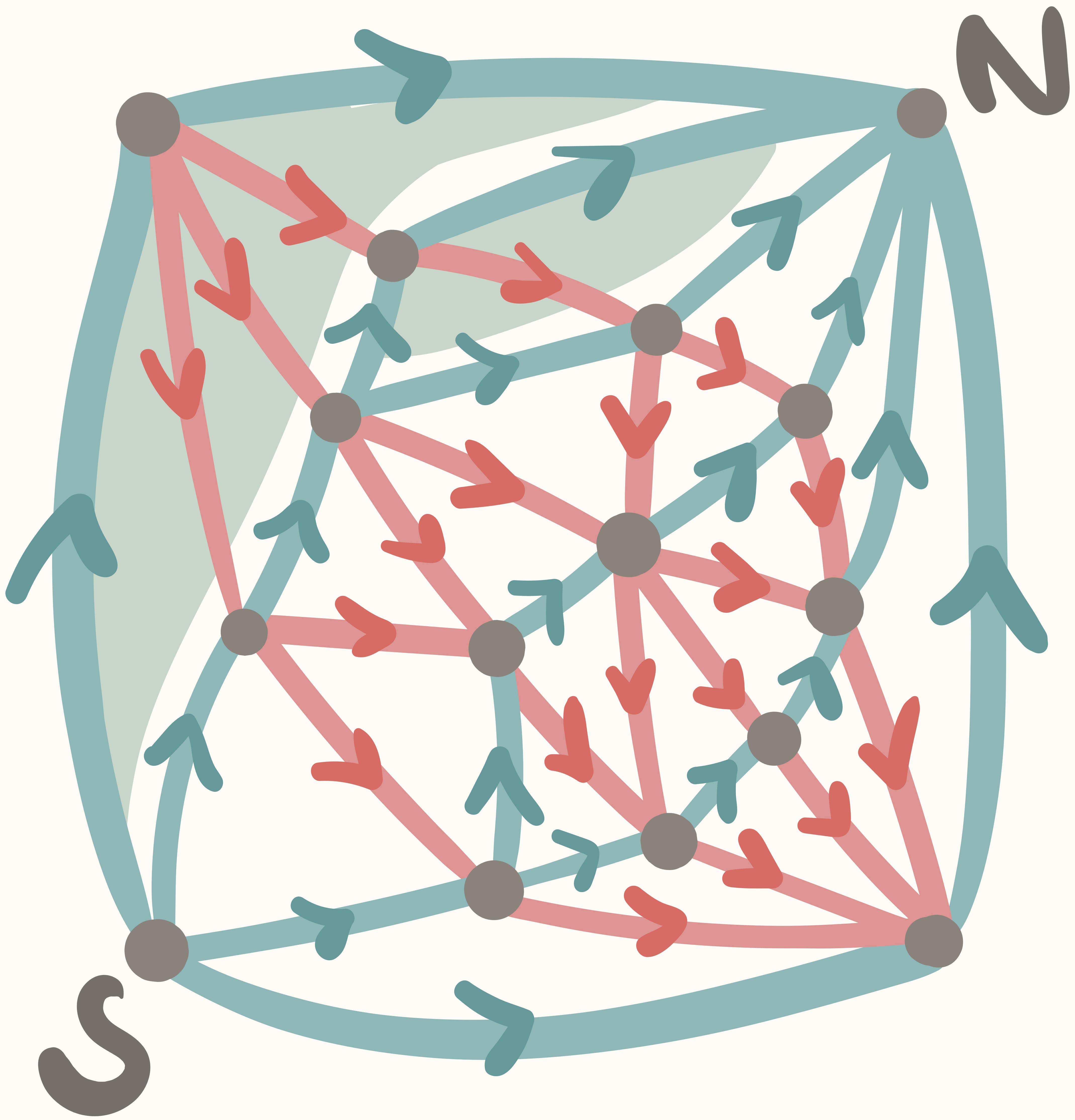
Specialization to transversal structures



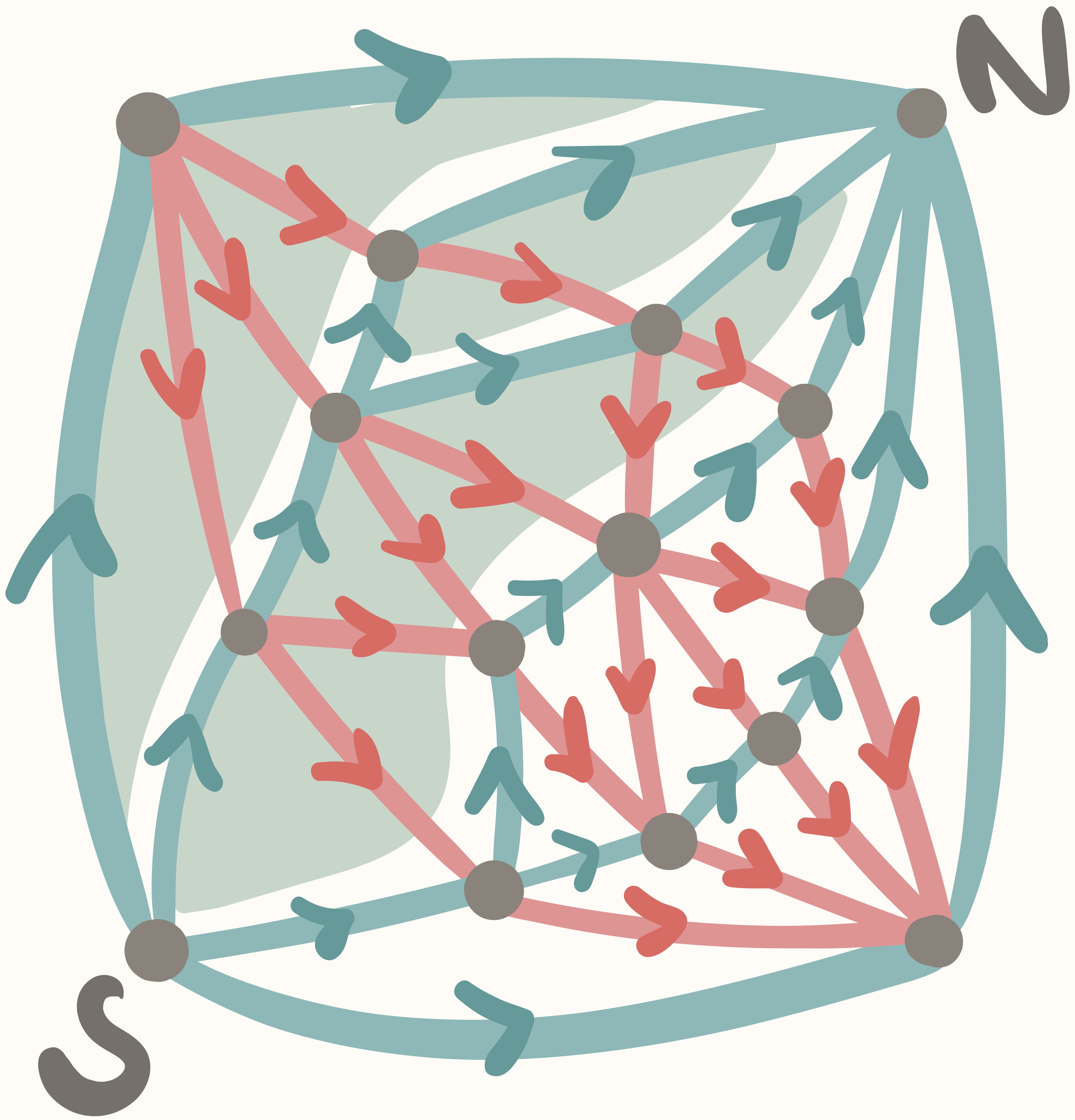
Specialization to transversal structures



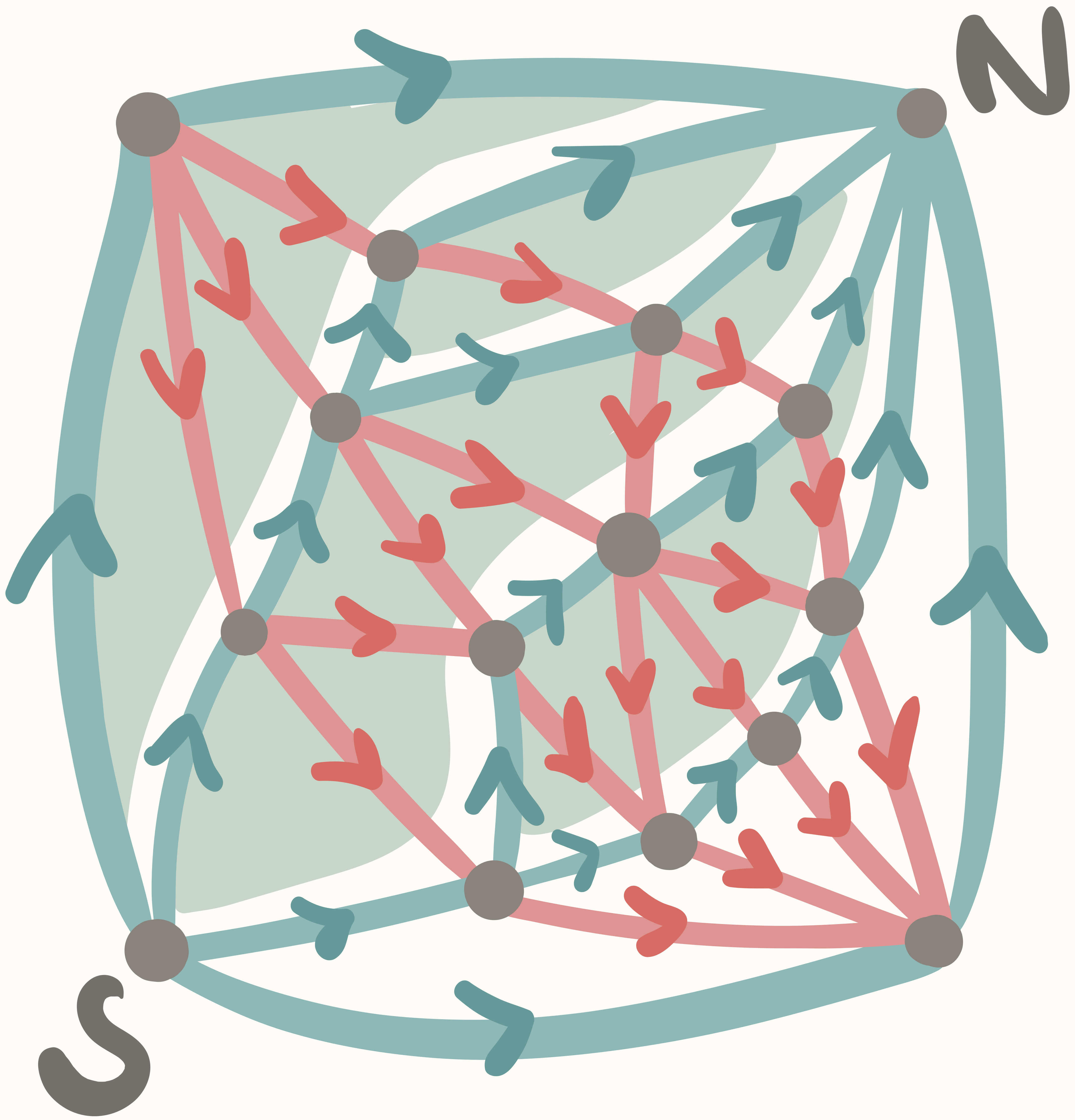
Specialization to transversal structures



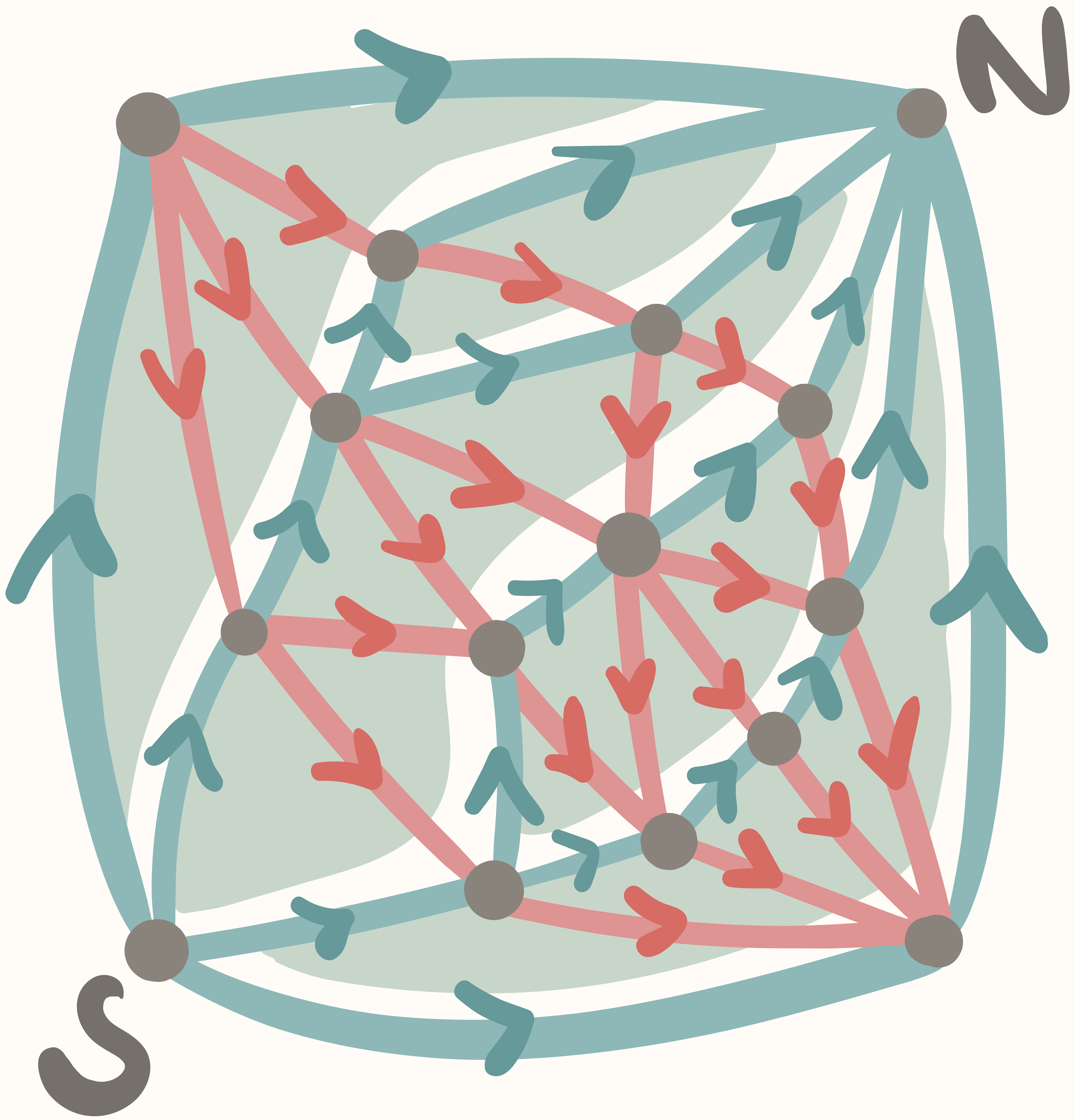
Specialization to transversal structures



Specialization to transversal structures



Specialization to transversal structures



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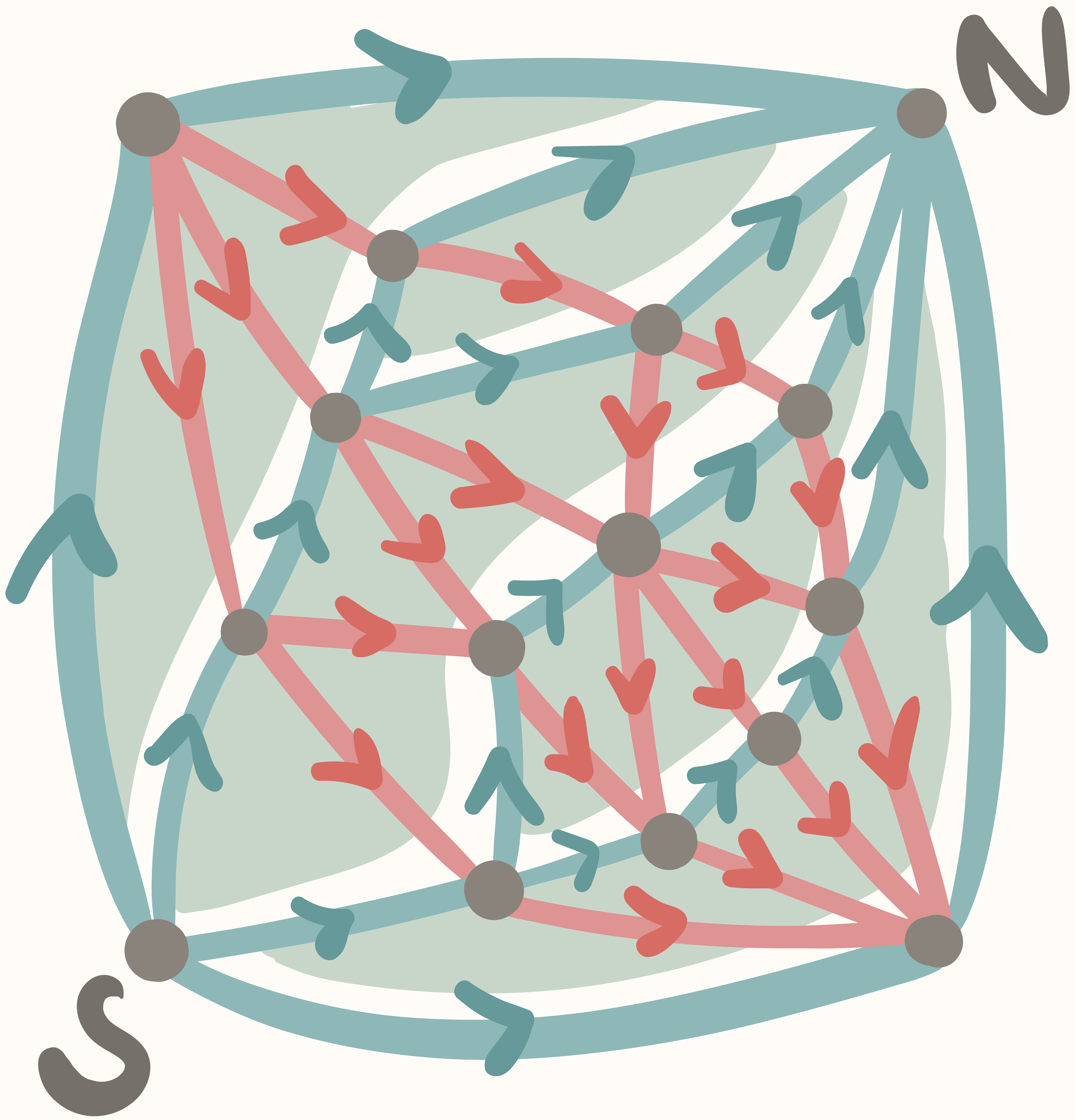

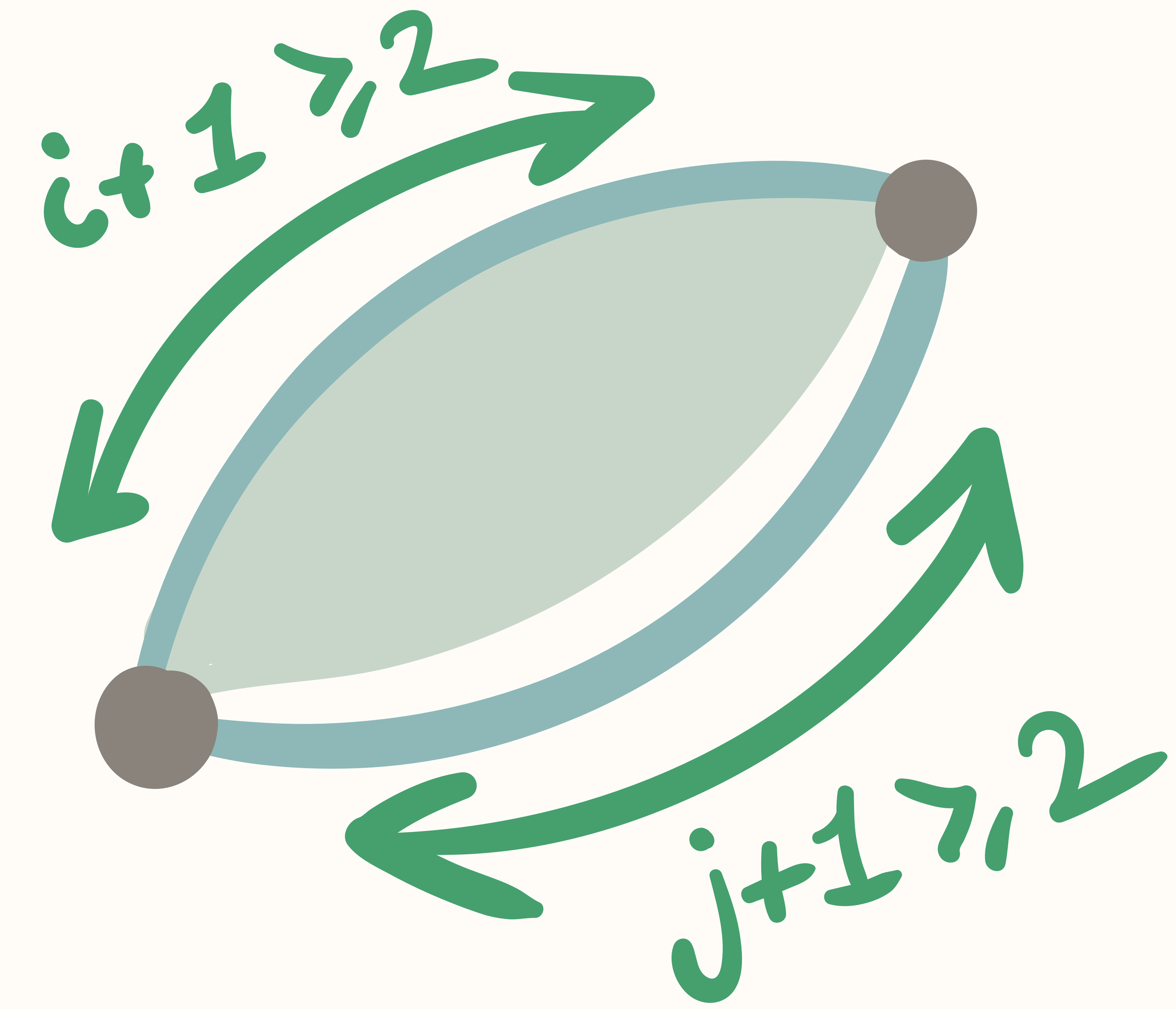

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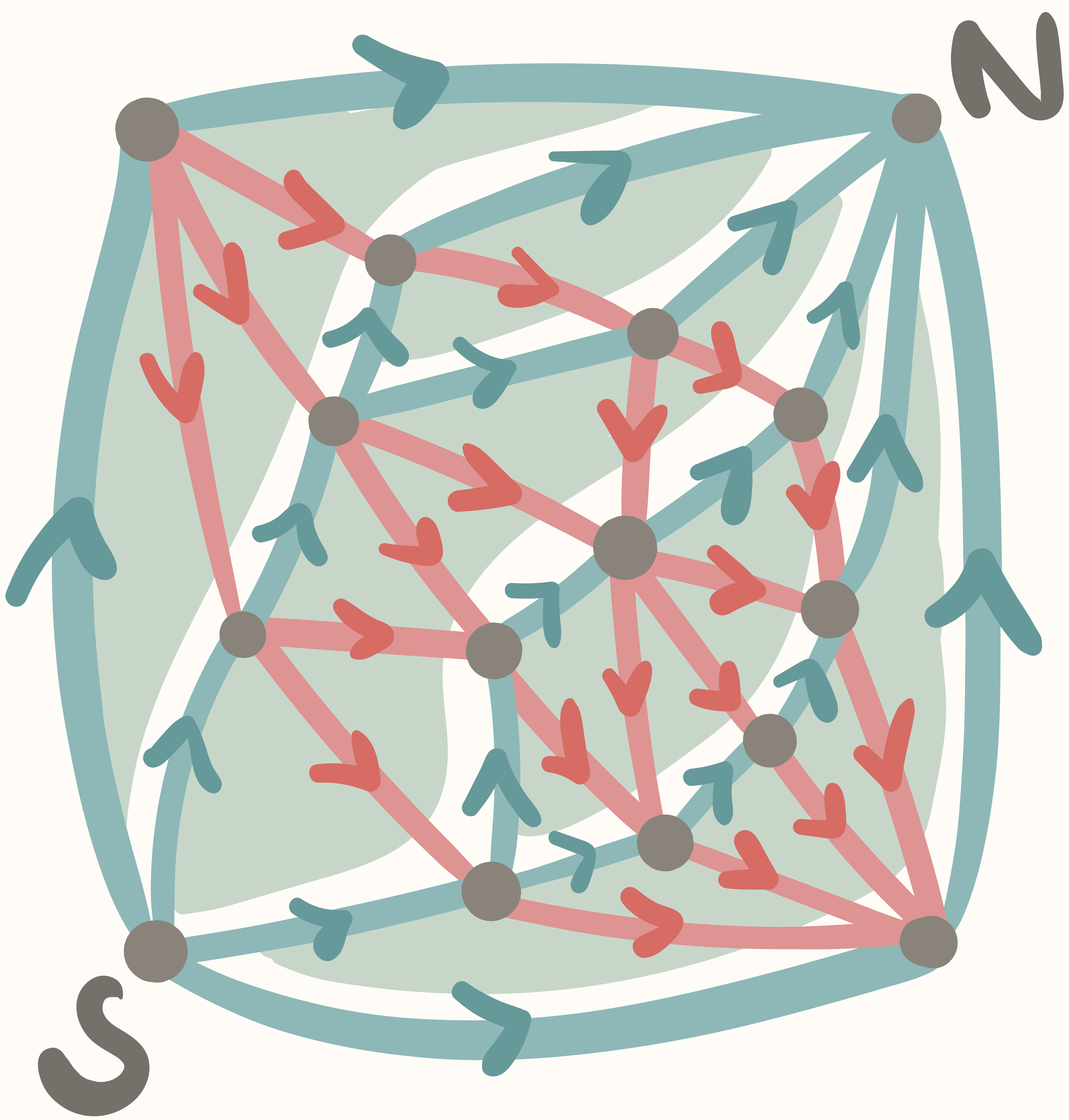
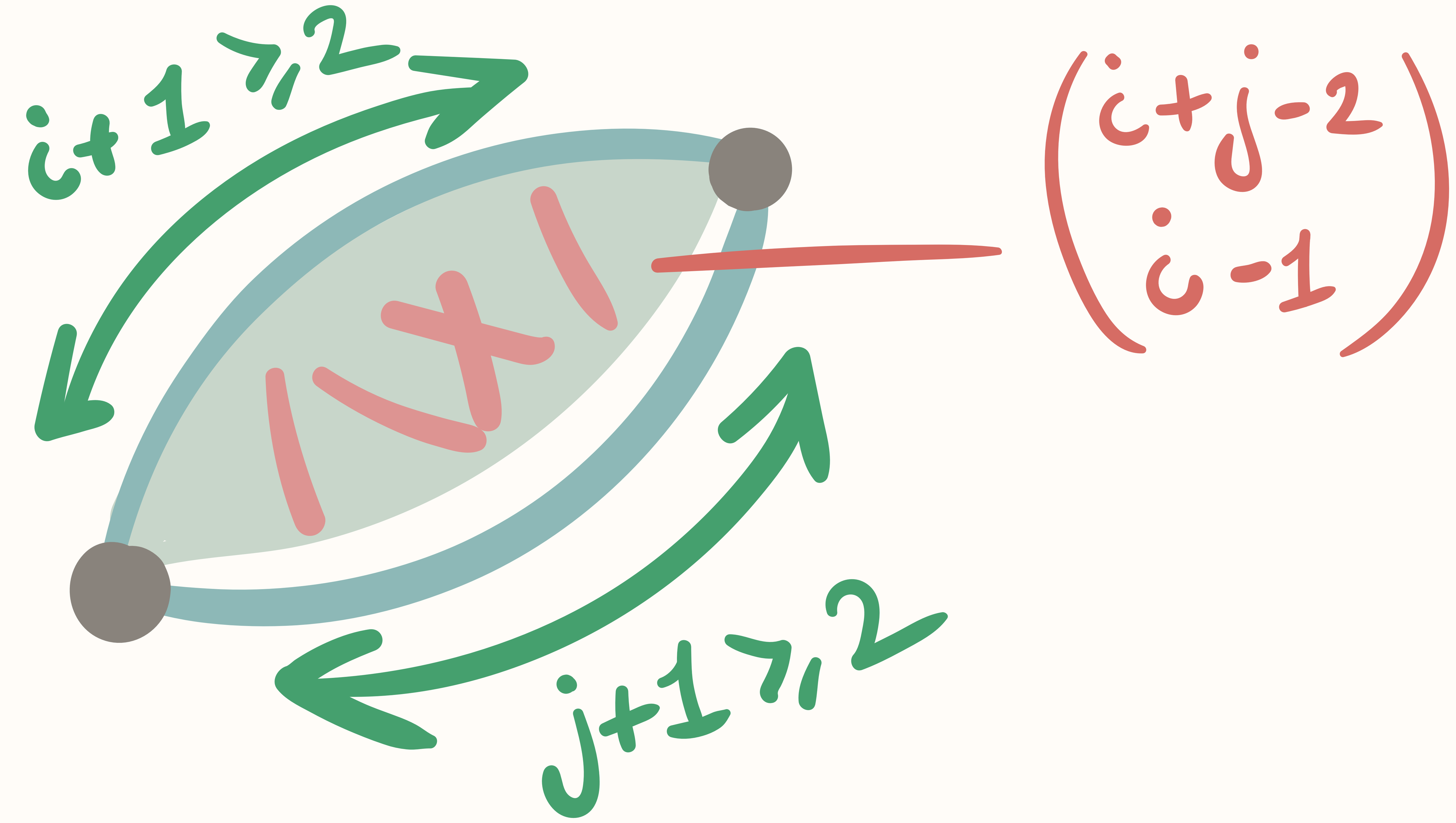
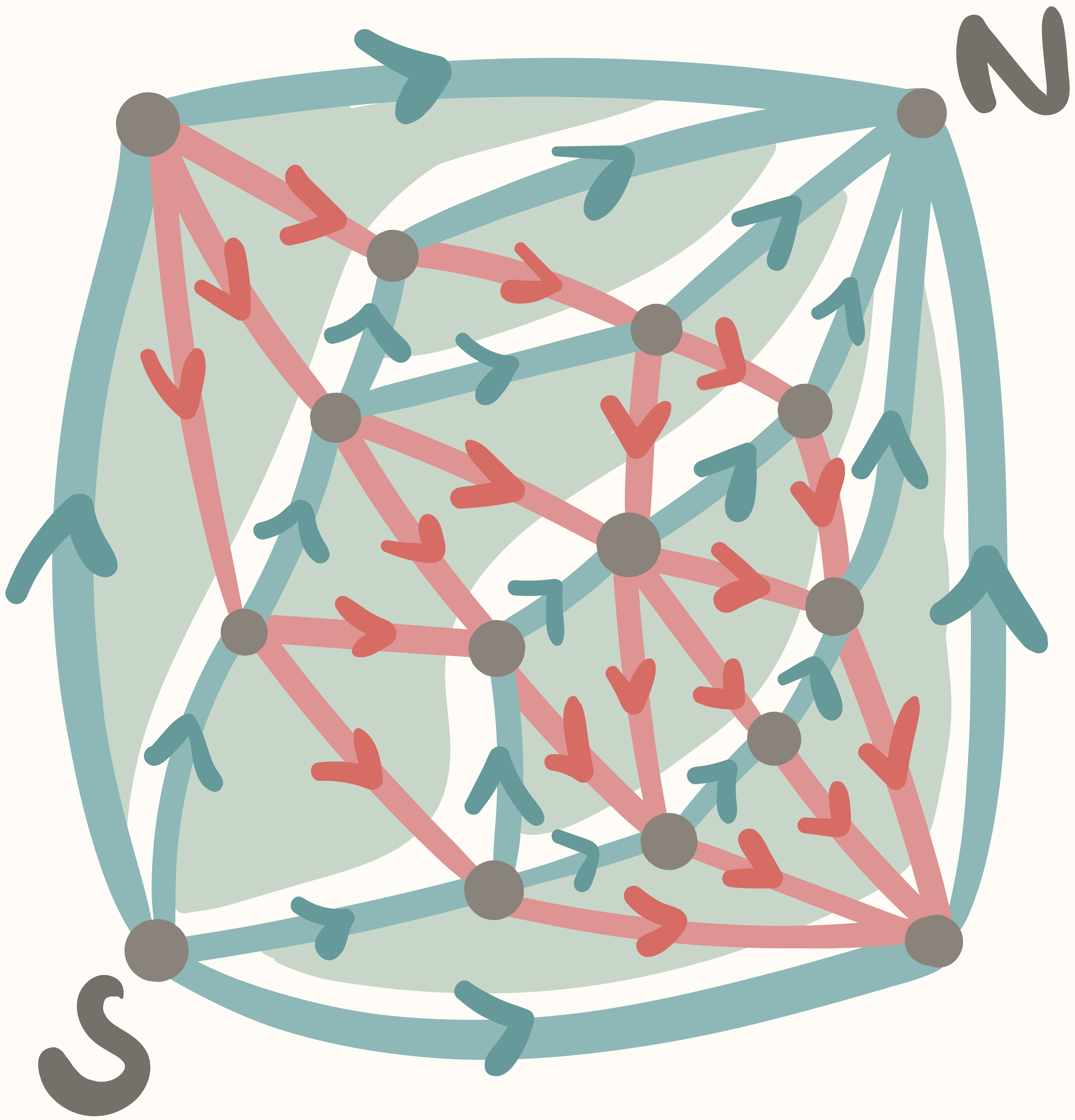
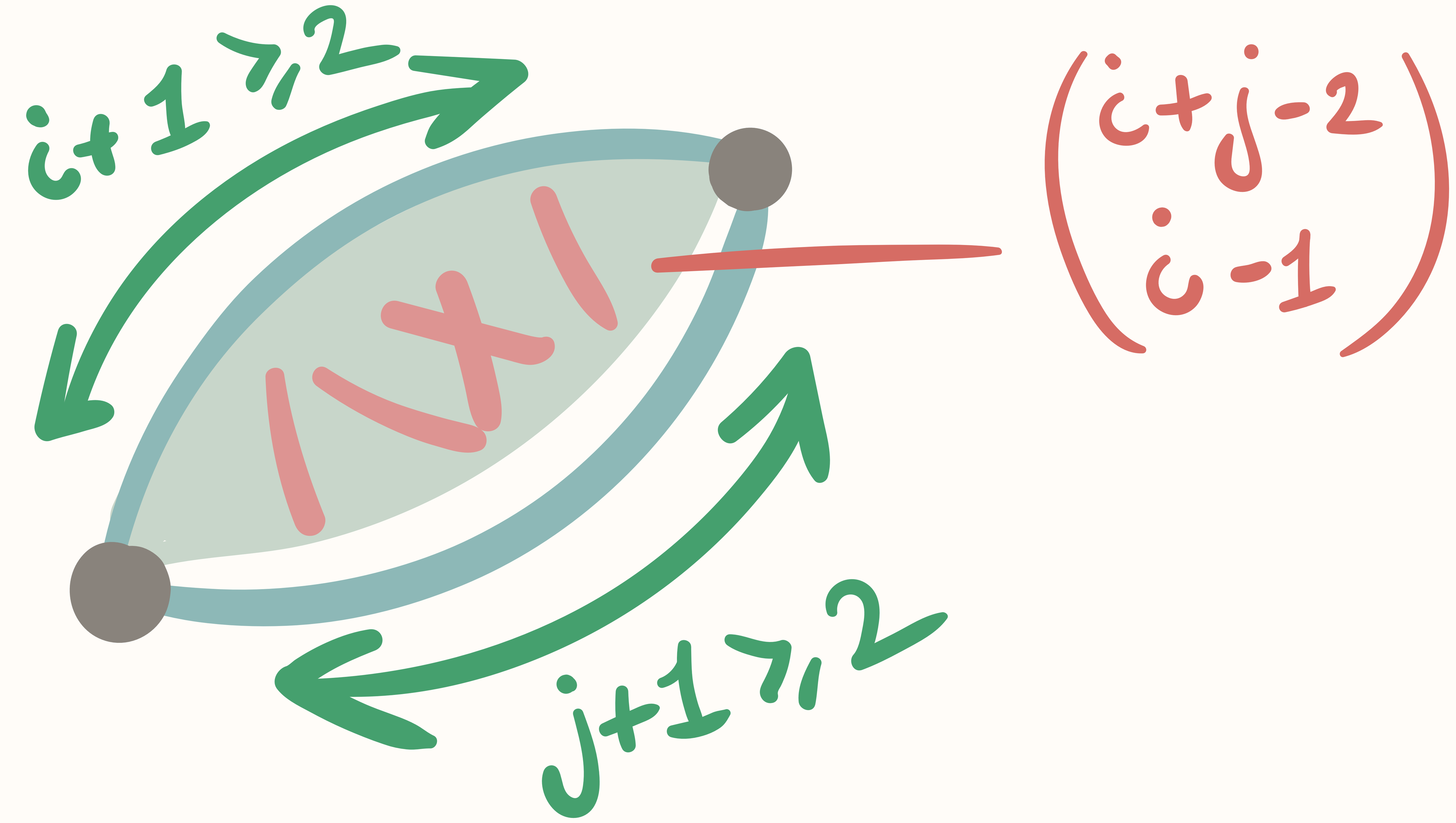
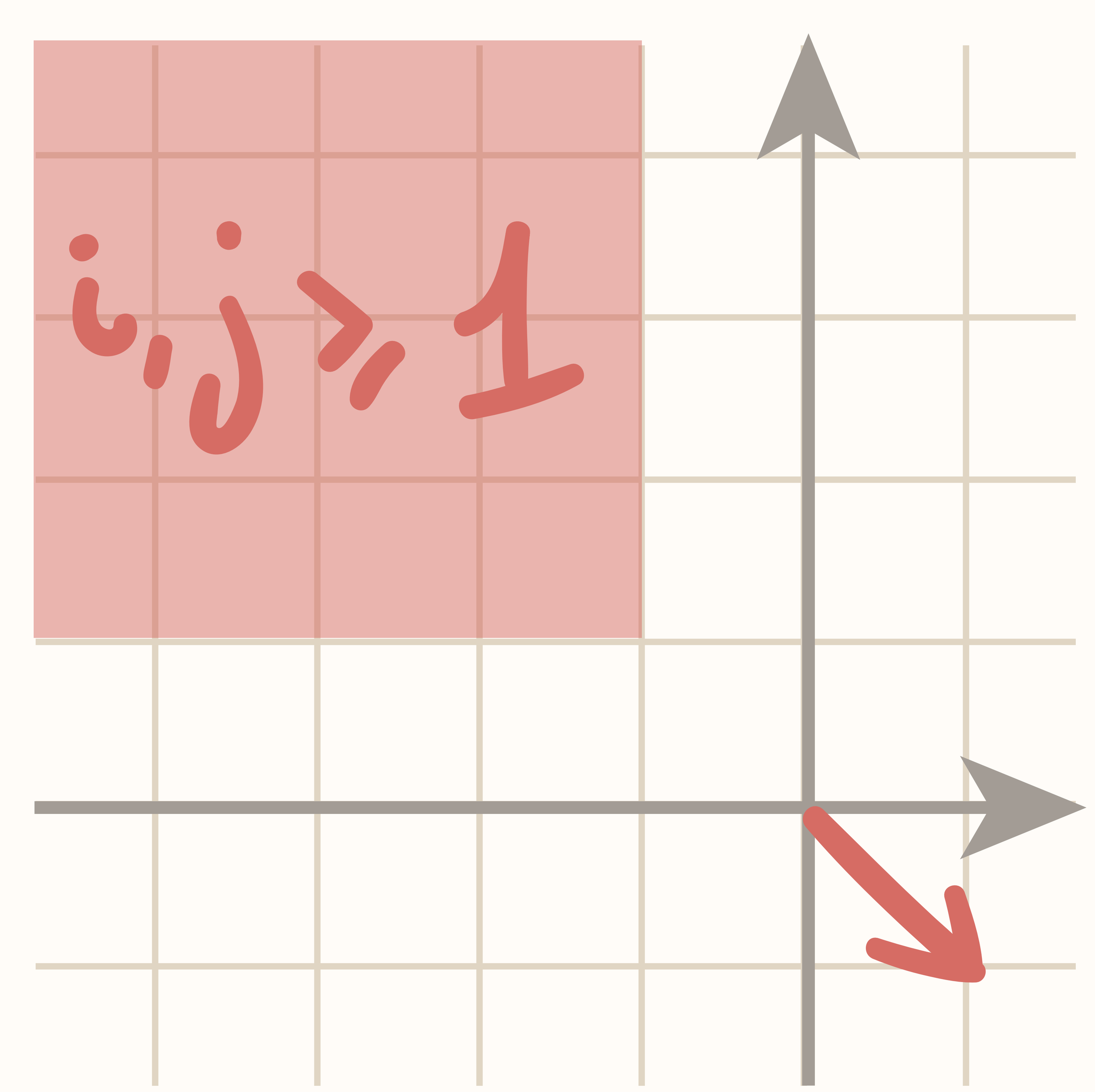




Specialization to transversal structures







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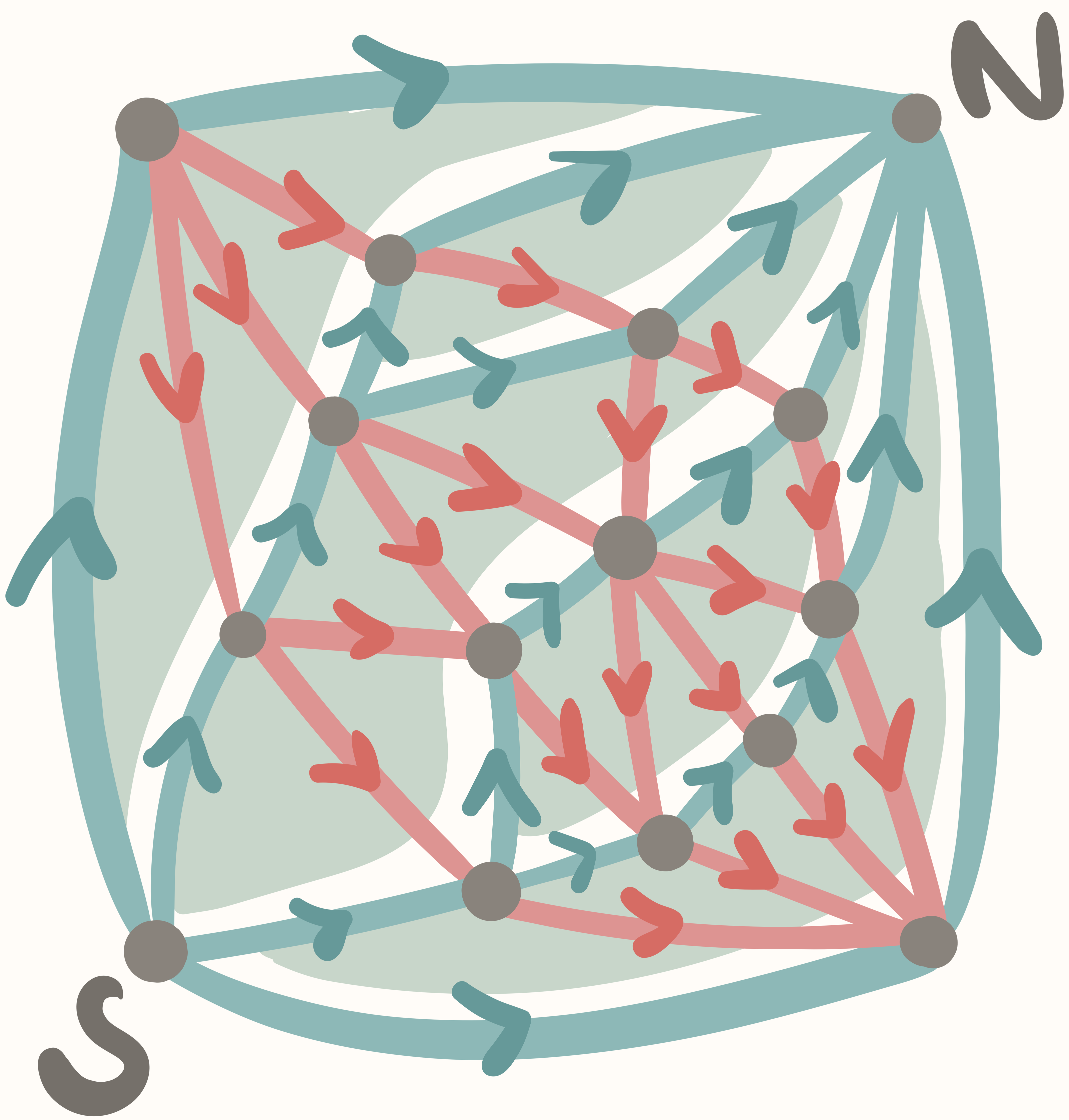

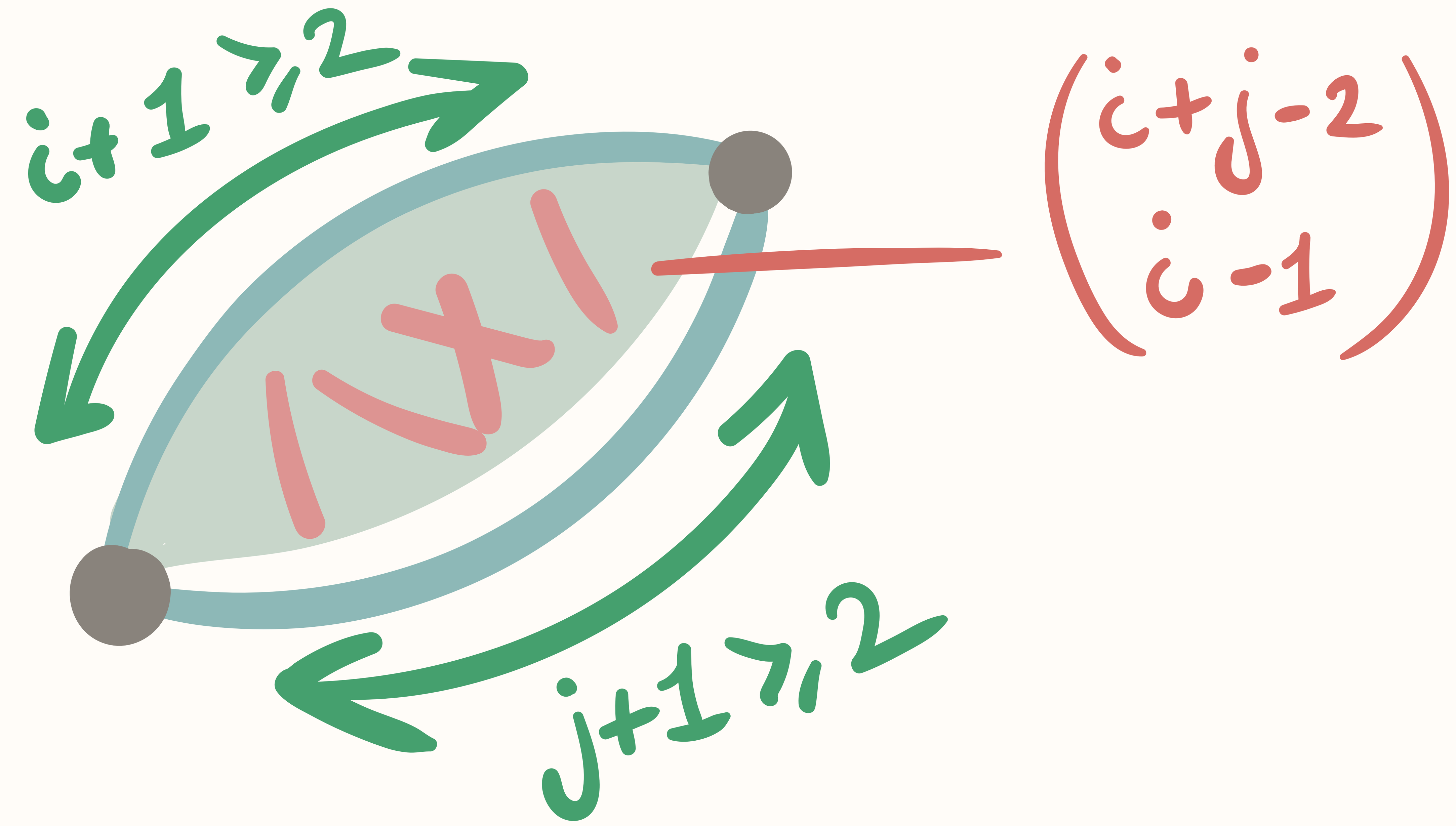

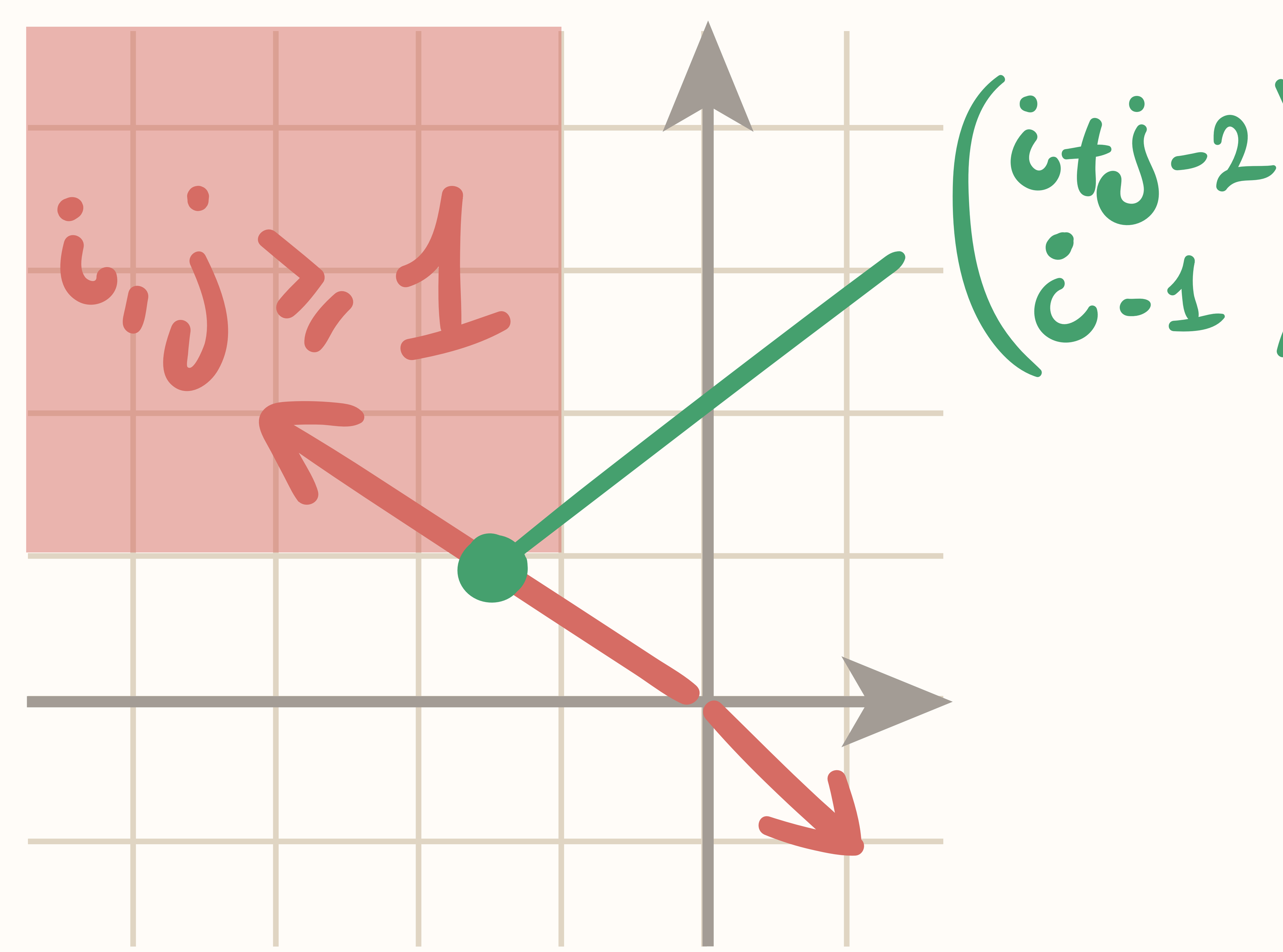

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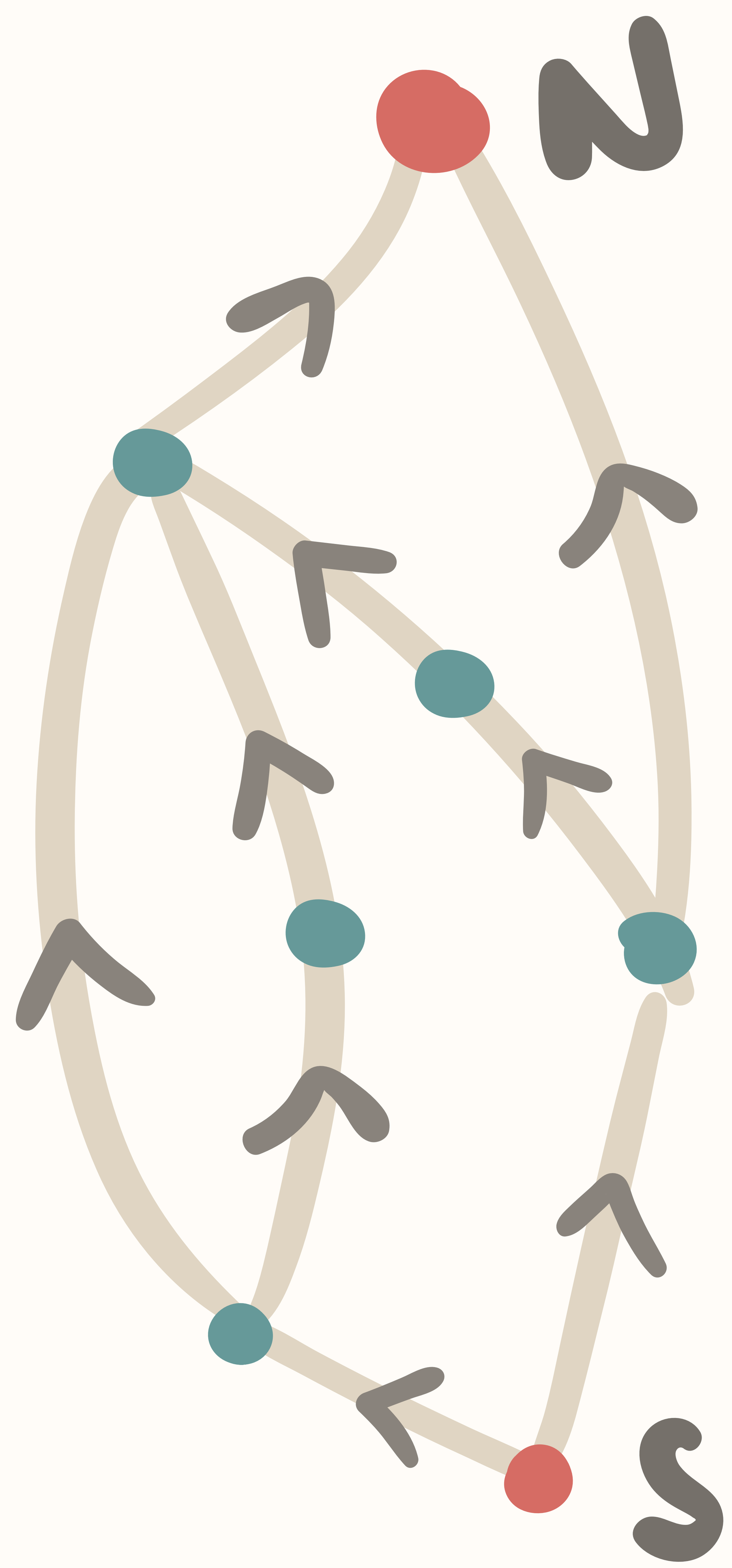
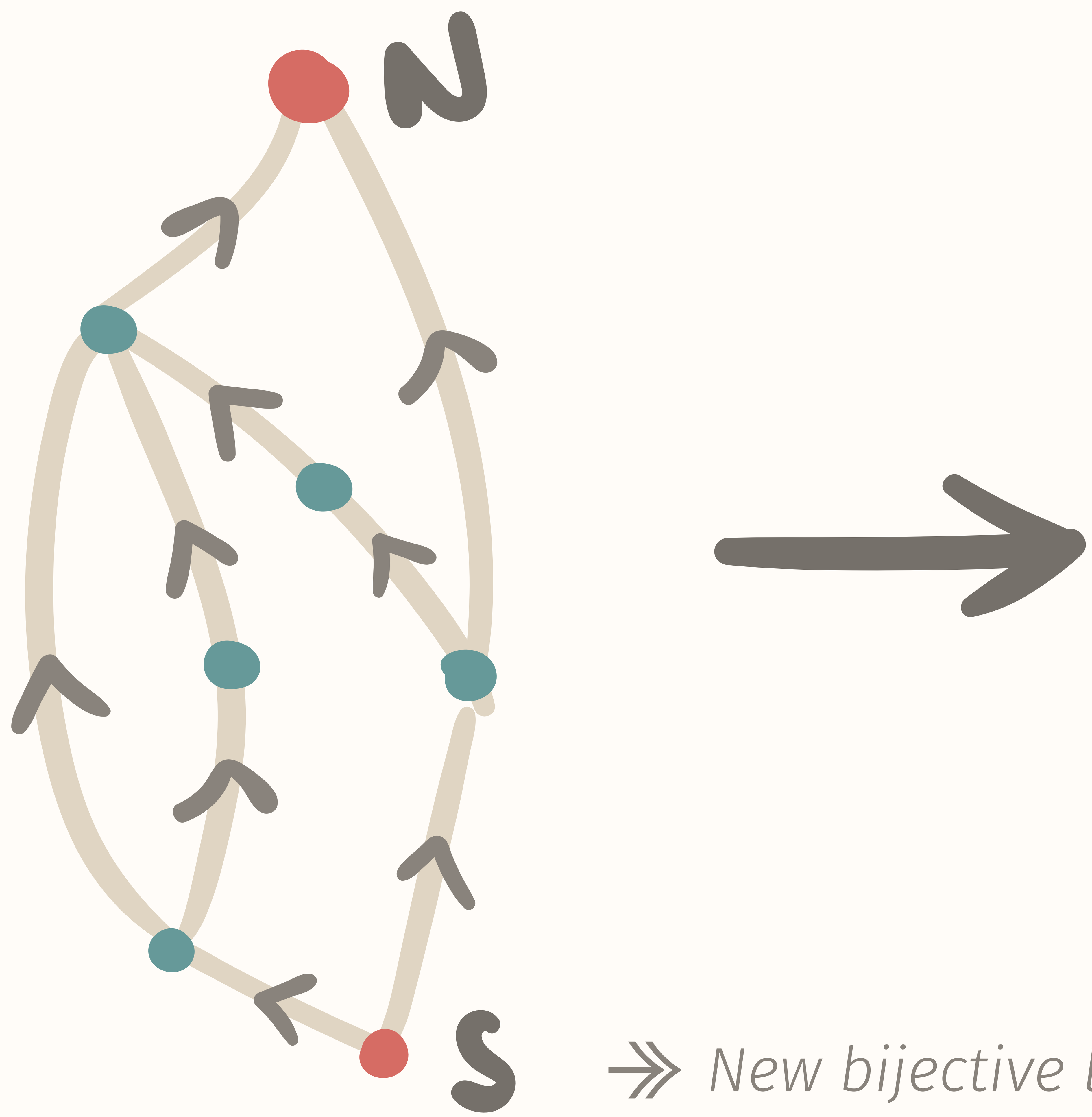
Specialization to Posets by vertices

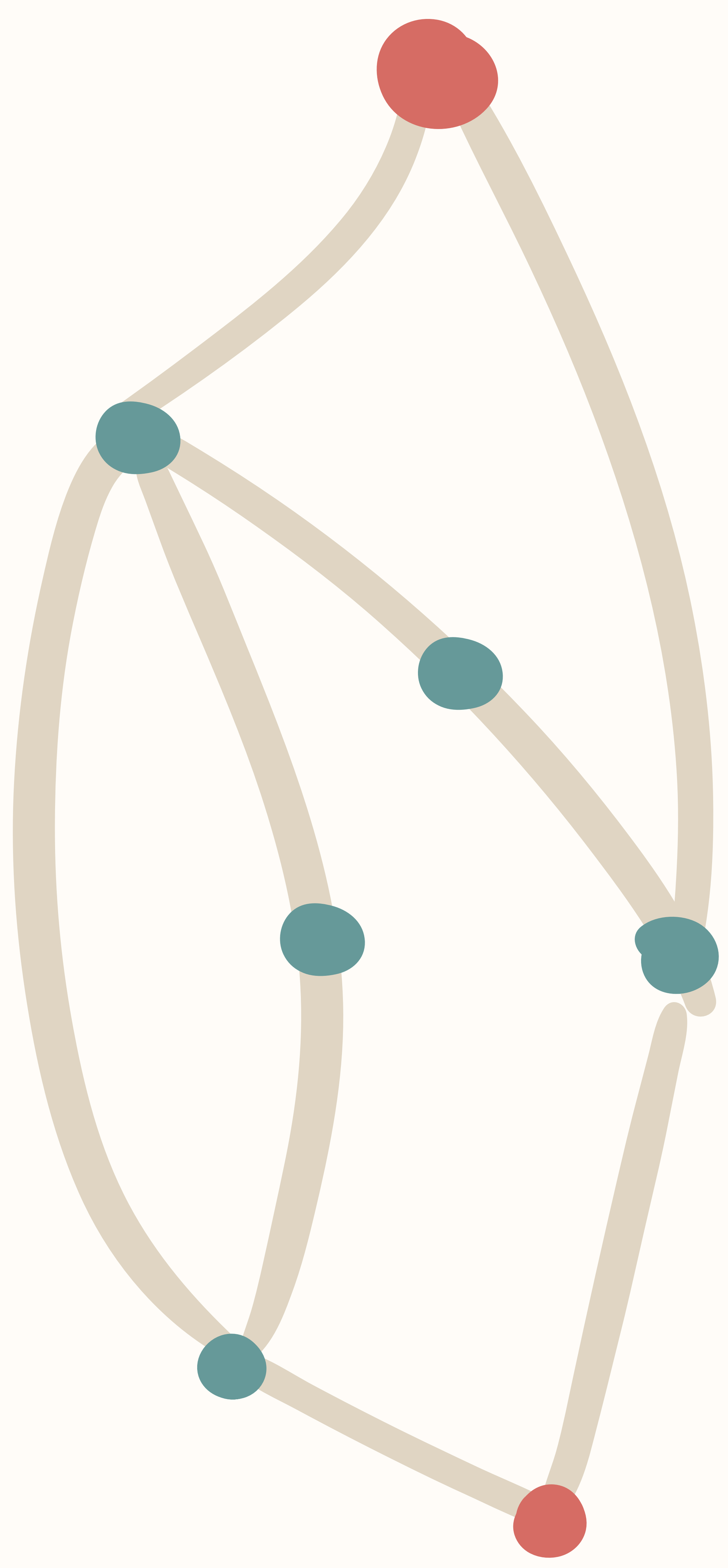
Bipolar orientation



Specialization to Posets by vertices

Bipolar orientation

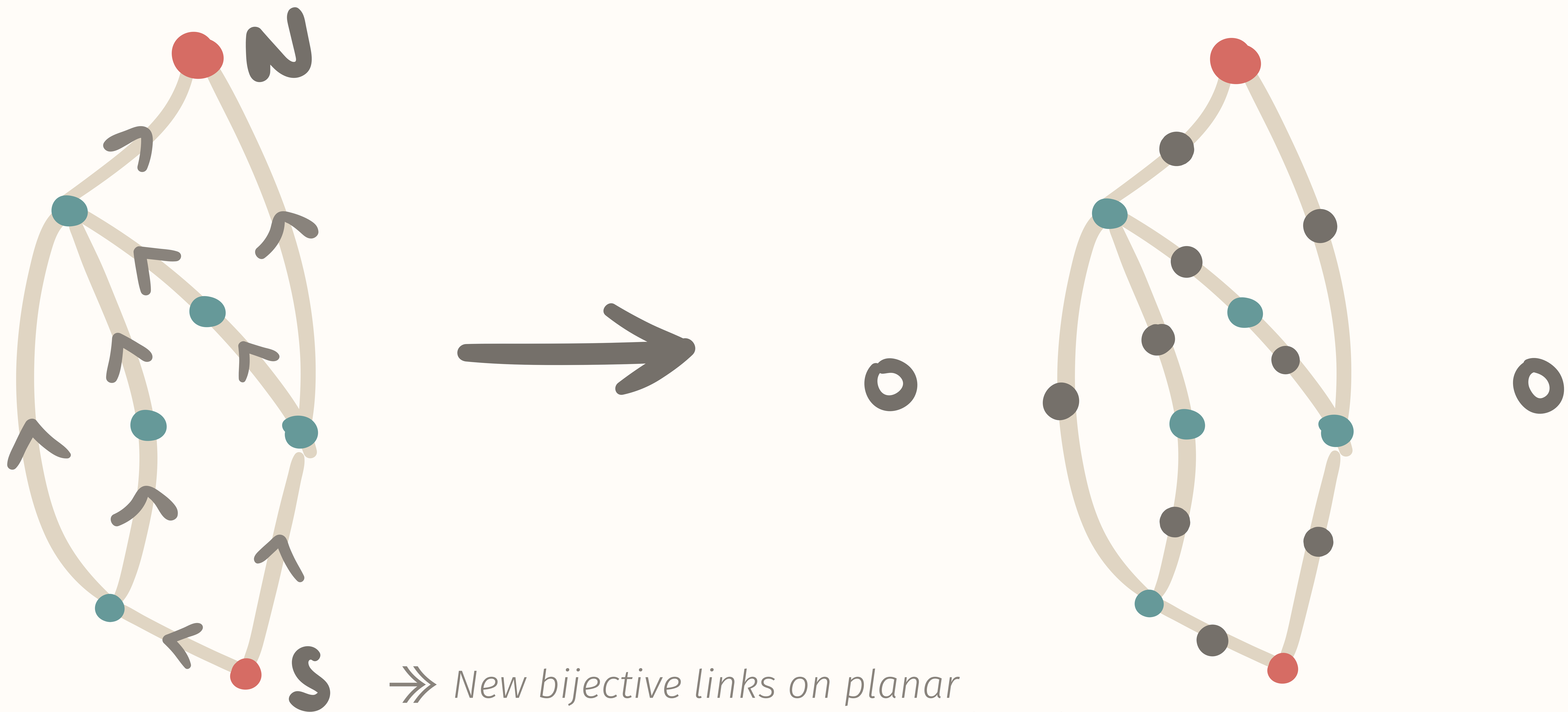




New bijective links on planar maps via orientation, E. Fusy (2010)

Specialization to Posets by vertices

Bipolar orientation



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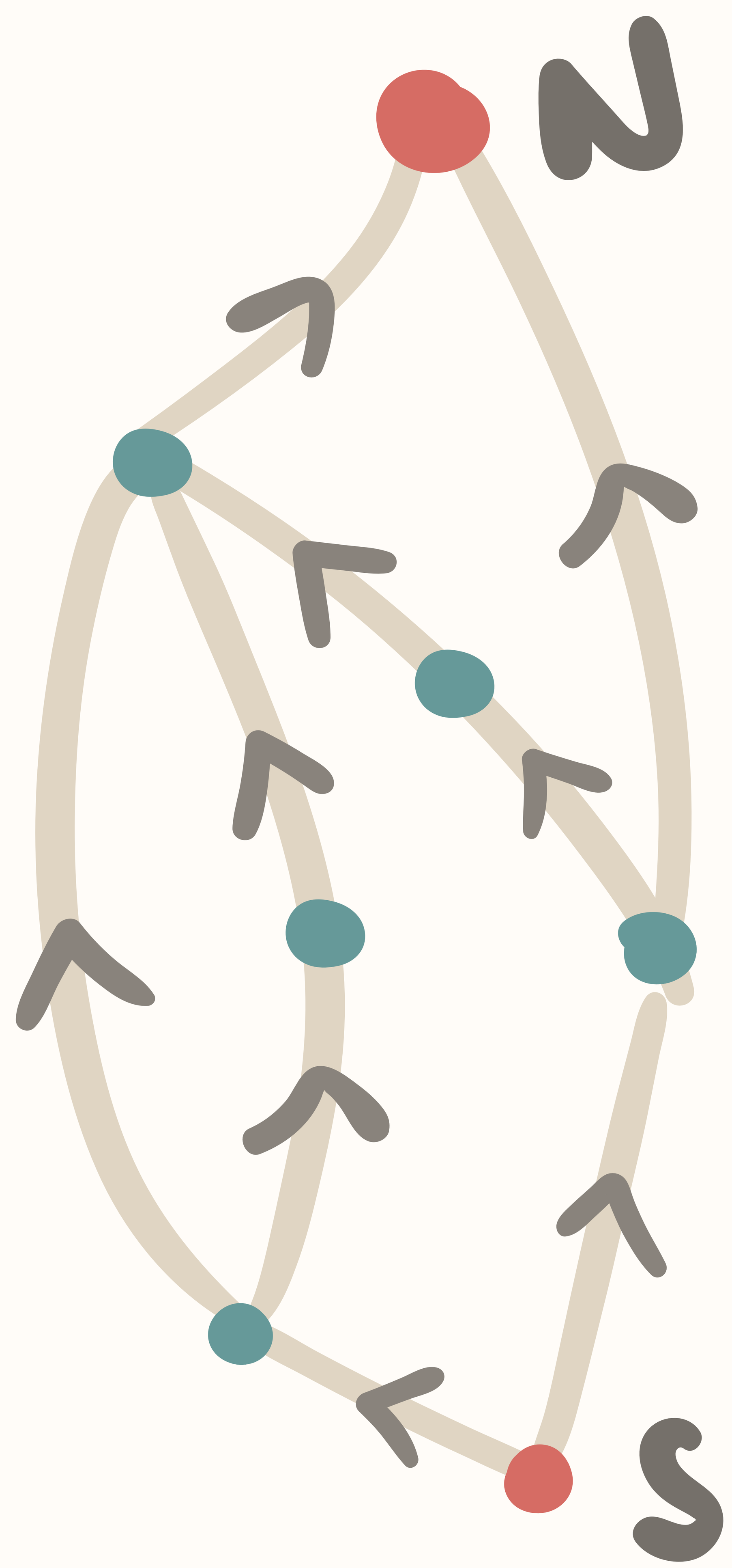
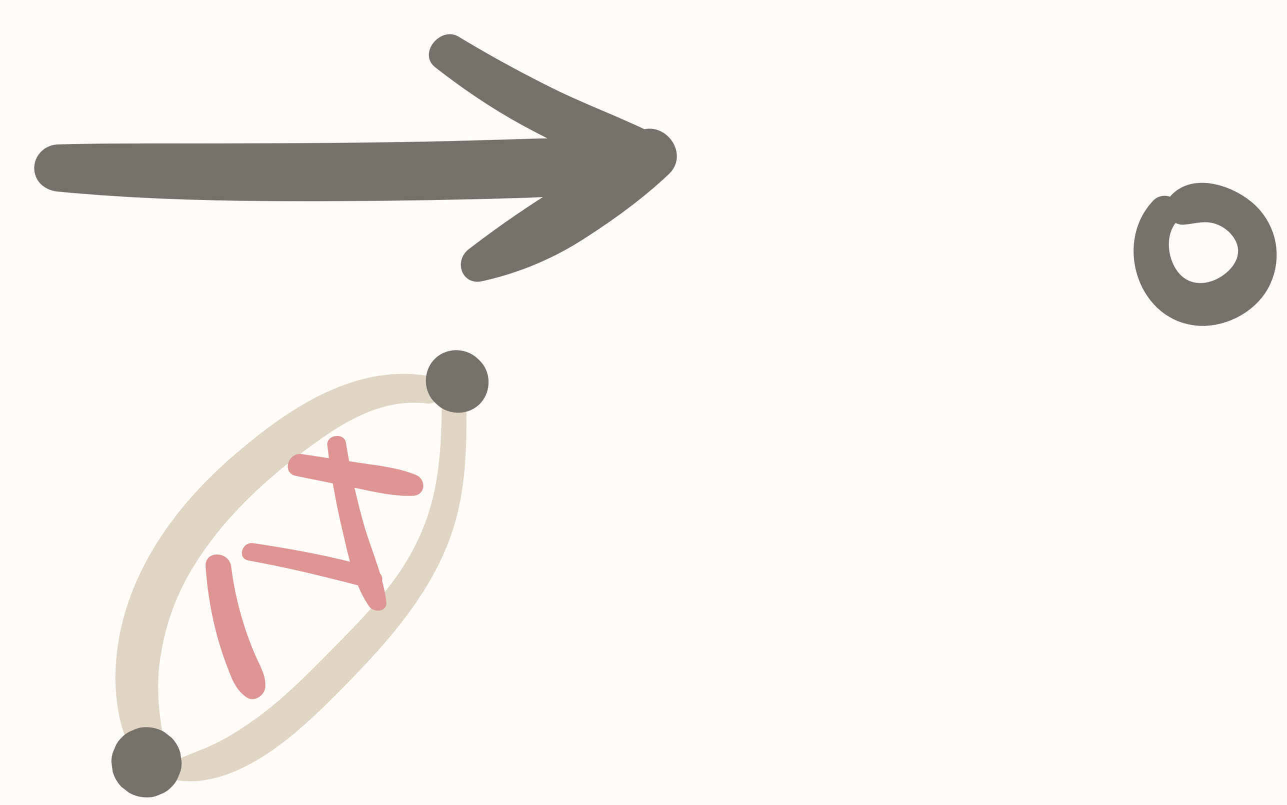
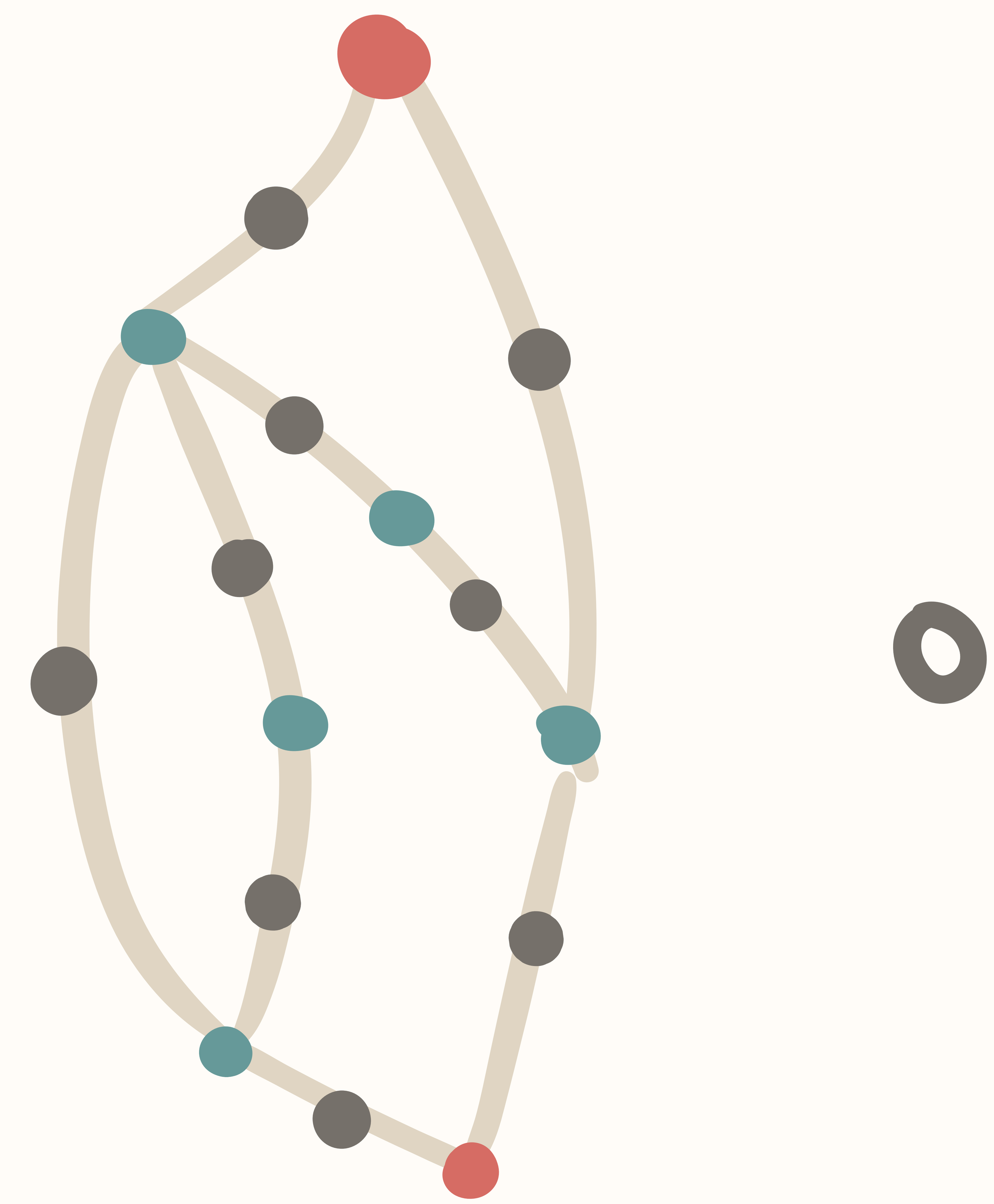
Specialization to Posets by vertices

Bipolar orientation







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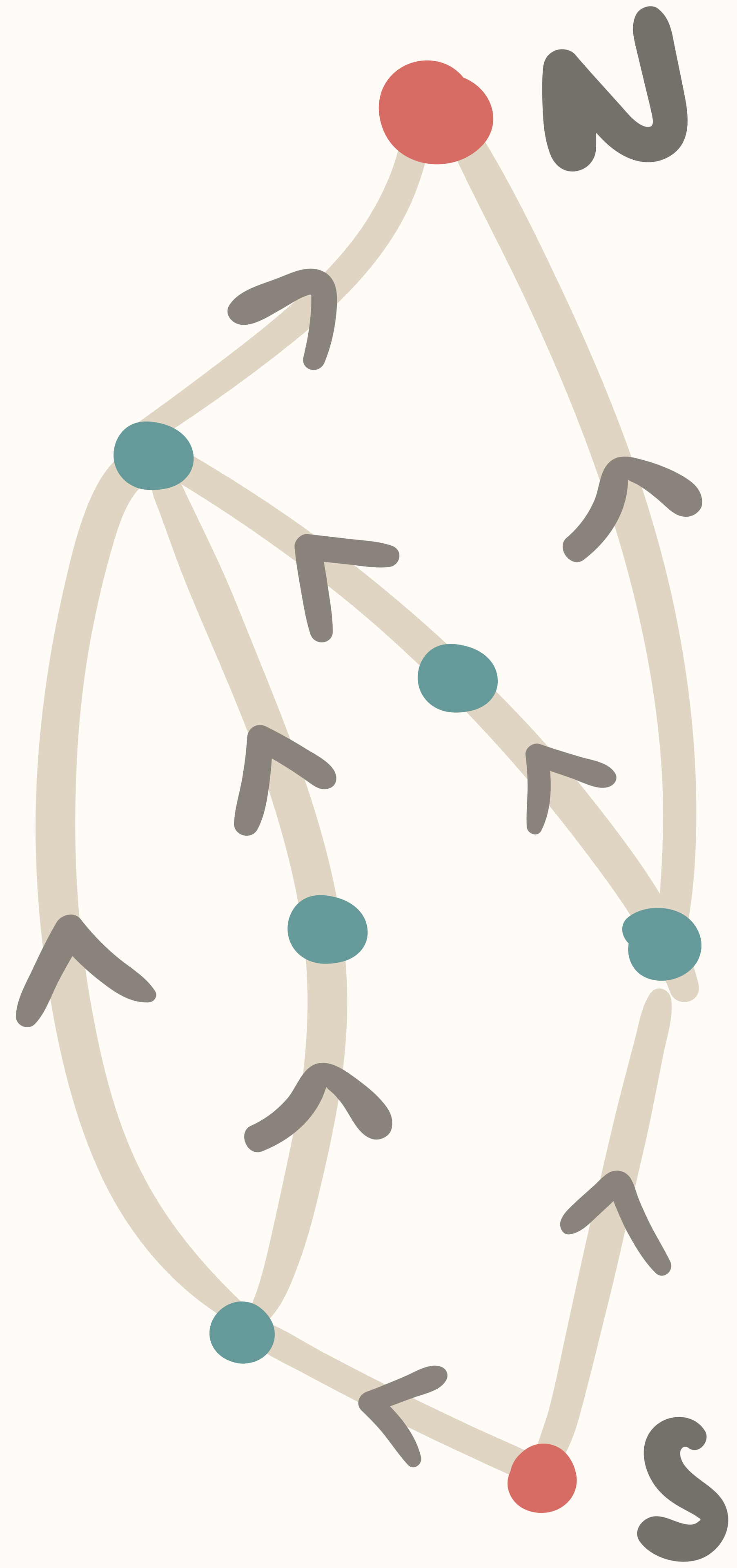
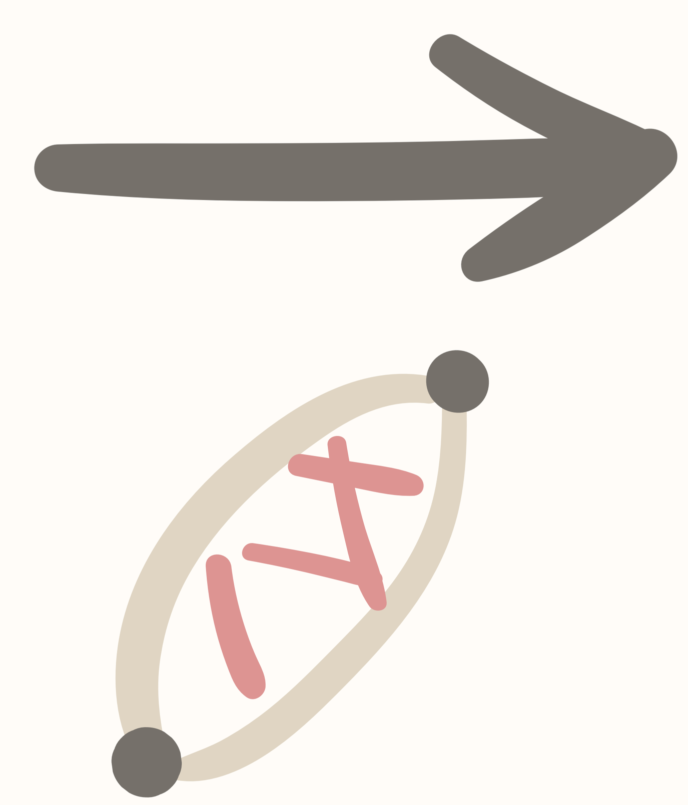
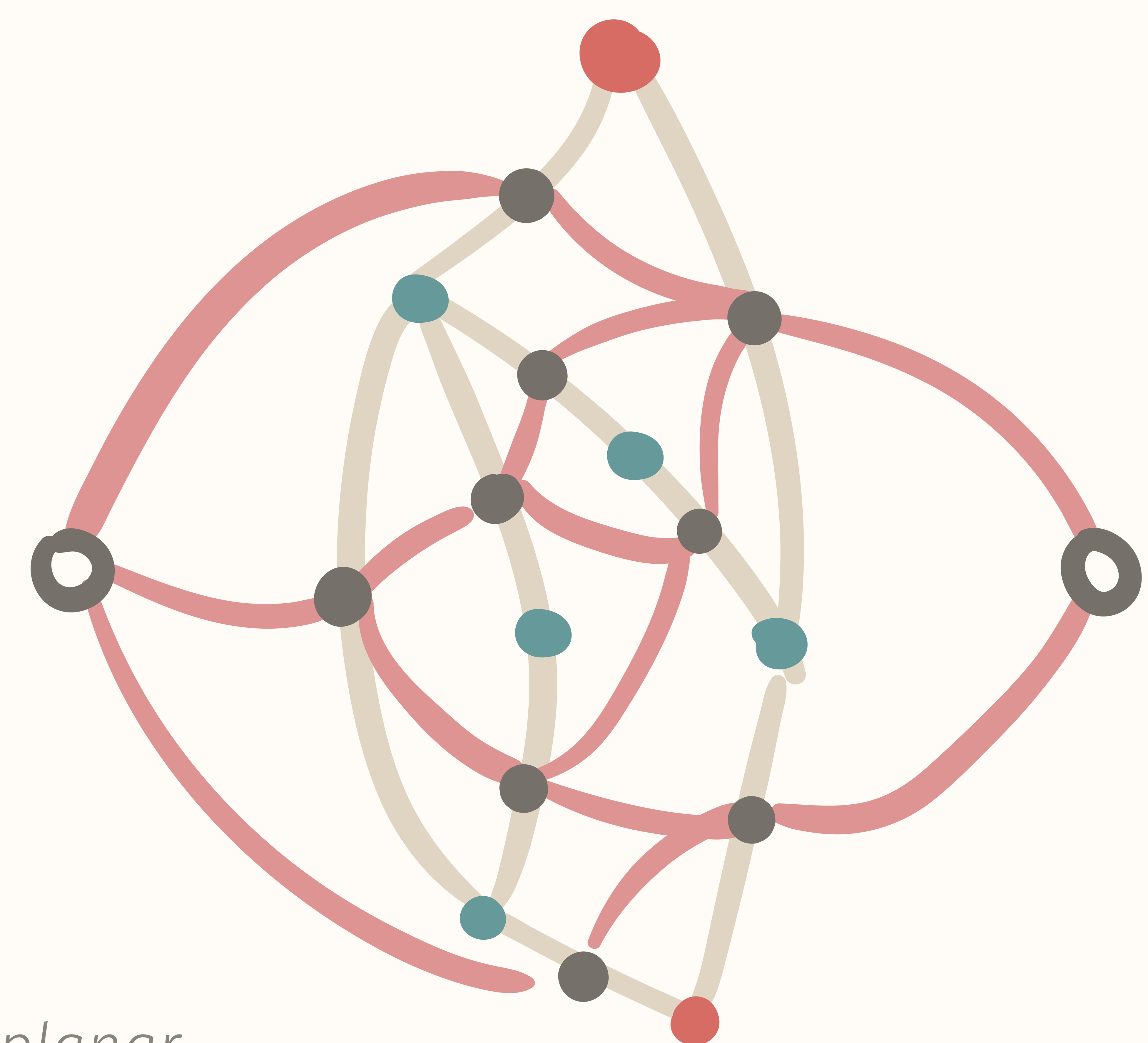
Specialization to Posets by vertices

Bipolar orientation



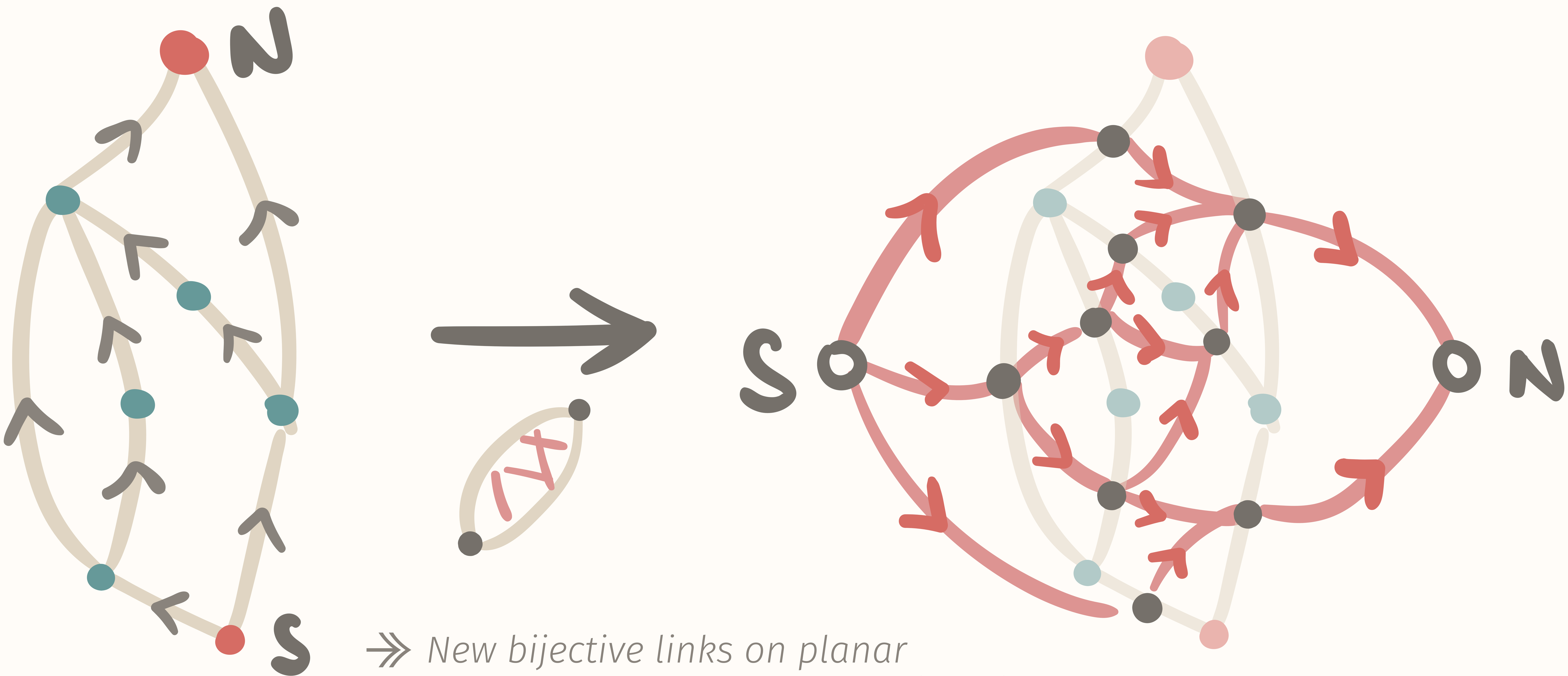




⇛ *New bijective links on planar maps via orientation*, E. Fusy (2010)

Specialization to Posets by vertices

Bipolar orientation *poset*



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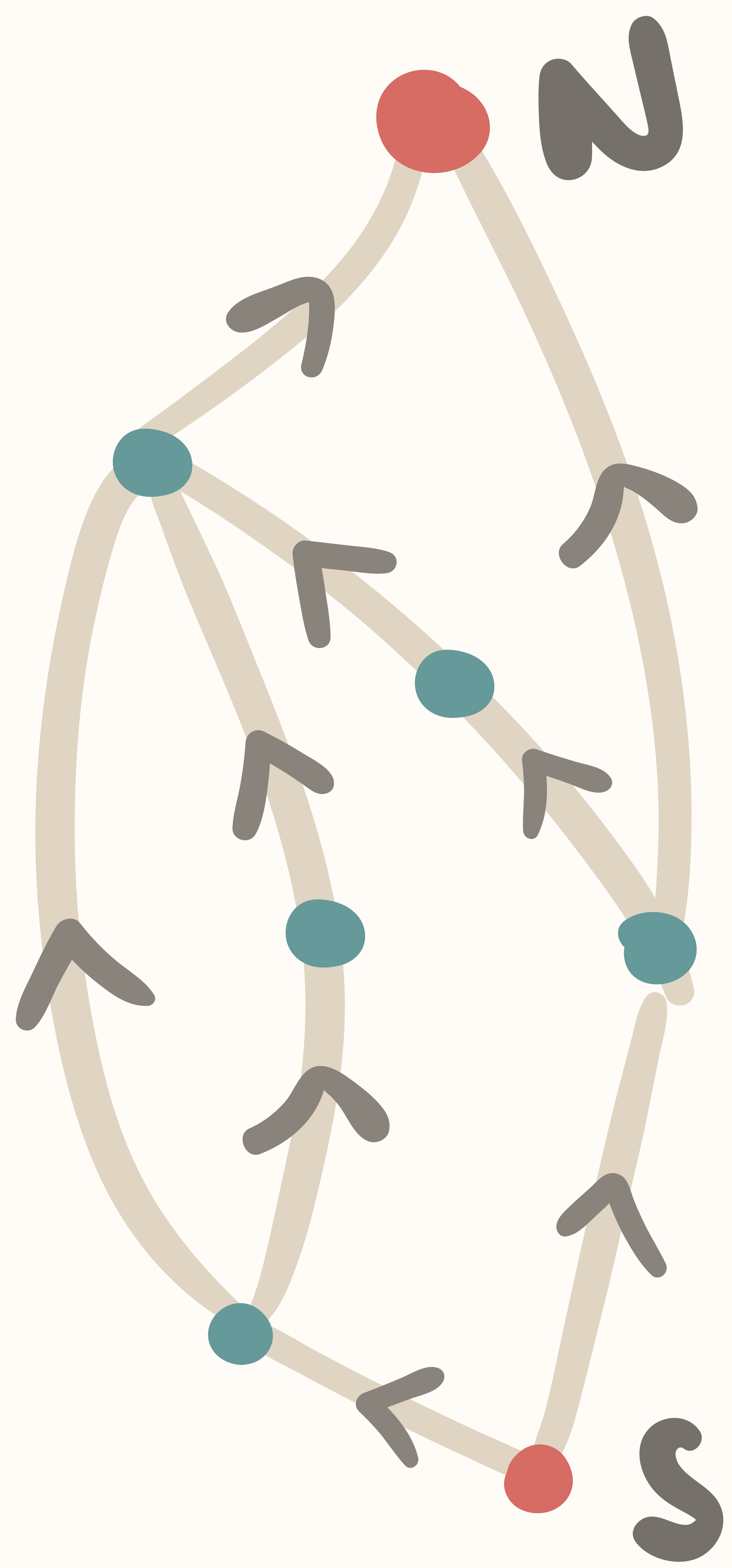
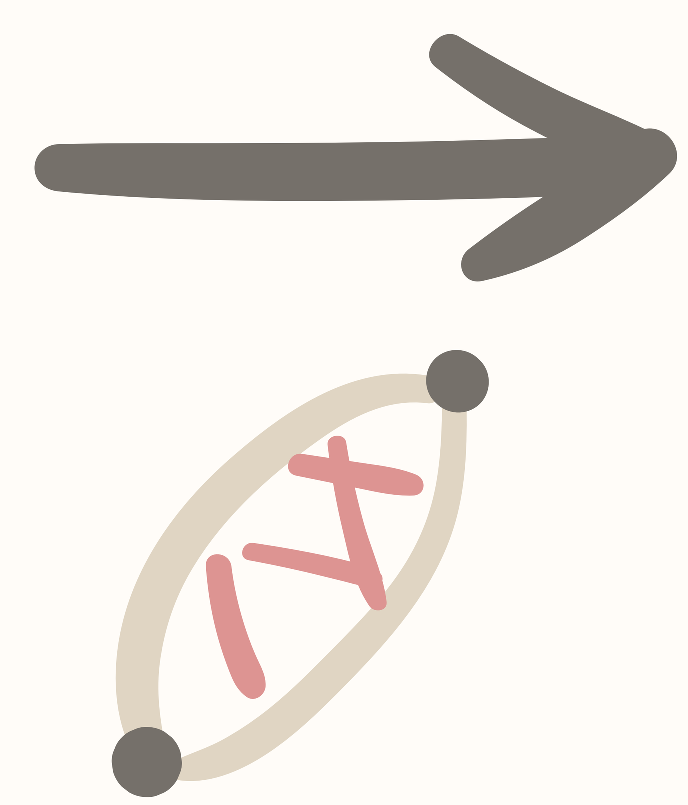
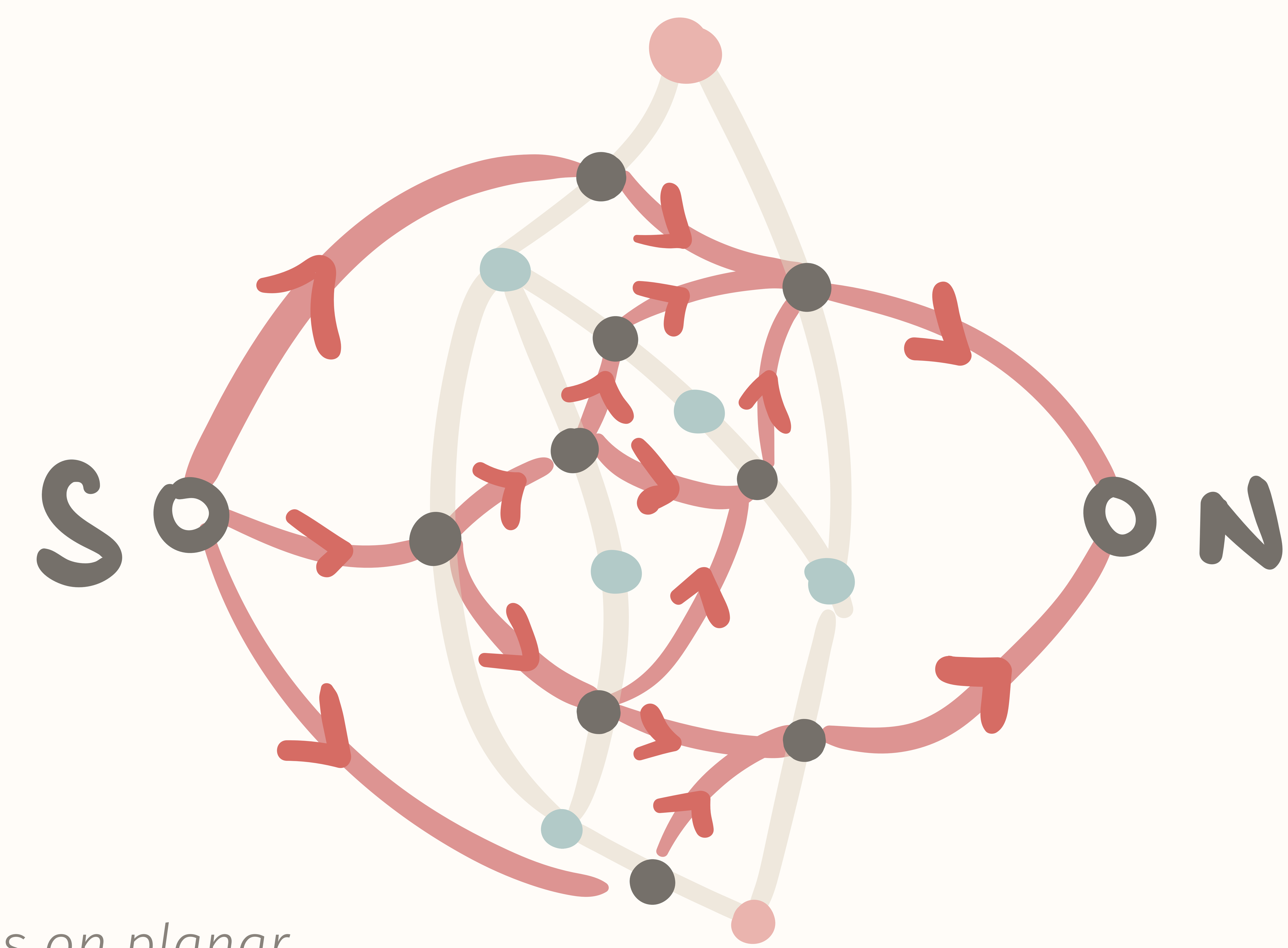
Specialization to Posets by vertices

Bipolar orientation





poset



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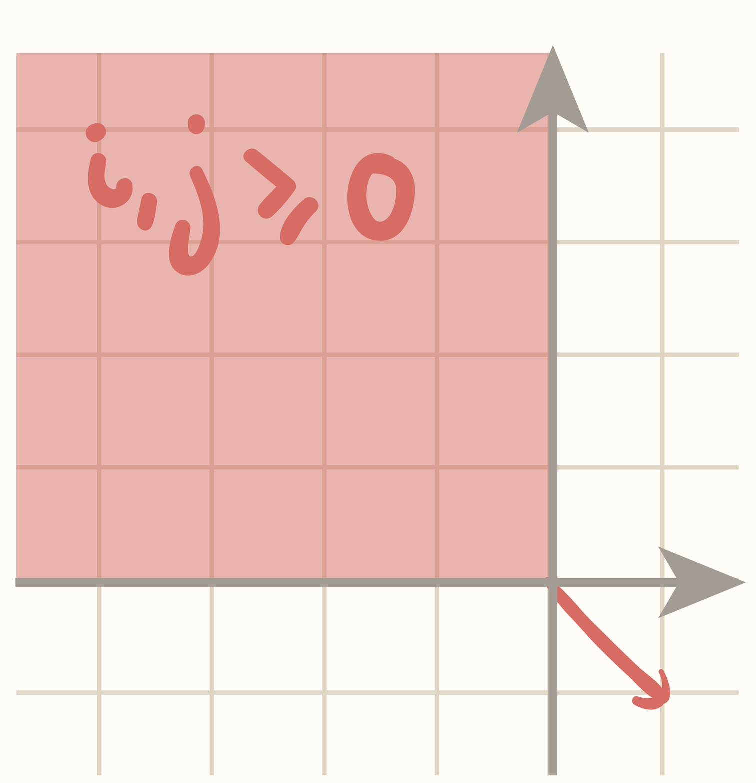
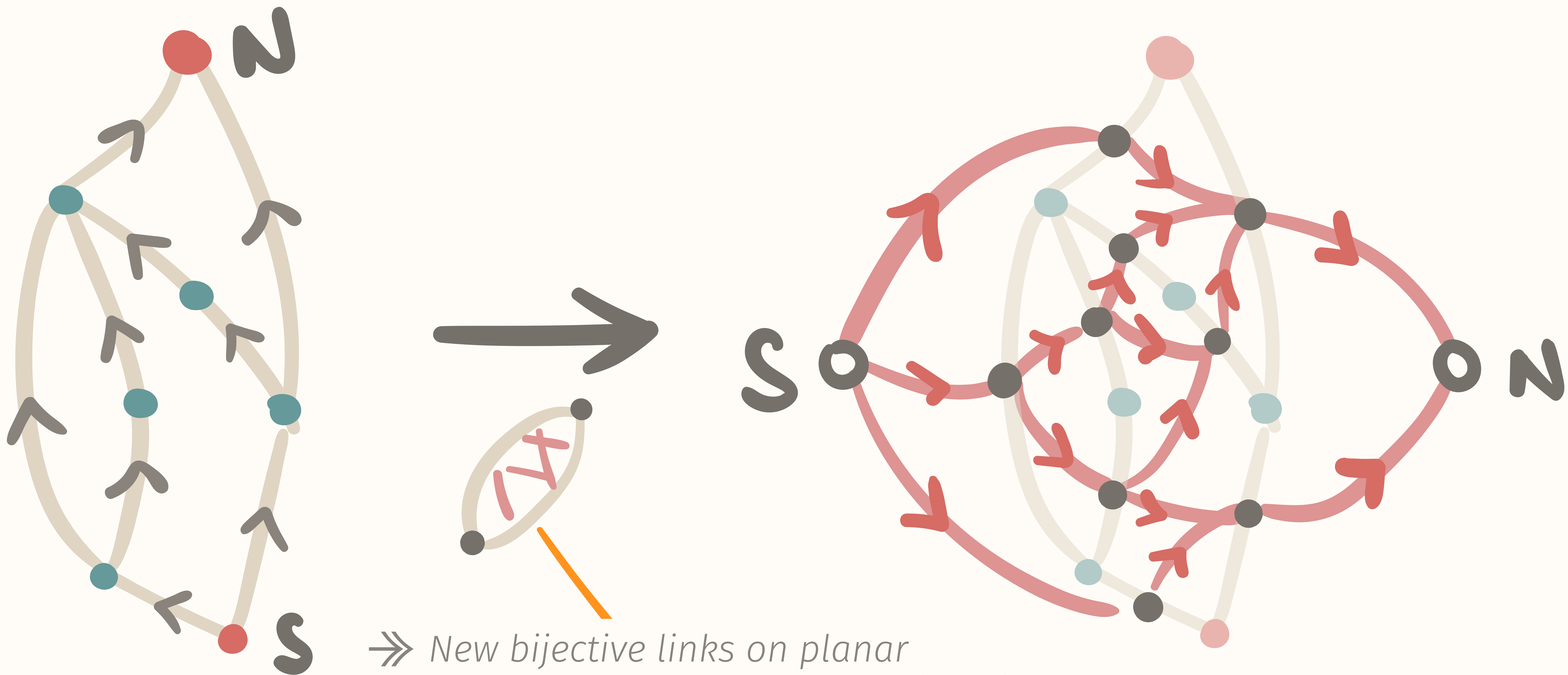


Specialization to Posets by vertices

Bipolar orientation *poset*



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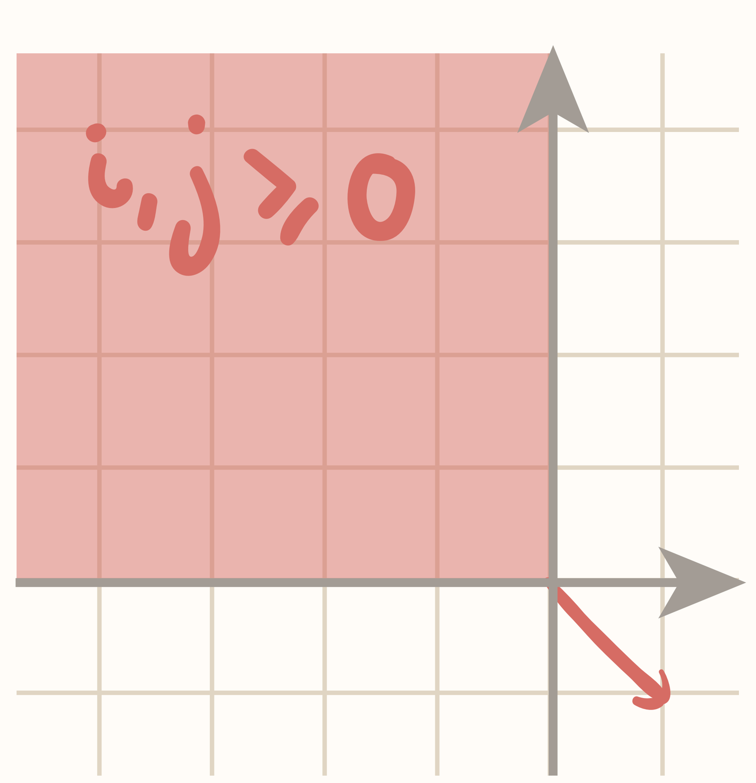
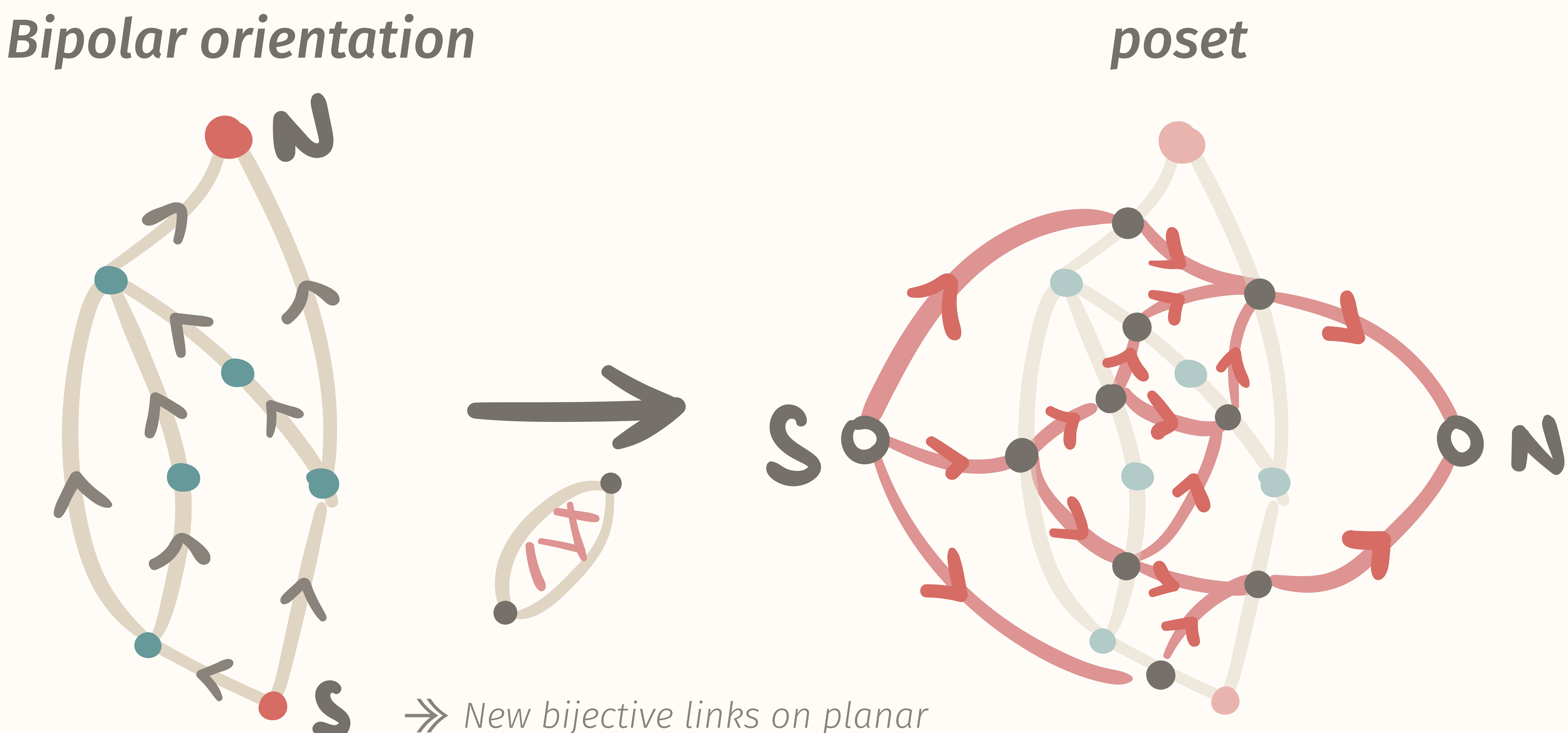


$\binom{i+j}{i}$

Specialization to Posets by vertices



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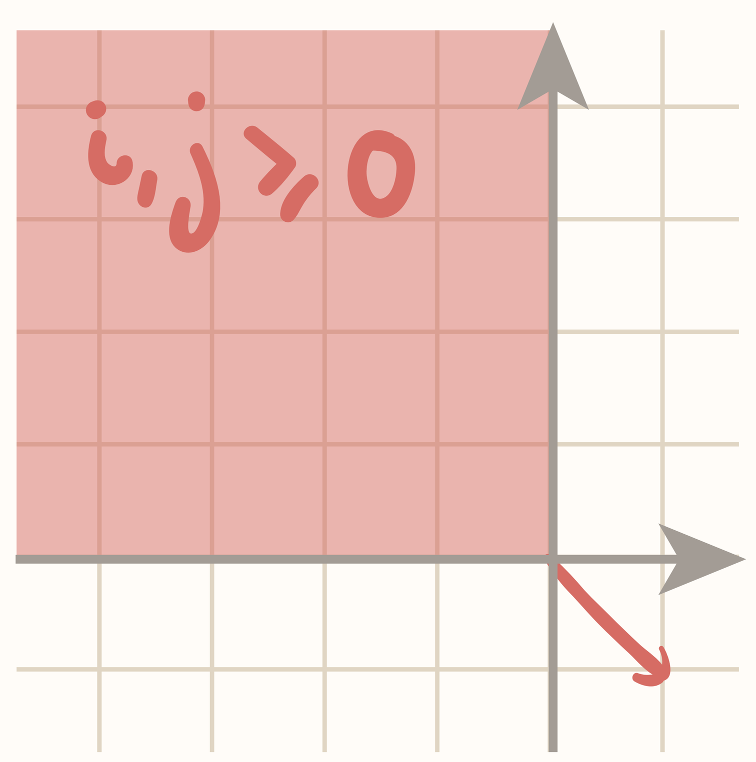
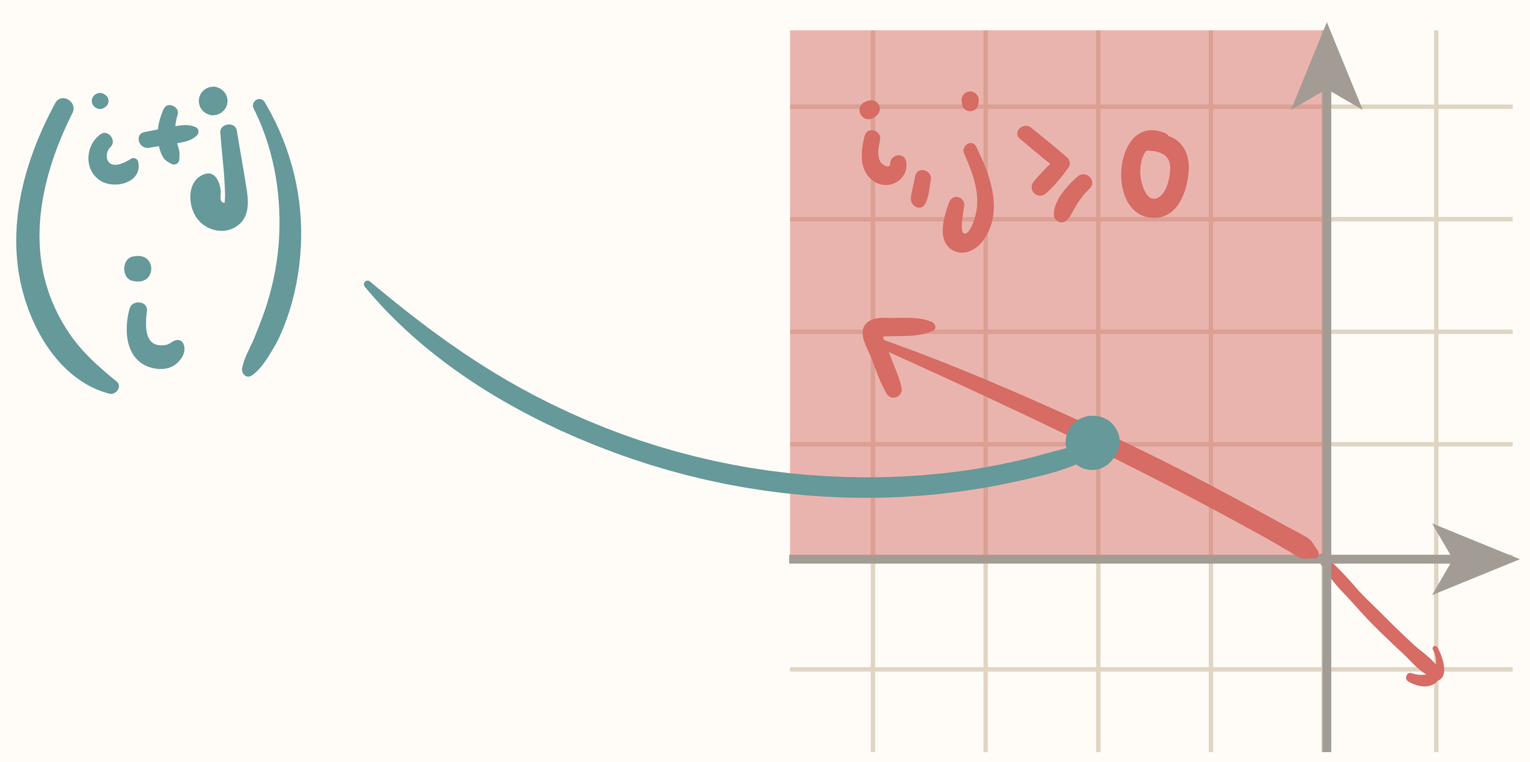
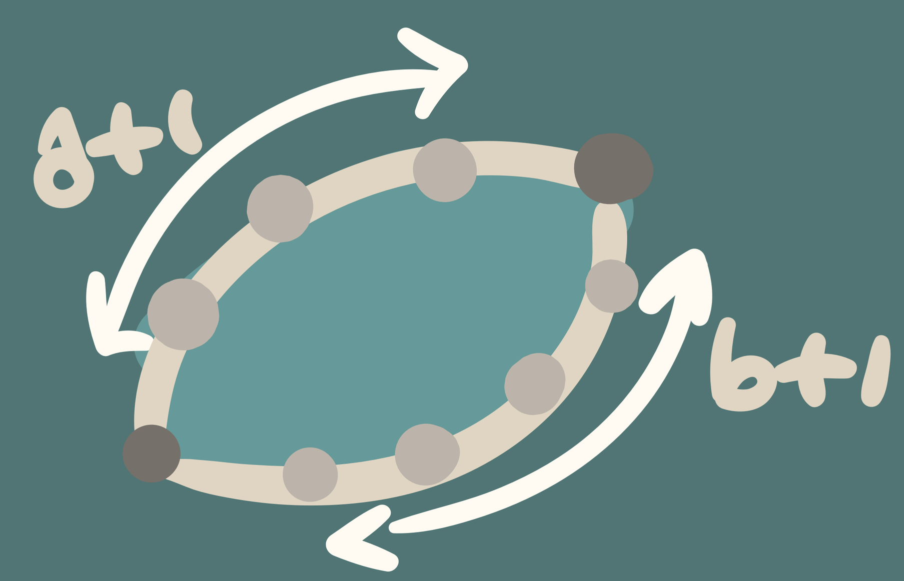
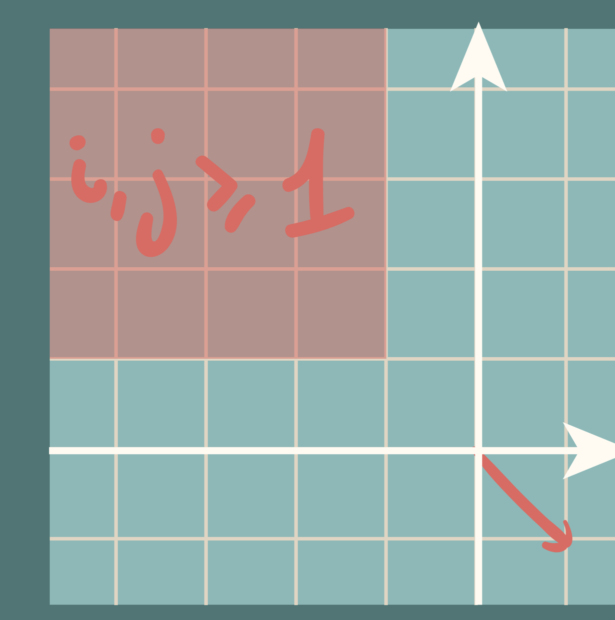
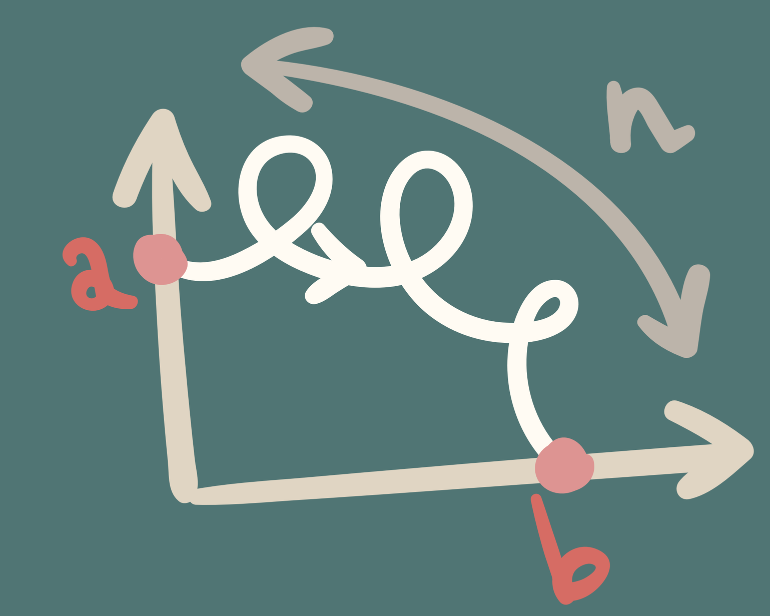




Specializations summary

Model	*Tandem walk*

Specializations summary

Model		*Tandem walk*
Posets *n+2 edges* 	←*bijection*→	 

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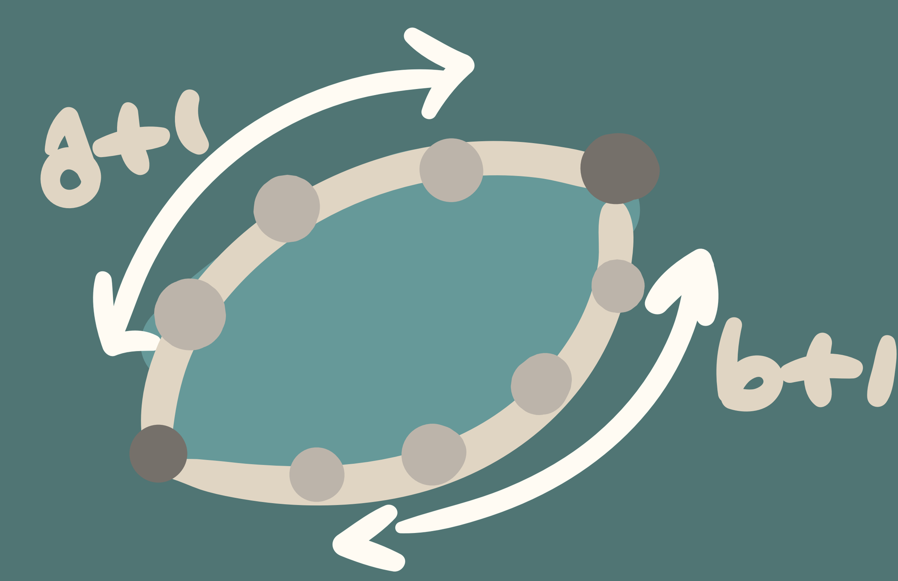
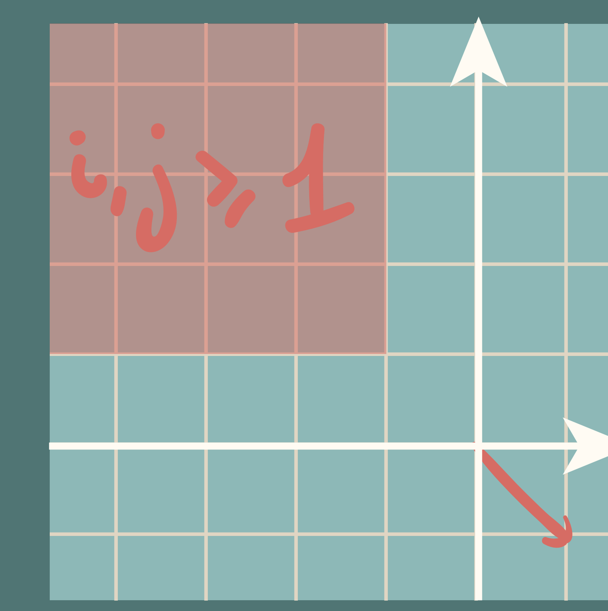
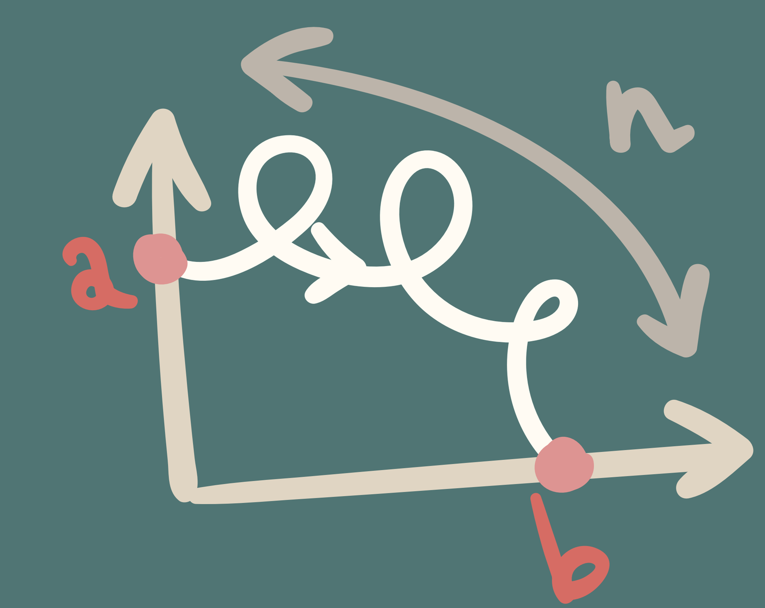
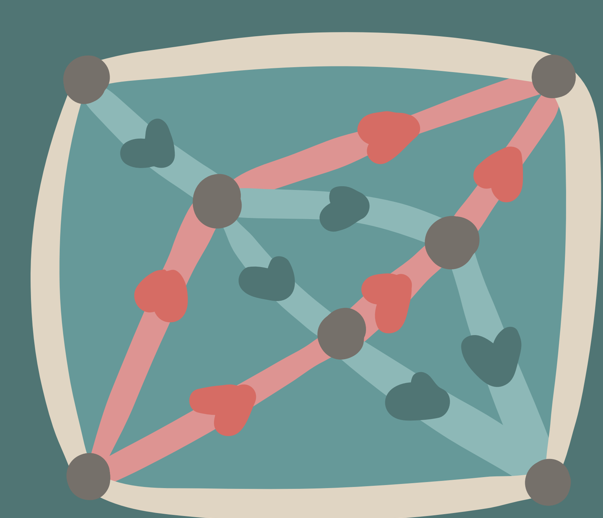
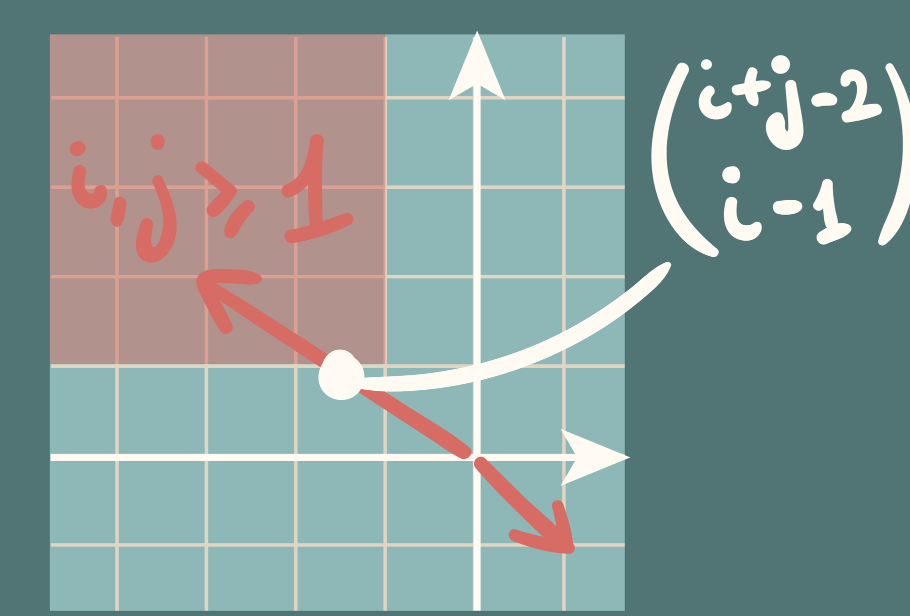
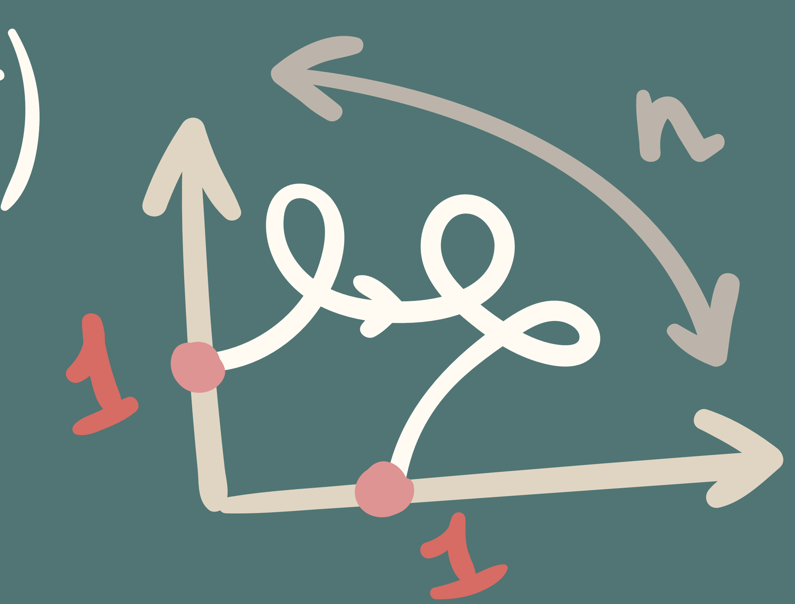
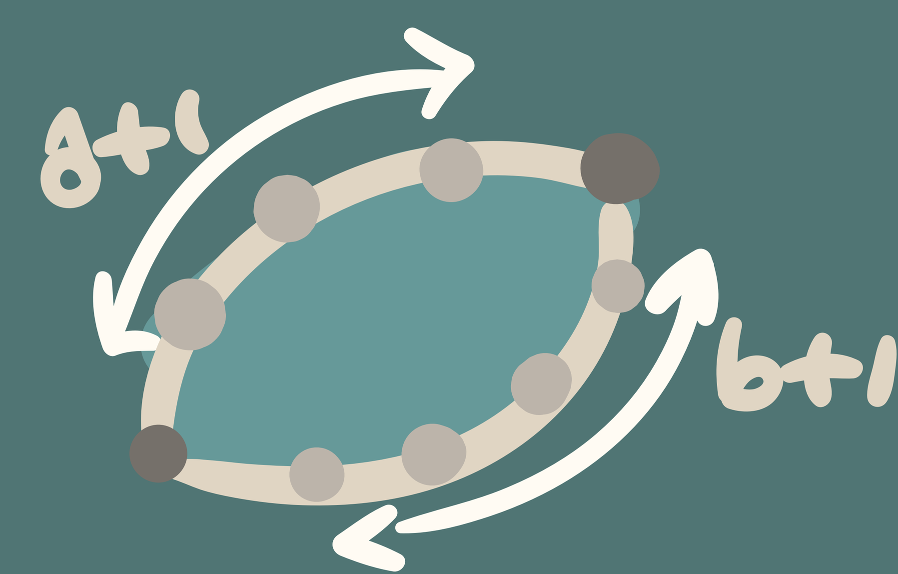
Posets *n+2 edges* 	←bijection→	 
Transversal structures *n blue edges* 	←bijection→	 

Specializations summary

Model	Tandem walk

Posets
n+2 edges



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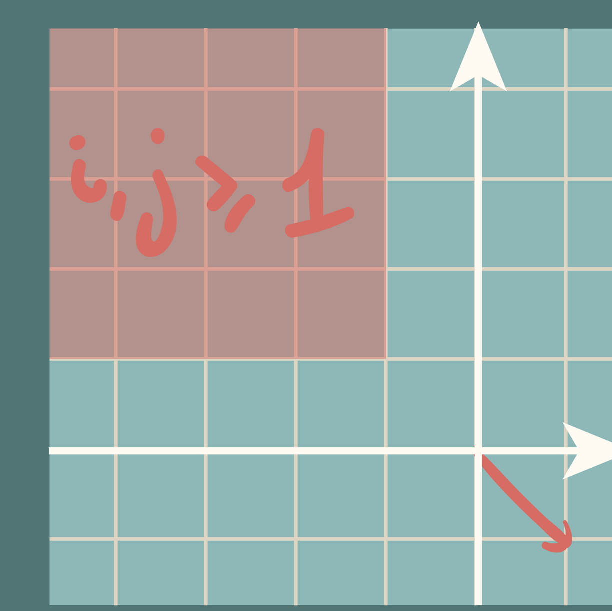
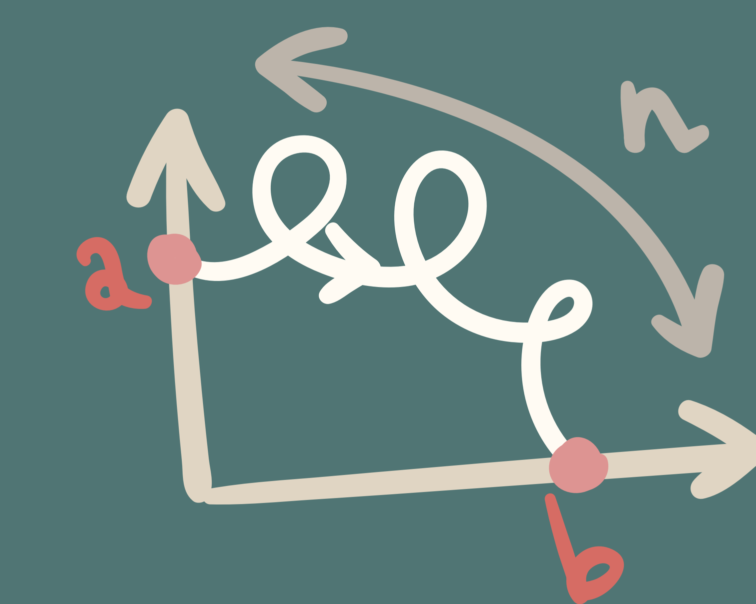
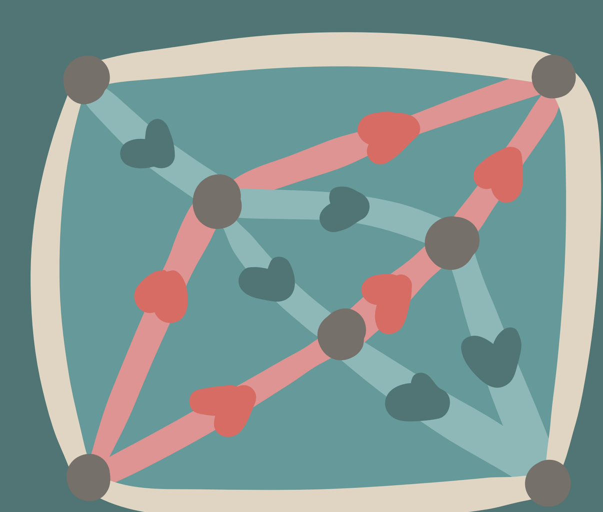
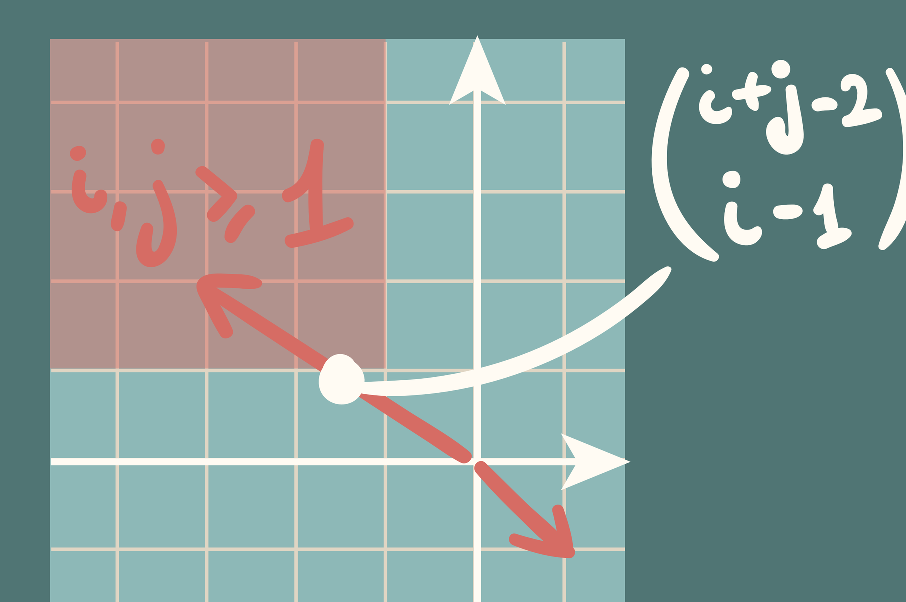


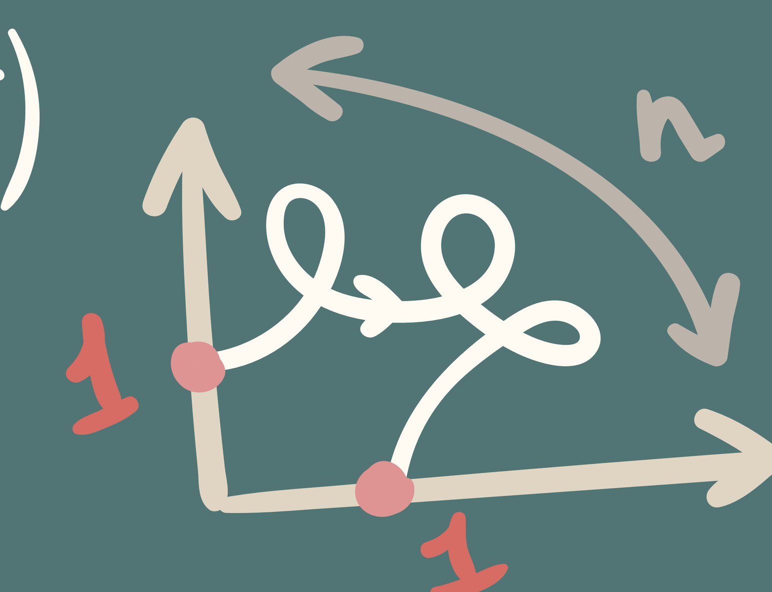


Transversal structures
n blue edges



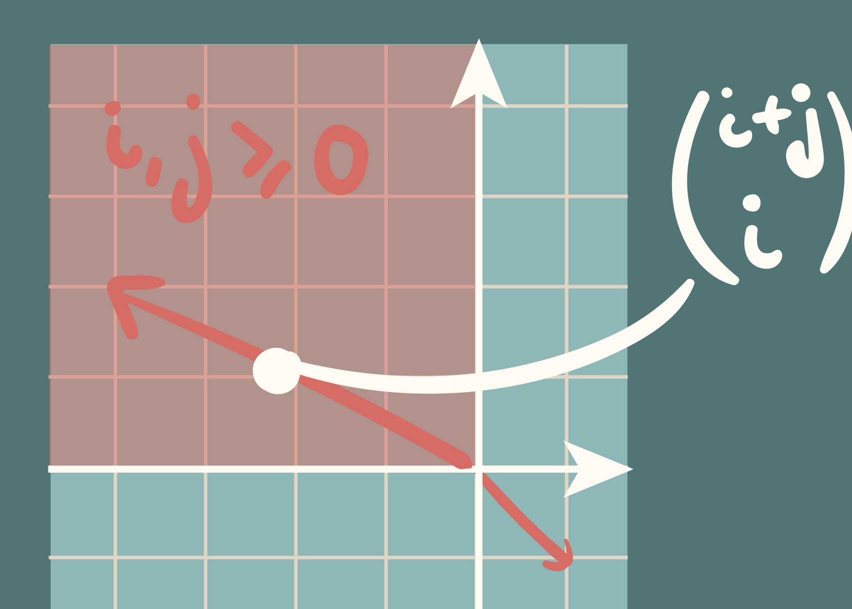
←bijection→

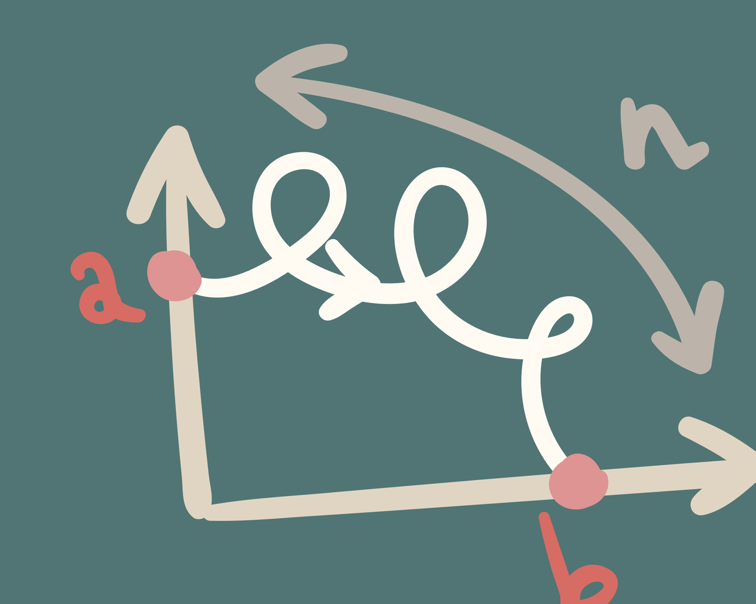




Posets
n vertices

←bijection→





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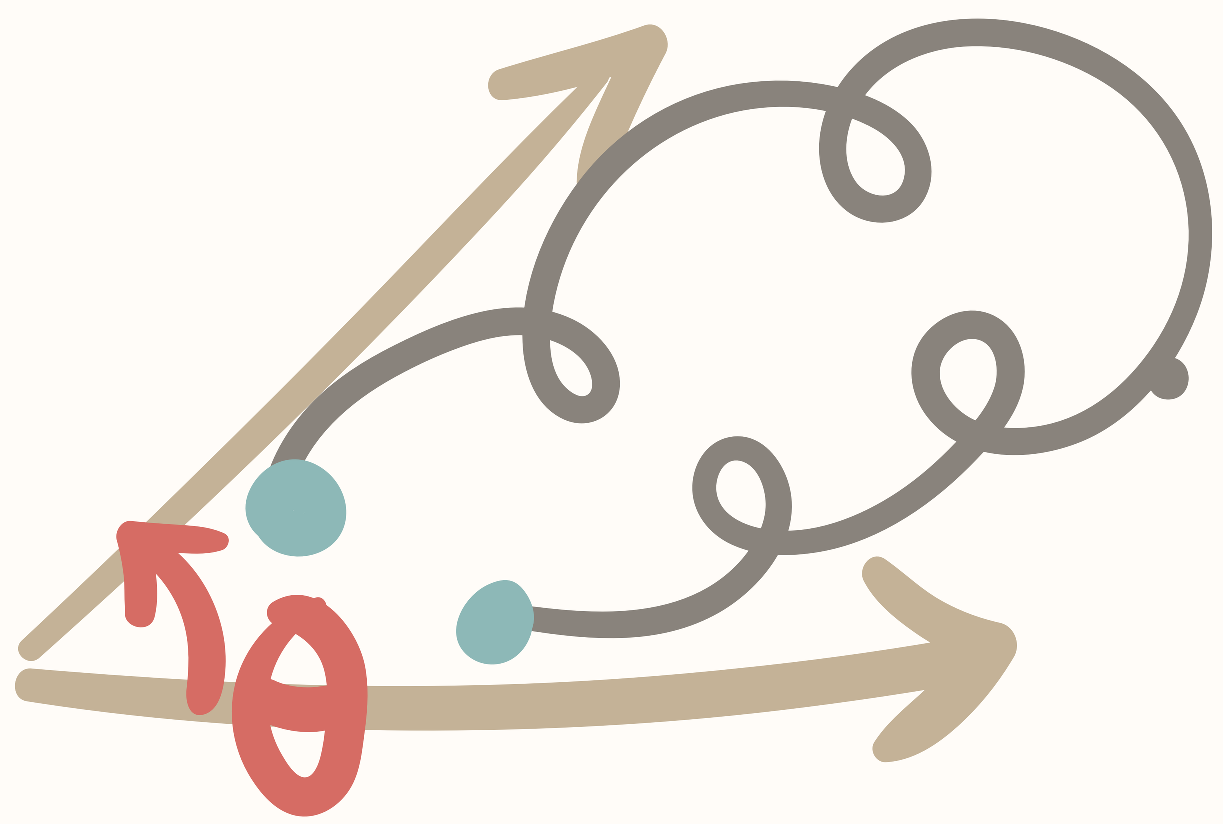
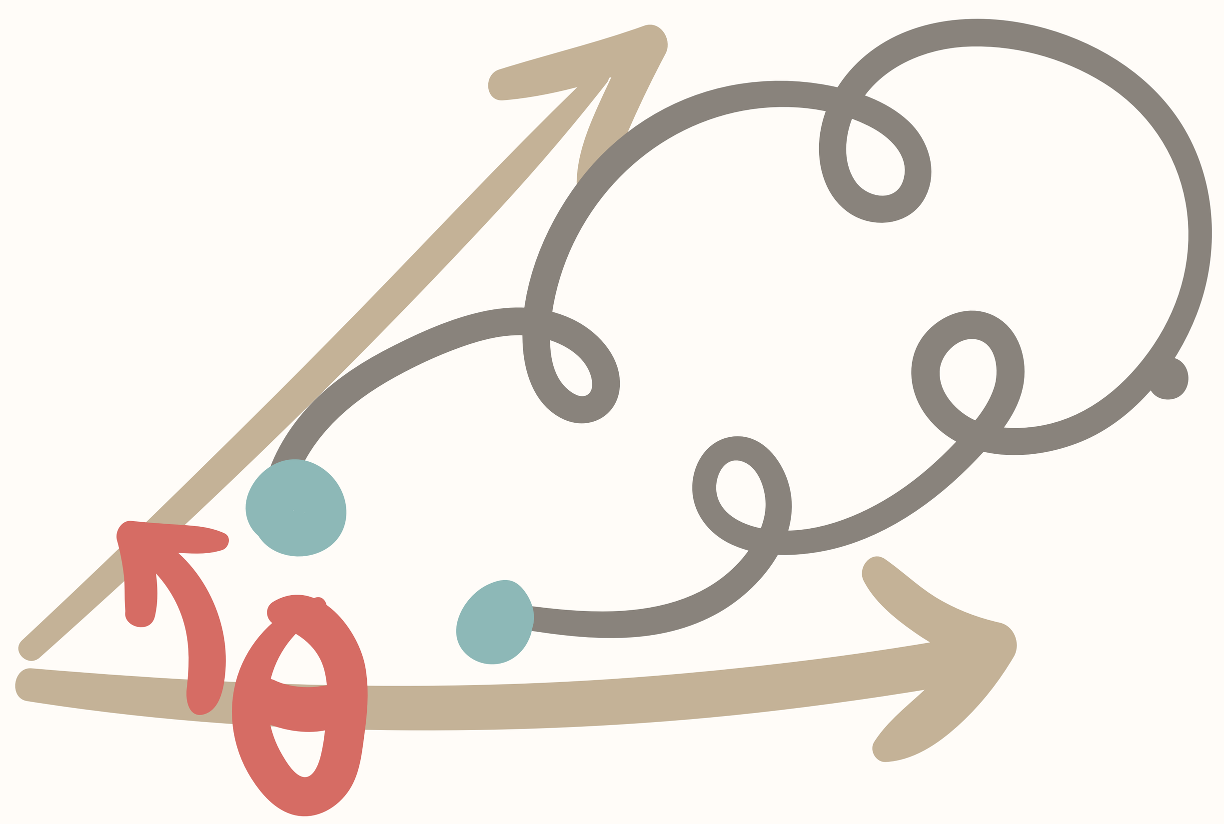
Asymptotic counting results



⇛ *Random walks in cone*, D. Denisov & V. Wachtel (2015)

$$a_n \sim \varkappa \cdot \gamma^n n^{-1-\frac{\pi}{\arccos(\theta)}}$$

Asymptotic counting results



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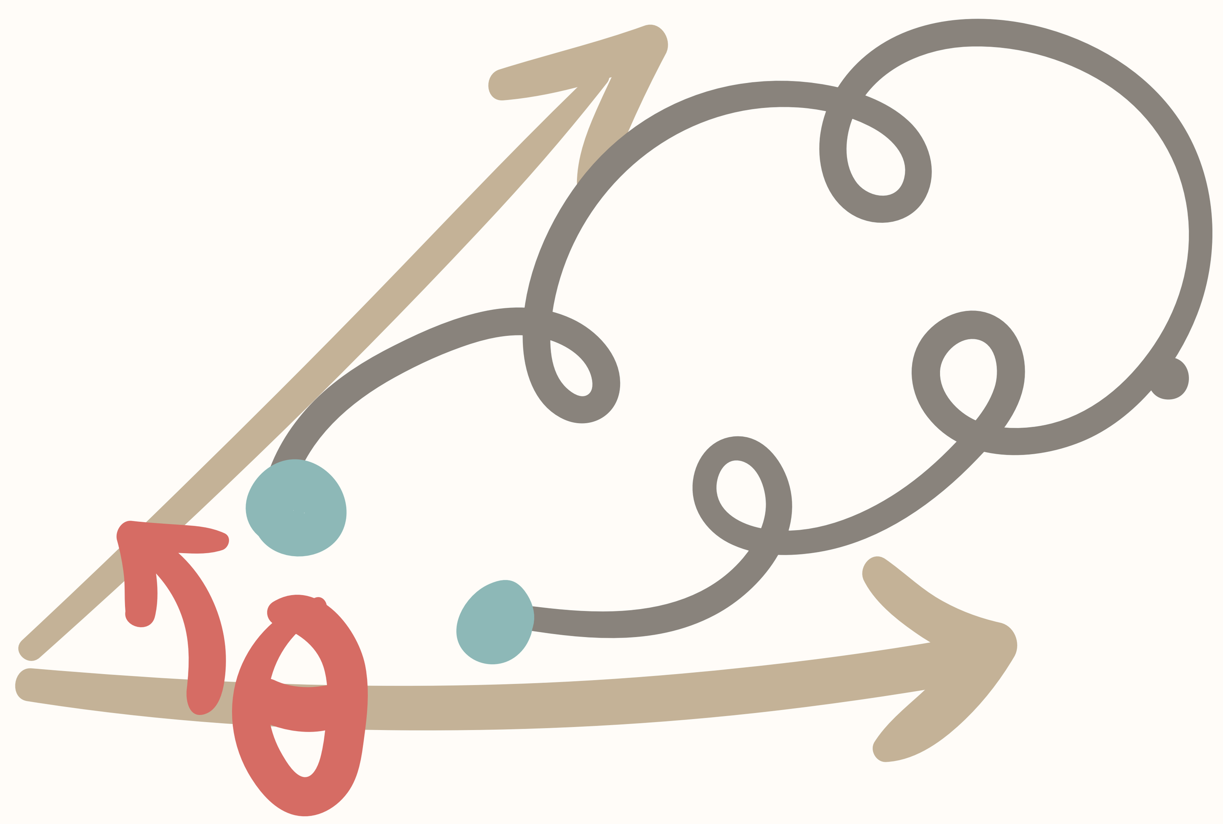
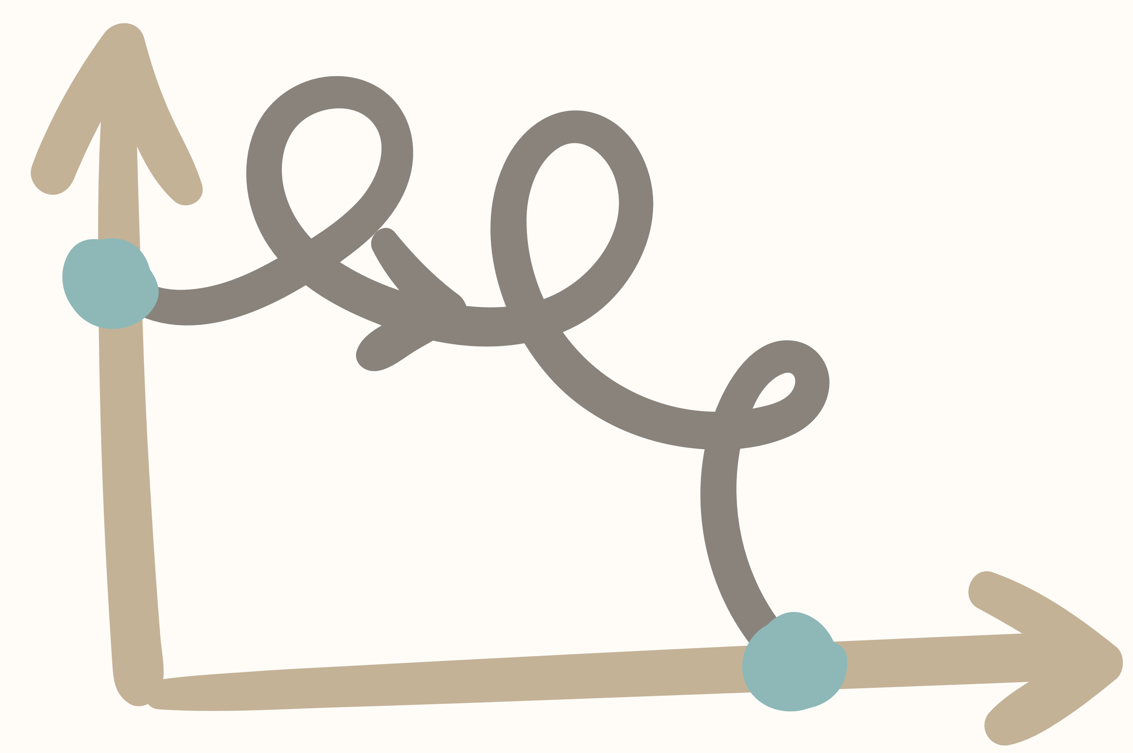
If the drift is zero, *i.e.* :

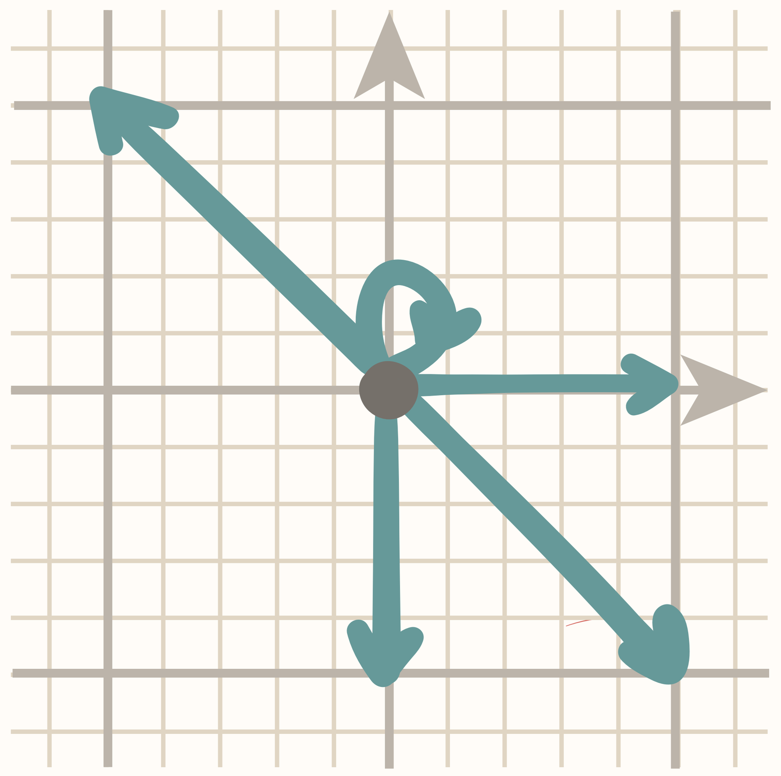
$$\mathbf{E}[X] = \mathbf{E}[Y] = 0$$

And the covariance matrix is identity.

Asymptotic counting results







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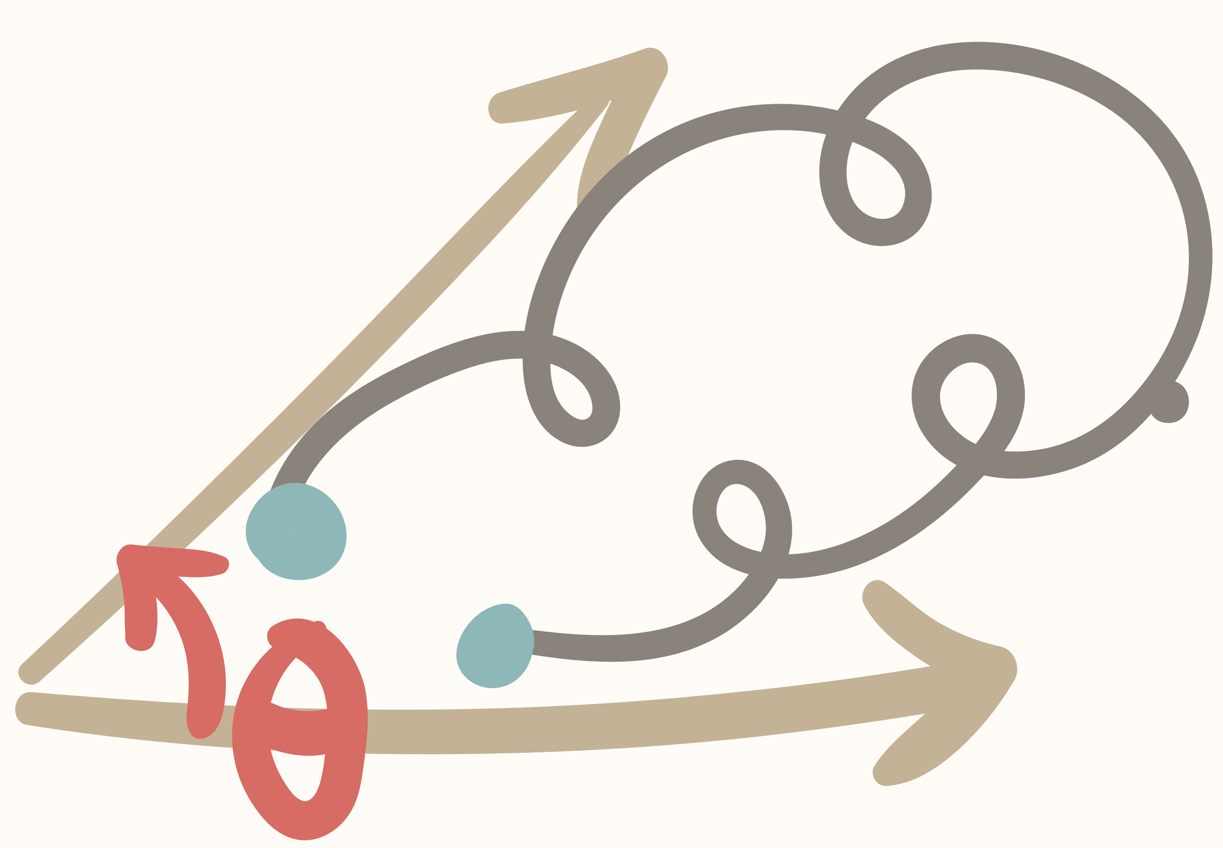

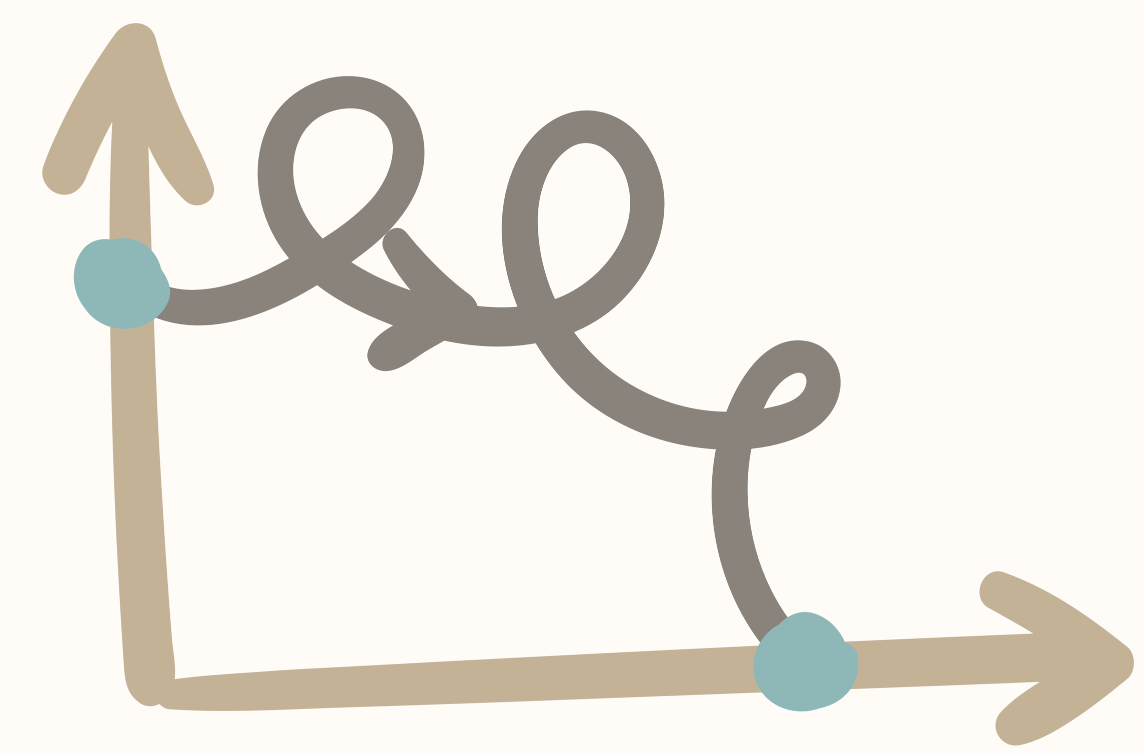

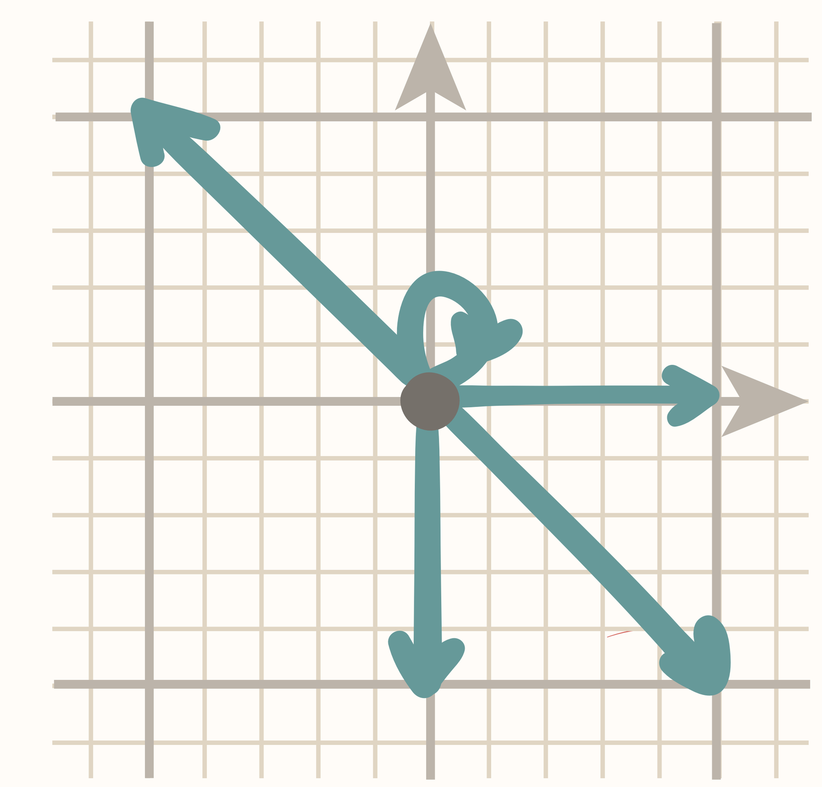

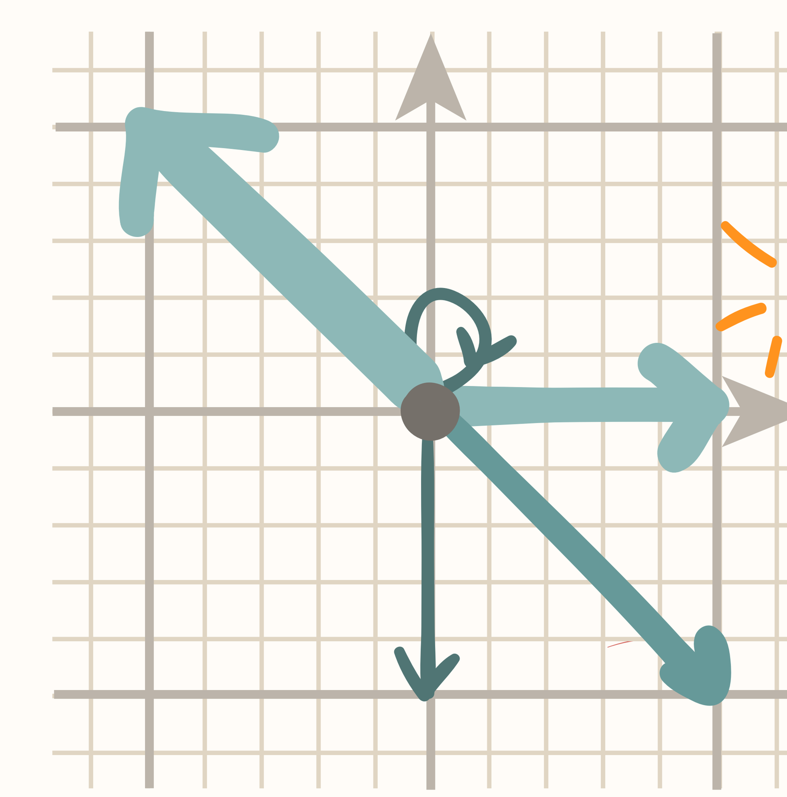

Weighted steps

→ *Random walks in cone*, D. Denisov & V. Wachtel (2015)

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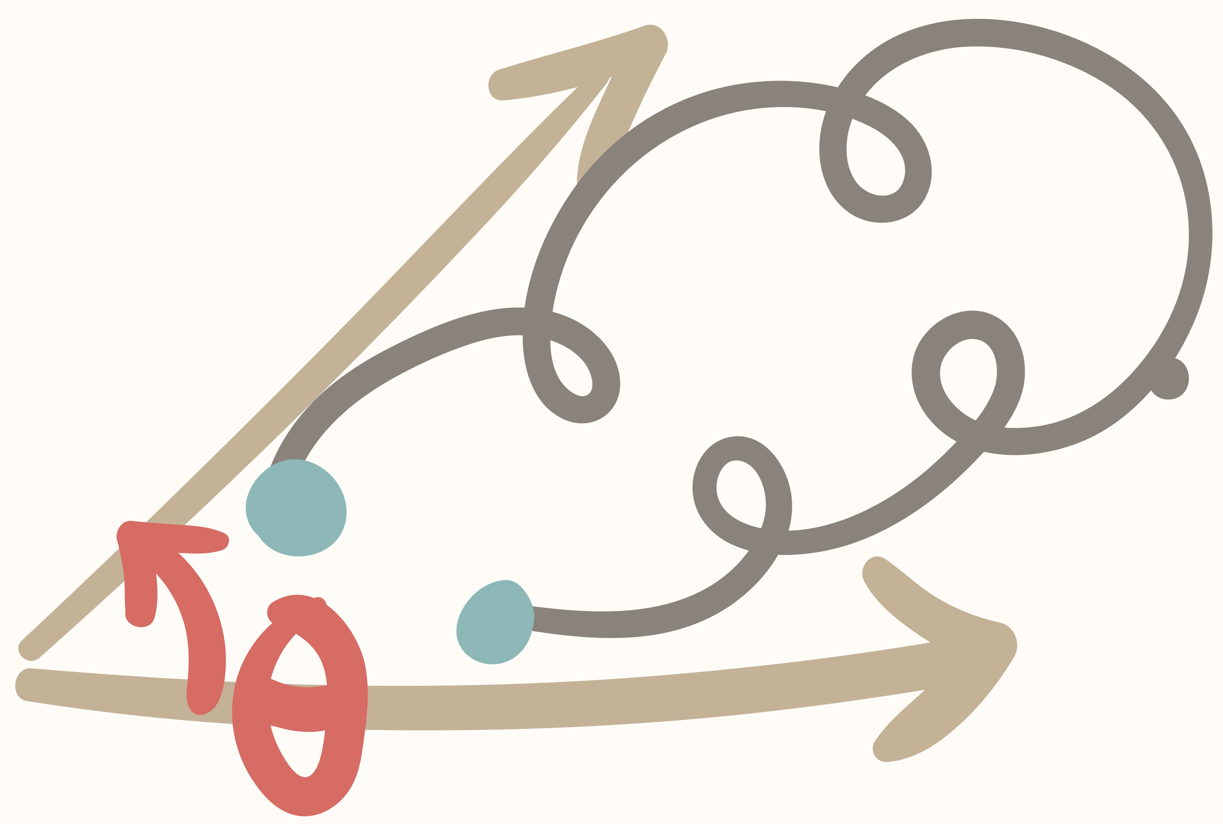
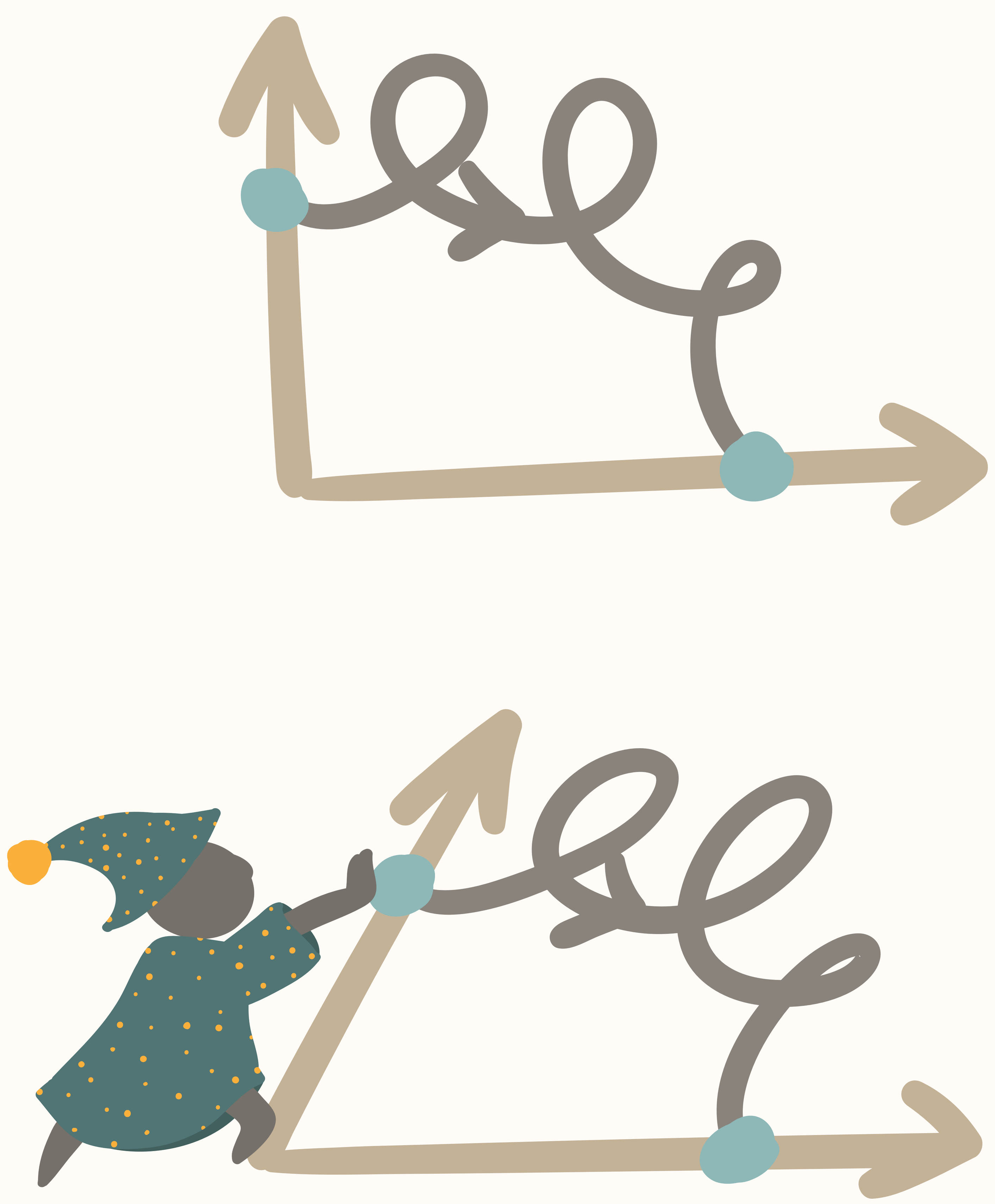
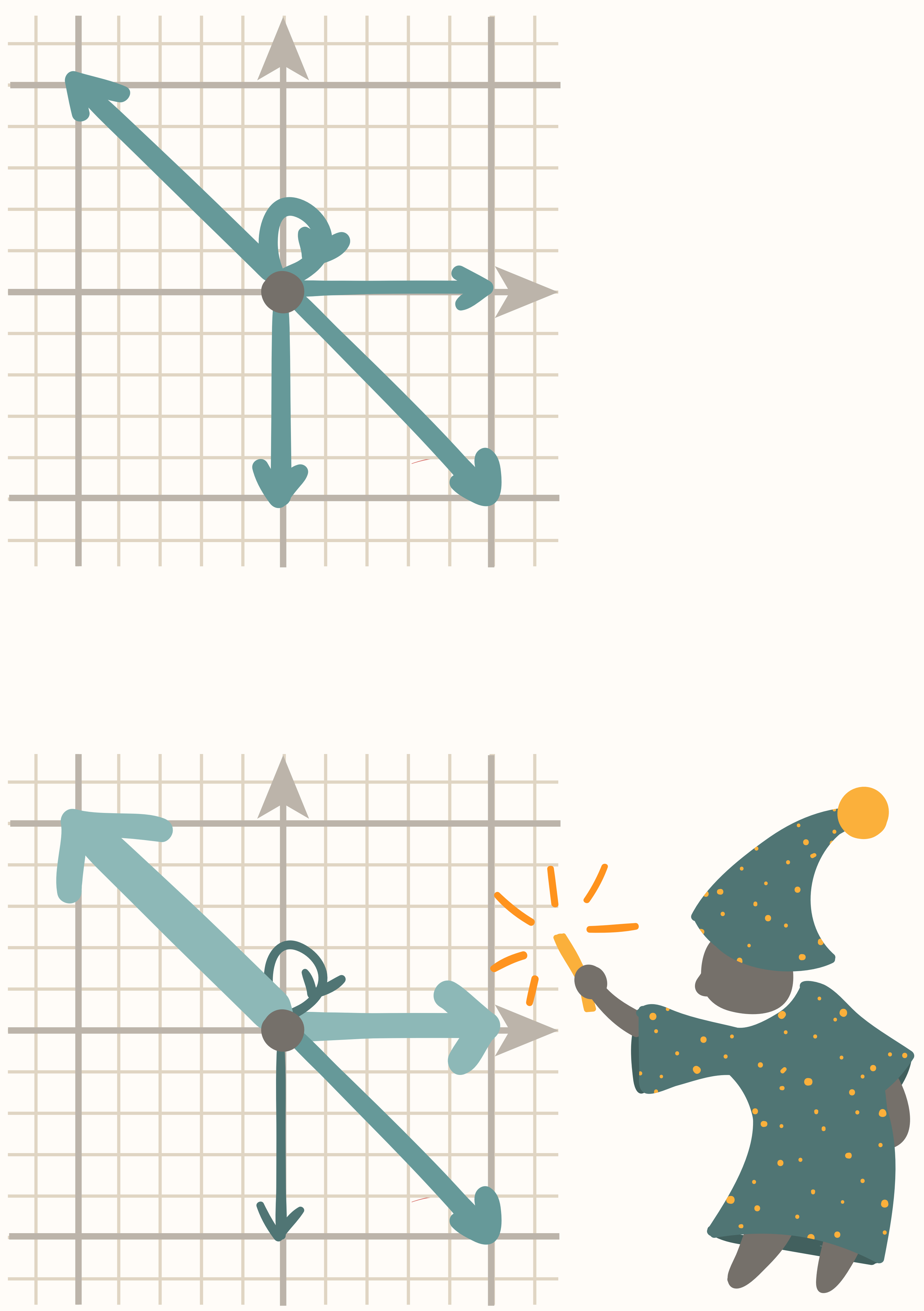




Shear transformation



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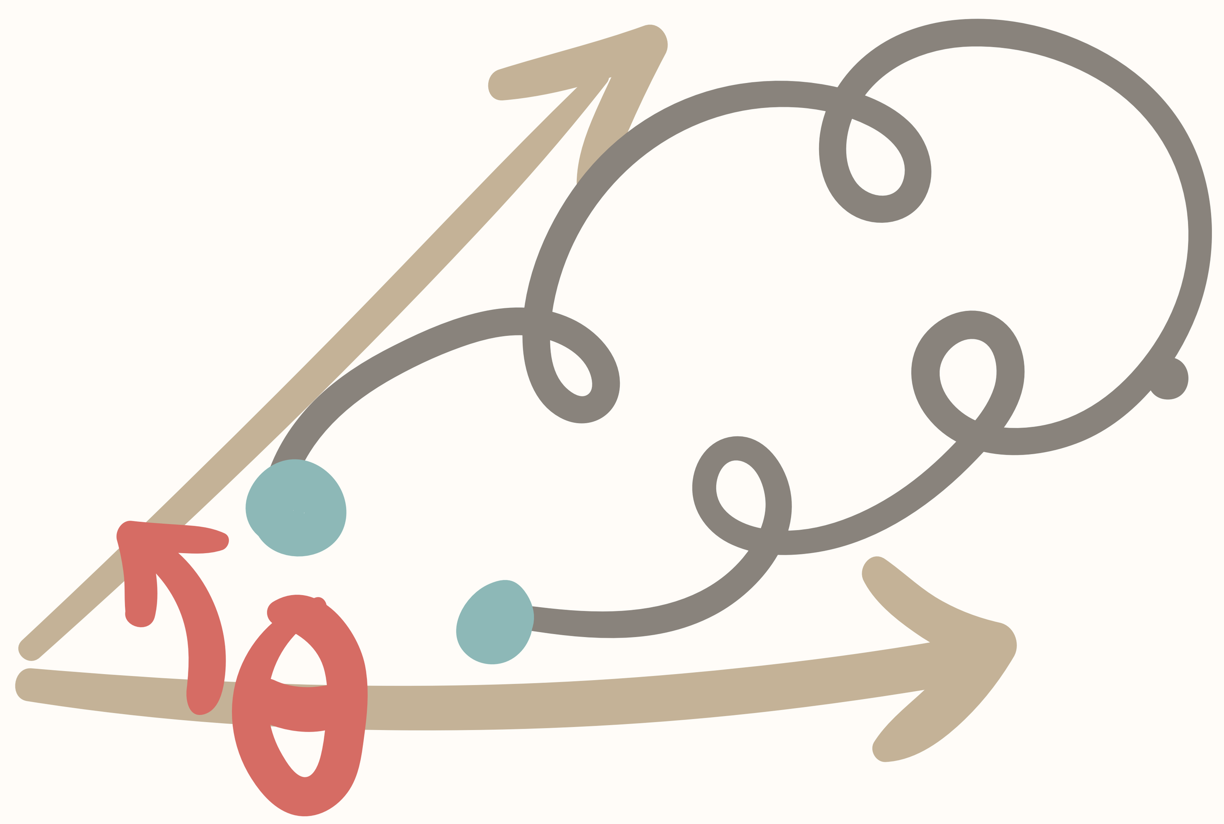
Asymptotic counting results



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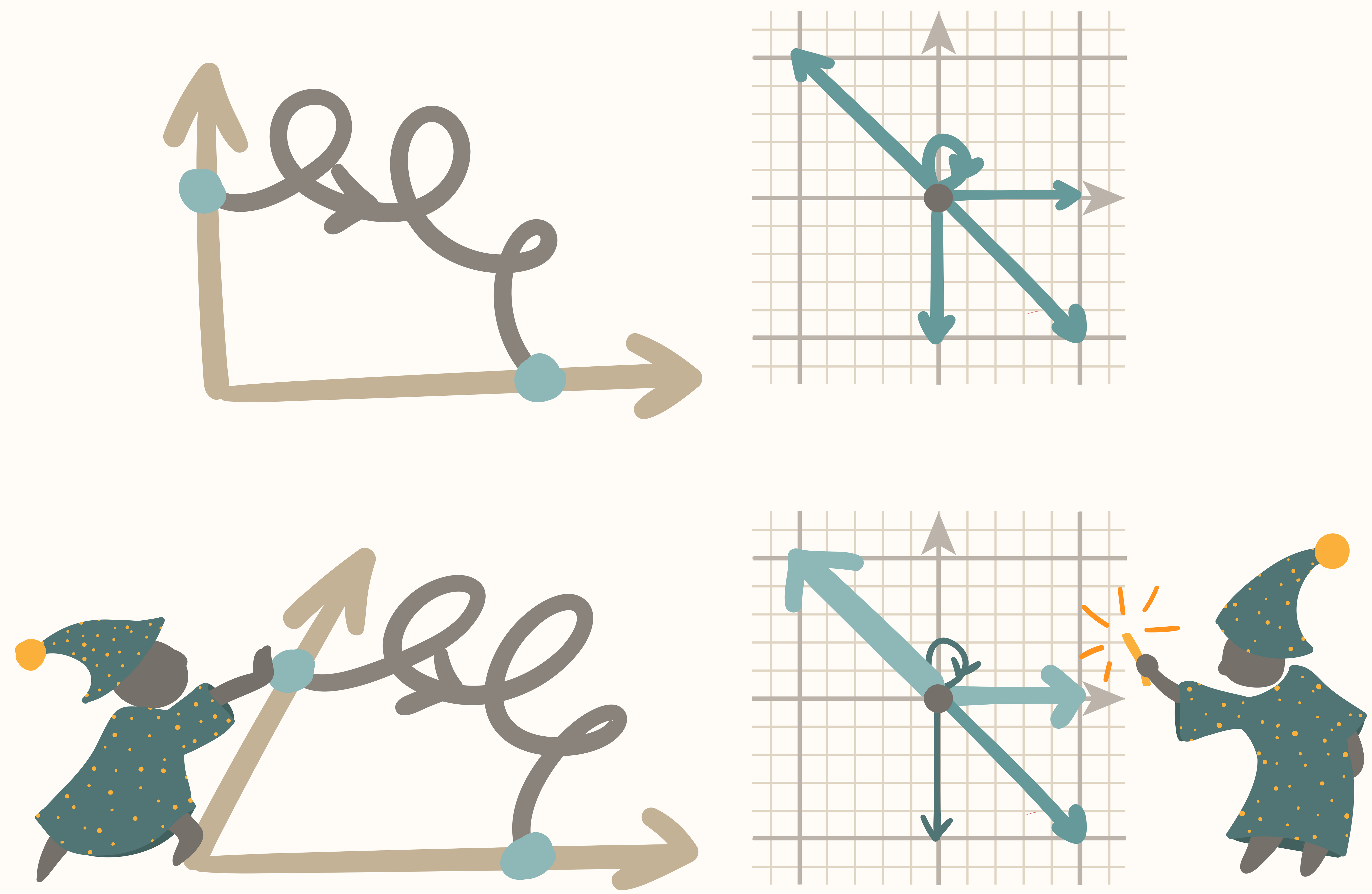
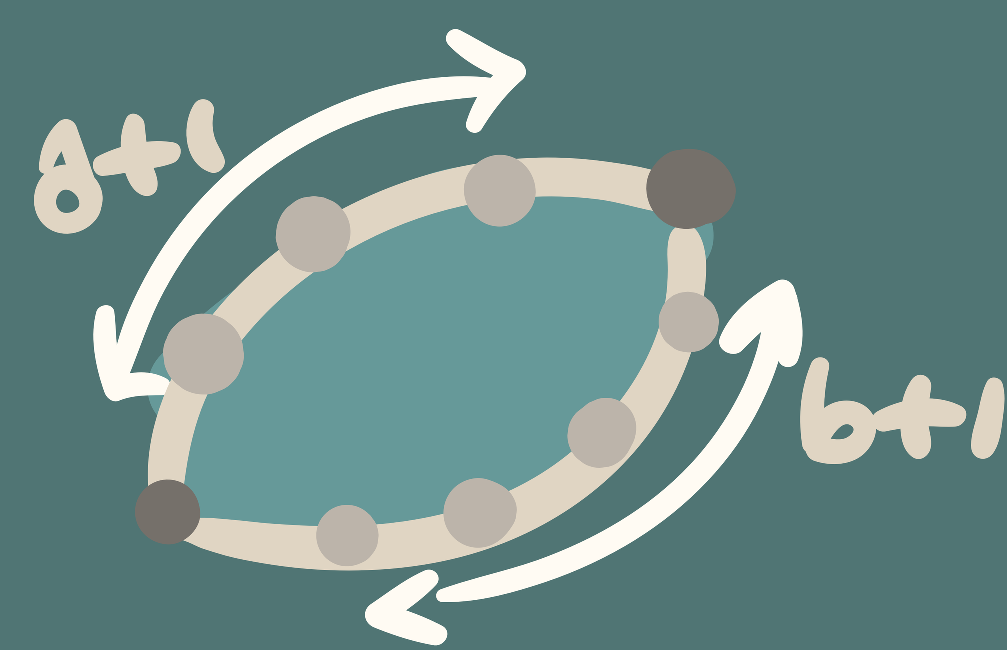


Shear transformation

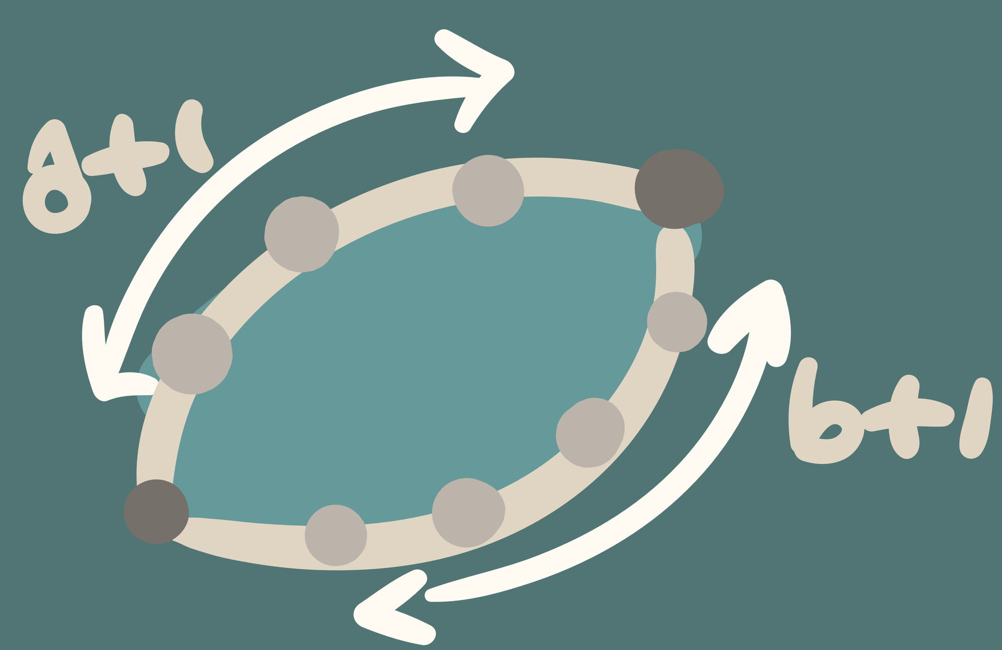
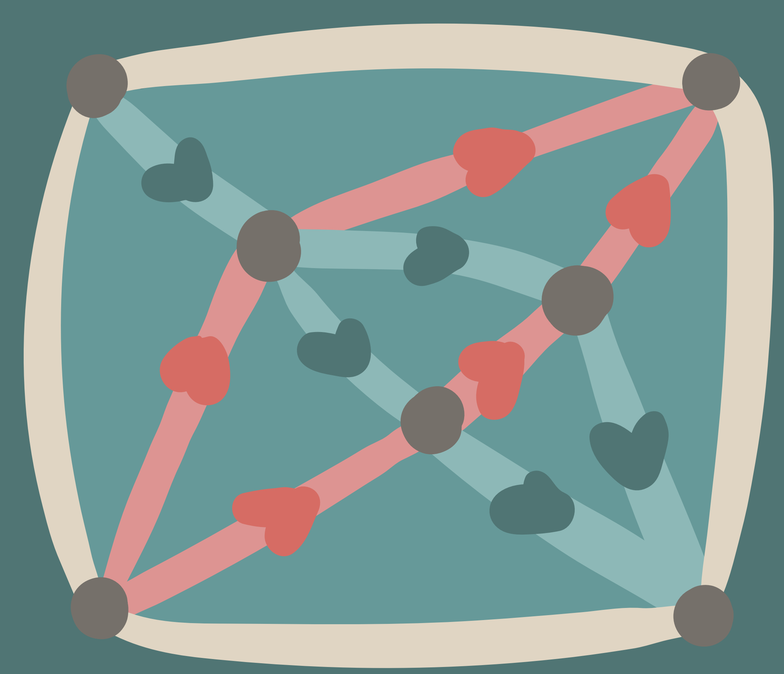
Weighted steps

Asymptotics

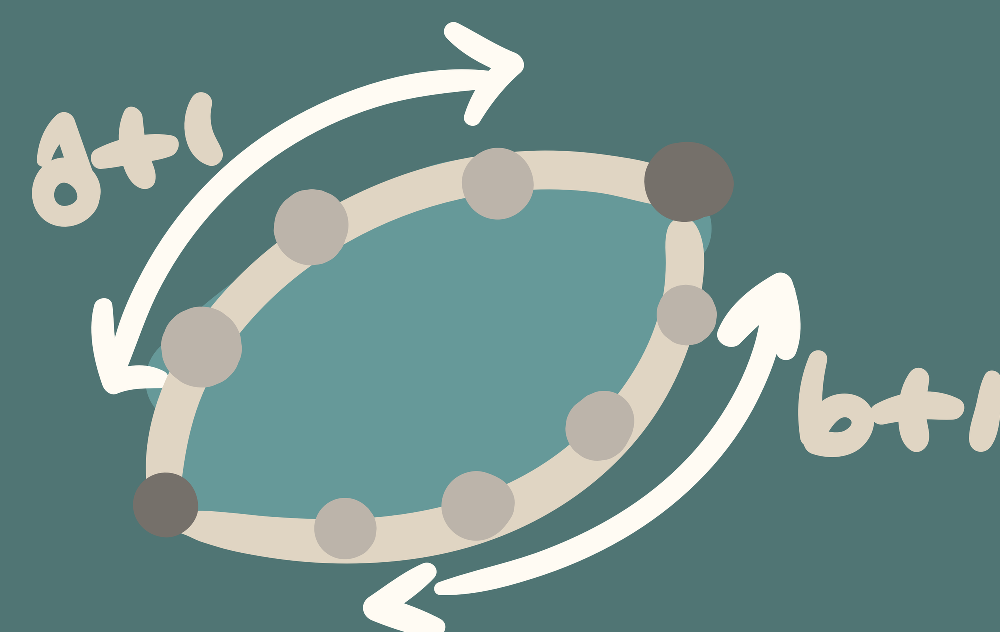
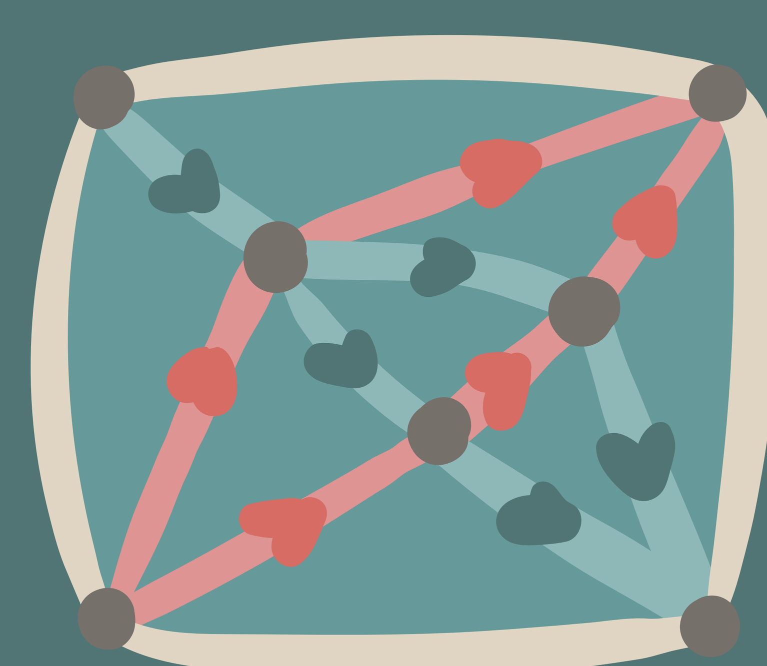
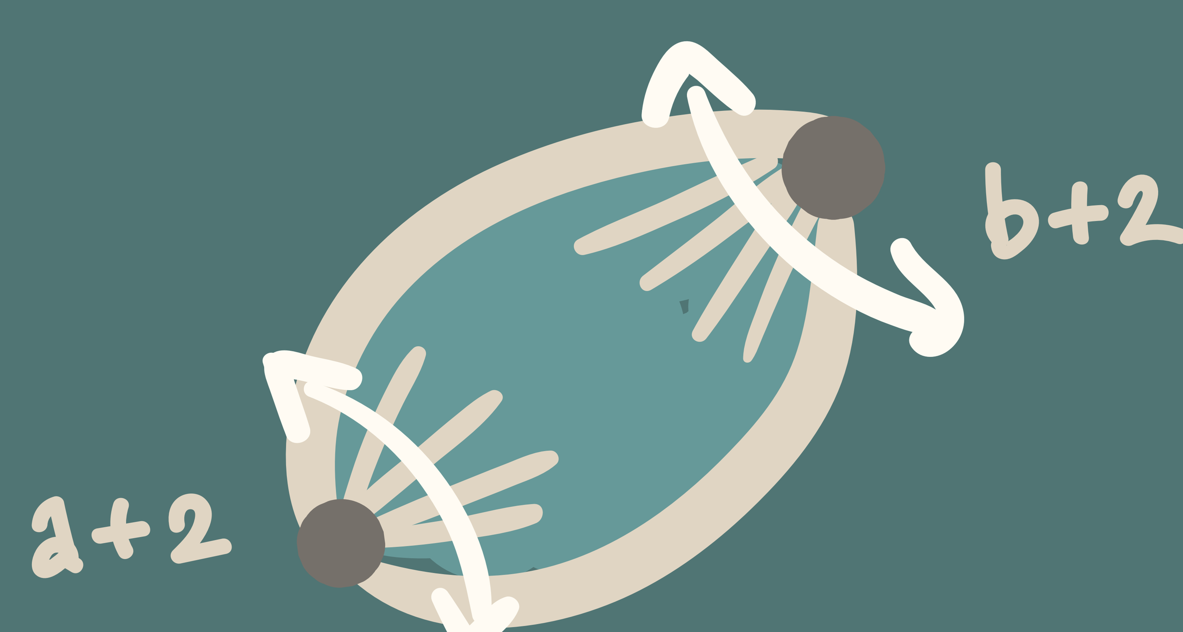
Asymptotic counting results

Model	*Asymptotics*
Posets *n+2 edges* 	$e_n \sim \varkappa\, \gamma^n n^{-\alpha}$ γ *and* α *are explict analytic constants* $\gamma \approx 4.80\ldots$ $\alpha \approx -5.14\ldots$

Asymptotic counting results

Model		*Asymptotics*
Posets *n+2 edges*		$e_n \sim \varkappa\, \gamma^n n^{-\alpha}$ γ *and* α *are explict analytic constants* $\gamma \approx 4.80\ldots \quad \alpha \approx -5.14\ldots$
Transversal structures *n vertices*		$t_n \sim \varkappa \left(\frac{27}{2}\right)^n n^{-1-\frac{\pi}{\arccos(7/8)}}$ ↠ *Counting rectangular drawings,* Y. Inoue, T. Takahashi & R. Fujimaki (2009)

Asymptotic counting results

Model		*Asymptotics*
Posets *n+2 edges*		$e_n \sim \varkappa\, \gamma^n n^{-\alpha}$ γ *and* α *are explict analytic constants* $\gamma \approx 4.80\ldots \quad \alpha \approx -5.14\ldots$
Transversal structures *n vertices*		$t_n \sim \varkappa \left(\frac{27}{2}\right)^n n^{-1-\frac{\pi}{\arccos(7/8)}}$ ↠ *Counting rectangular drawings,* Y. Inoue, T. Takahashi & R. Fujimaki (2009)
Posets *n vertices*		$b_n \sim \varkappa \left(\frac{11+\sqrt{5}}{2}\right)^n n^{-6}$

Asymptotic coun

Mod
n
Pose *n ver*

POSETS PER VERTEX

$$b_n \sim \varkappa \left(\frac{11+\sqrt{5}}{2}\right)^n \cdot n^{-6}$$

Asymptotic coun

Mod

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PLANE PERMUTATIONS

$$p_n \sim \varkappa \left(\frac{11+\sqrt{5}}{2}\right)^n \cdot n^{-6}$$

⇛ *Semi-Baxter and strong-Baxter: two relatives of Baxter Sequences,*
M. Bouvel, V. Guerrini, A. Rechnitzer & S. Rinaldi (2018)

Asymptotic coun

Mod

POSETS PER VERTEX

$$b_n \sim \varkappa \left(\frac{11+\sqrt{5}}{2}\right)^n \cdot n^{-6}$$

1, 1, 2, 6, 23, 104, 530, 2958, 17734, 112657, ...

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n

Pose

n ver

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Mod

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1, 1, 2, 6, 23, 104, 530, 2958,
17734, 112657, 750726, ...

≫ http://oeis.org/A117106

$\boxed{\text{TODO}}$

☐ <u>Bijection</u>

poset per vertices ⟷ plane permutations

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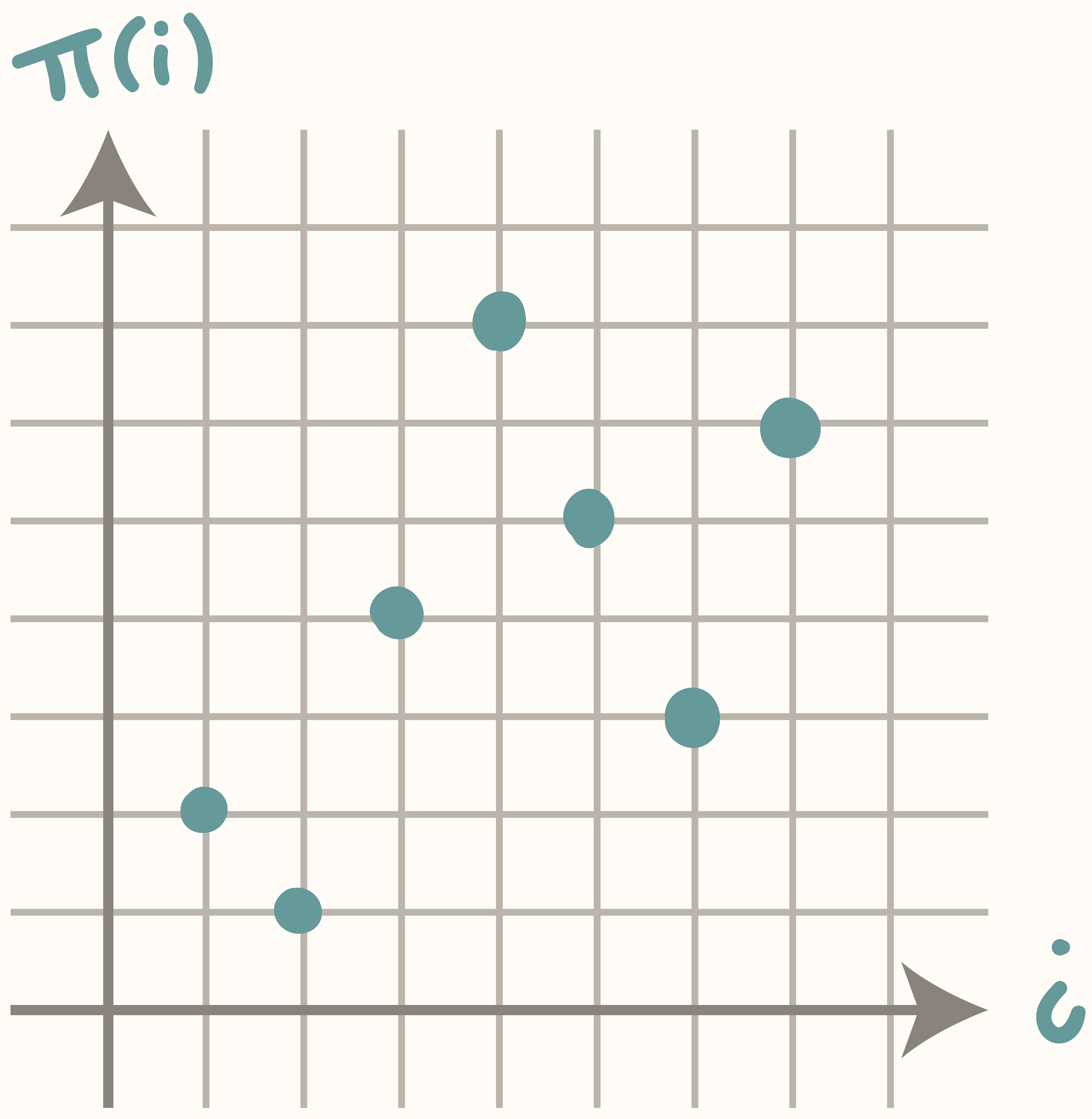
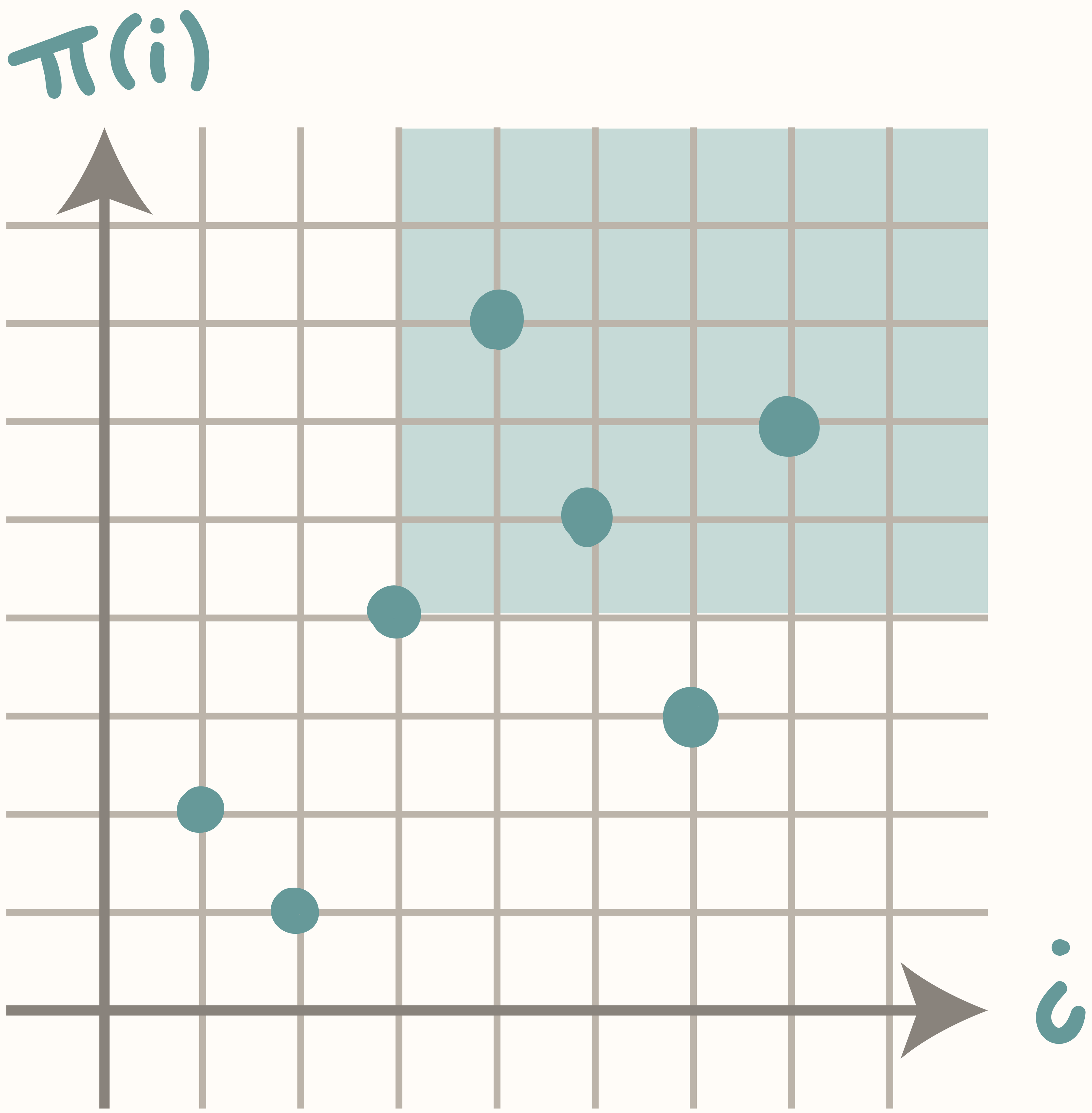
Plane permutations

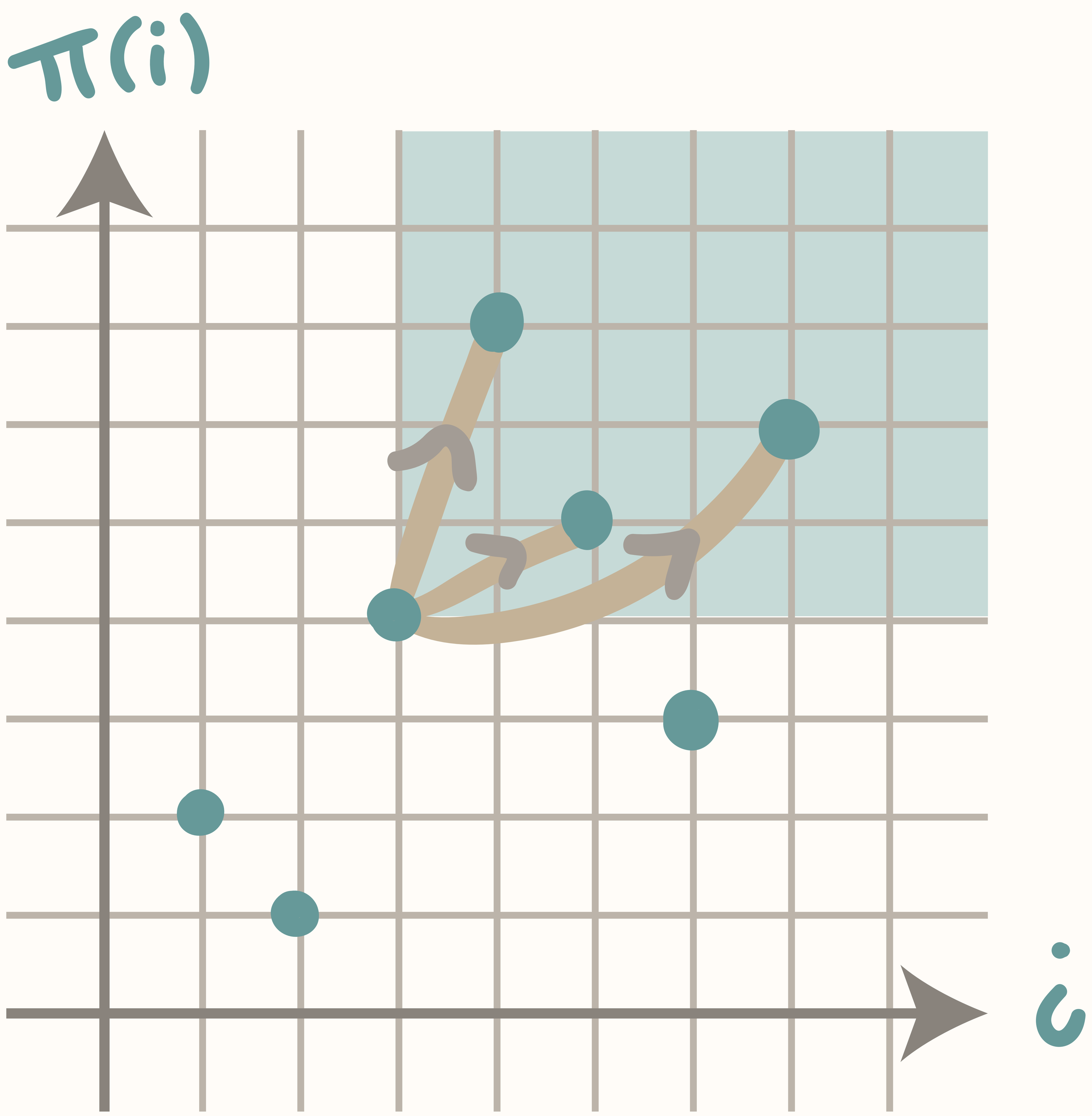


Plane permutations



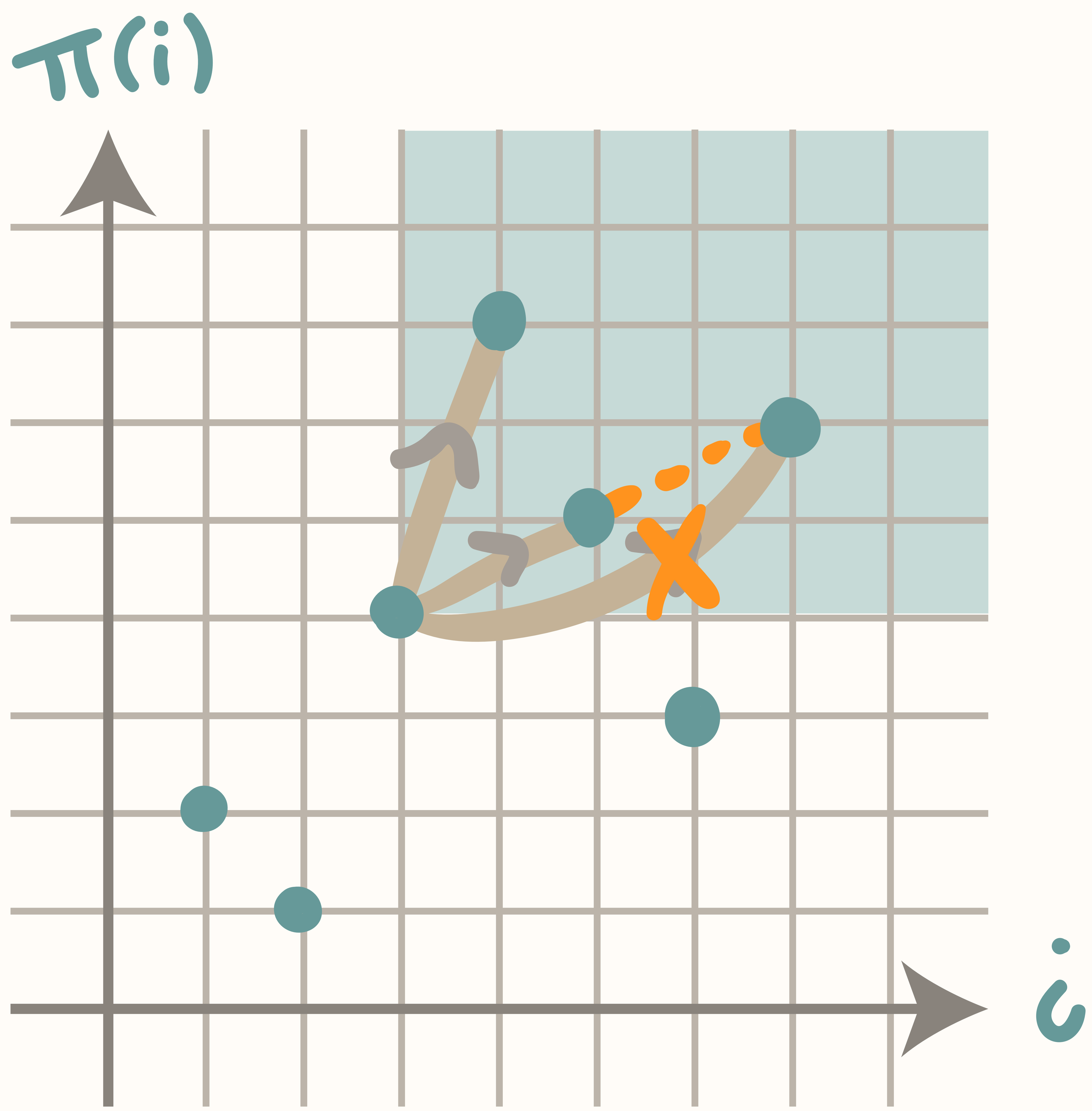
Dominance relation

Plane permutations



Dominance relation

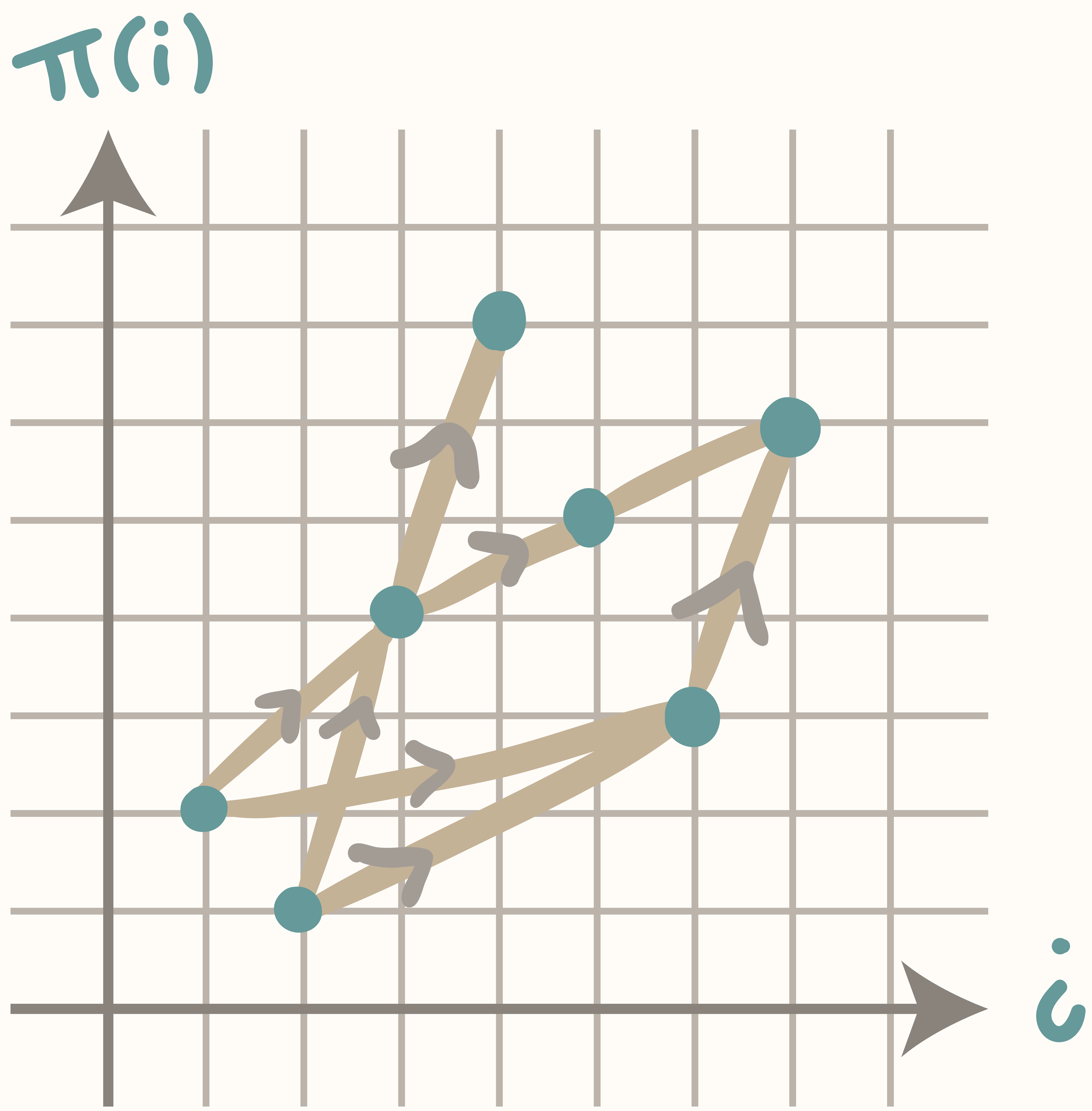
Plane permutations



Dominance diagram

= *Dominance relation with no transitive edges*

Plane permutations



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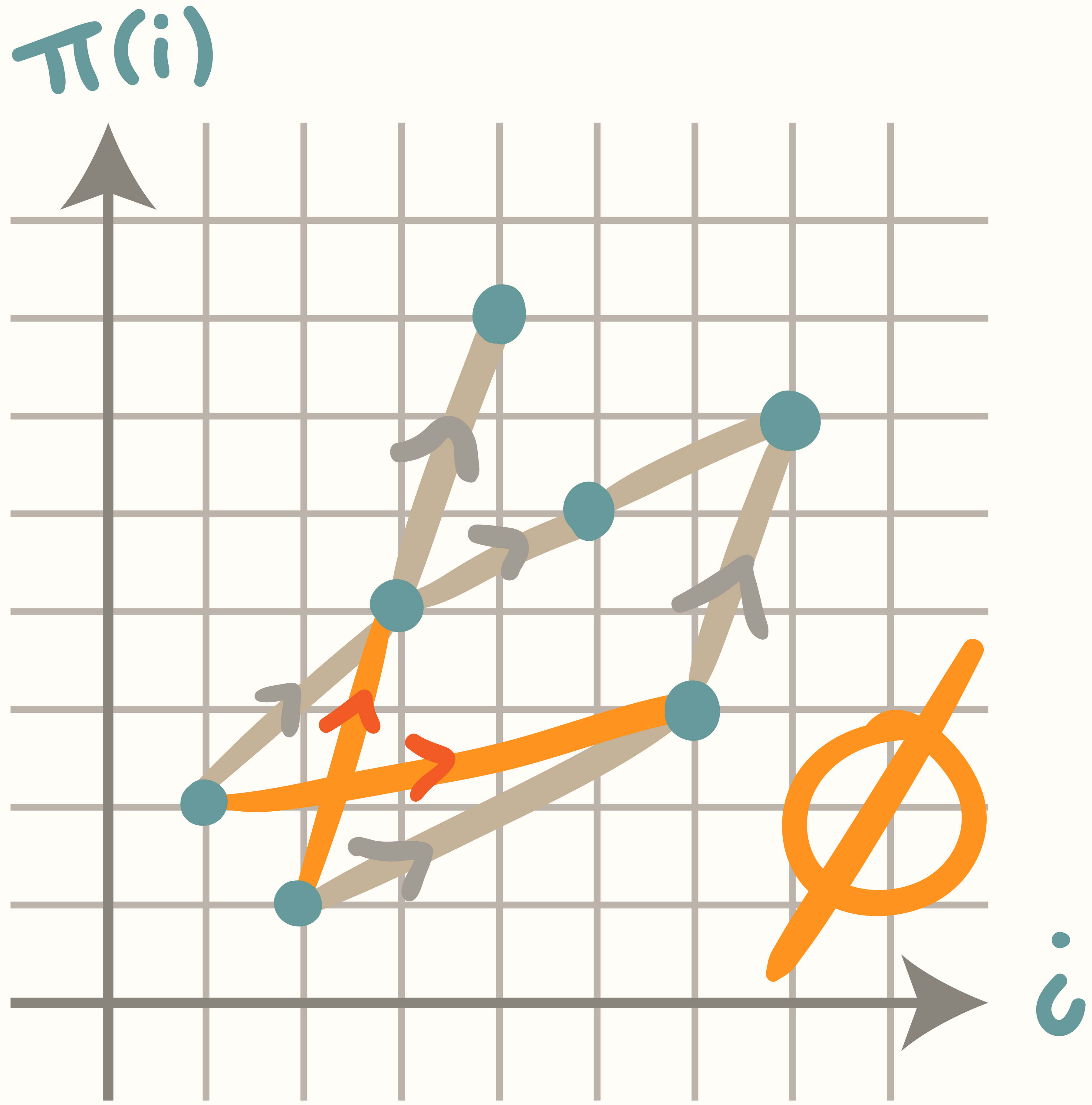
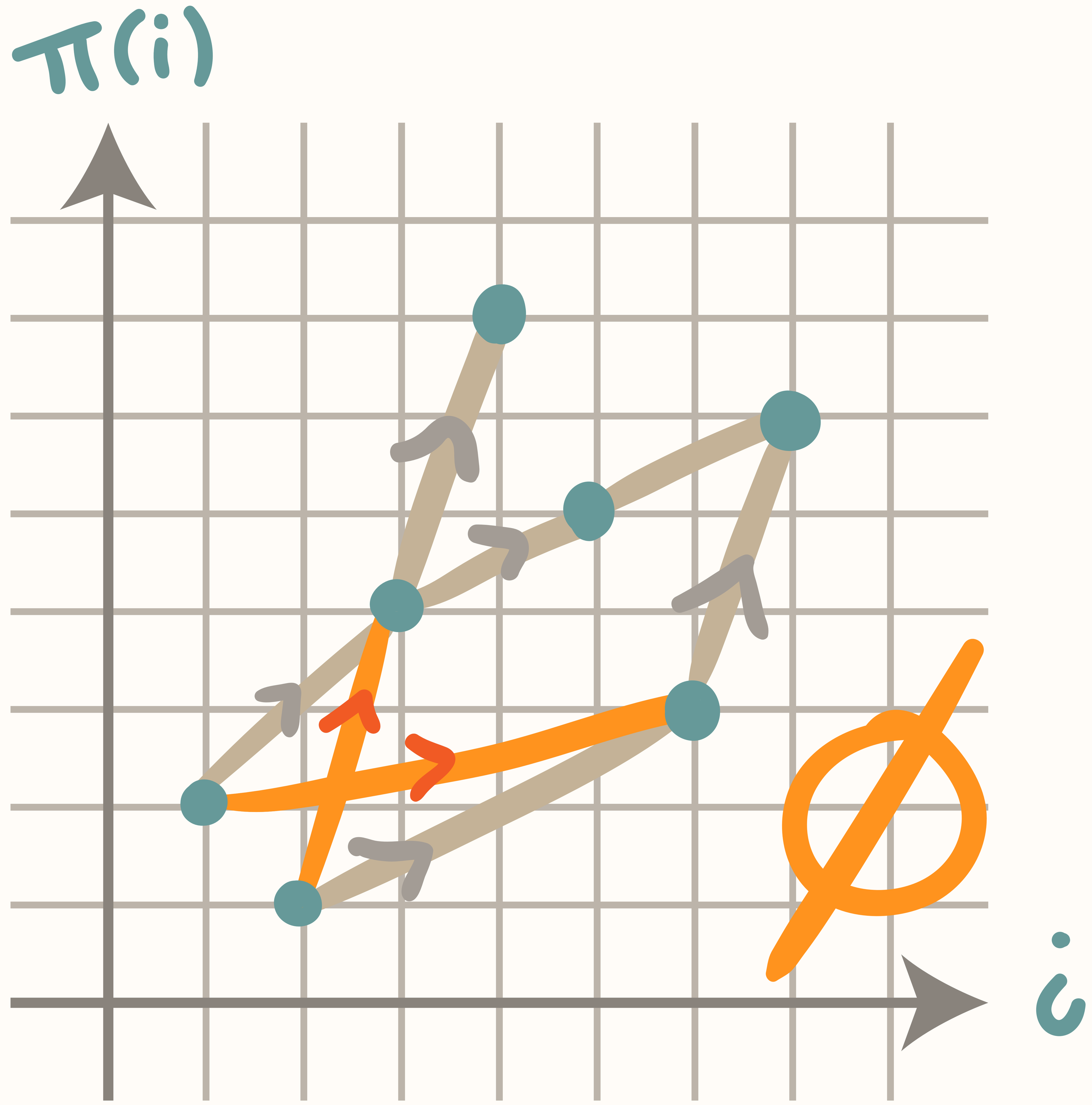
Plane permutations



Plane permutation

= *No edge crossing in the dominance diagram*

Plane permutations

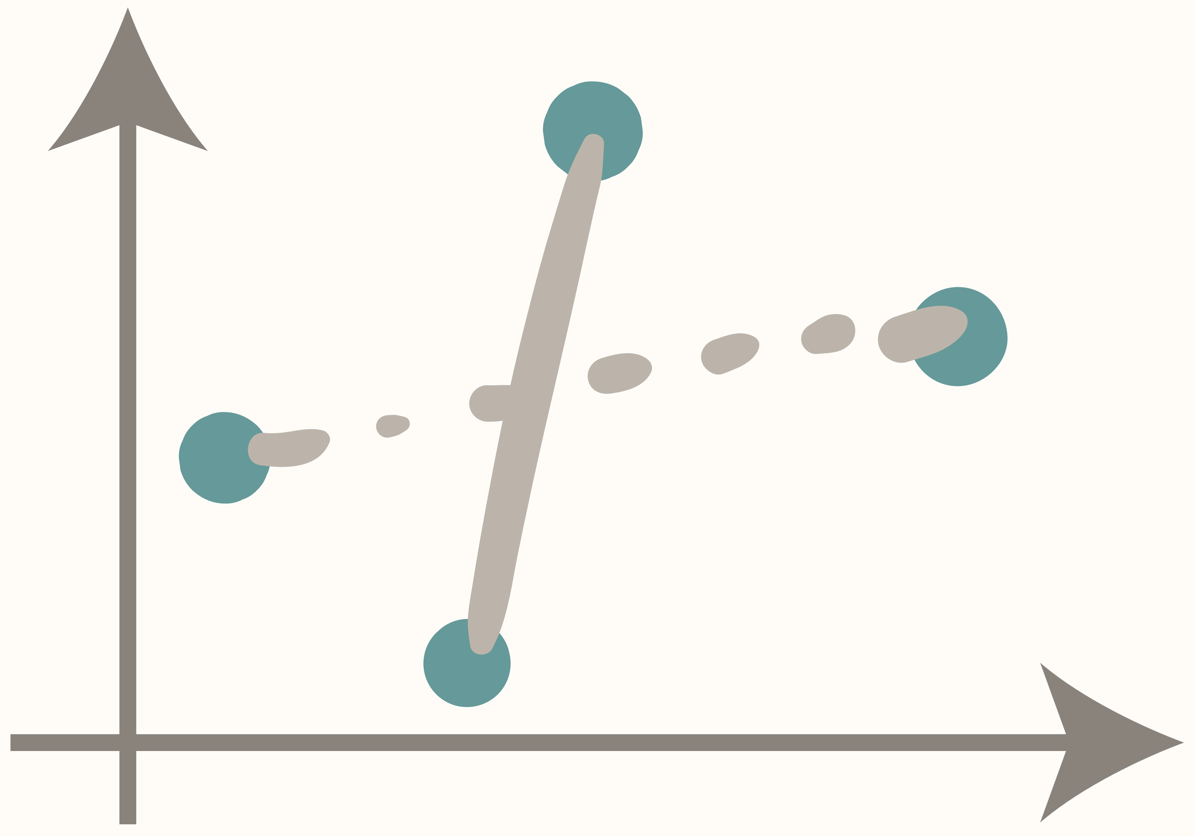


Plane permutation

= *No edge crossing in the dominance diagram*

= *Avoid the vincular pattern :*

$2\underbrace{14}3$



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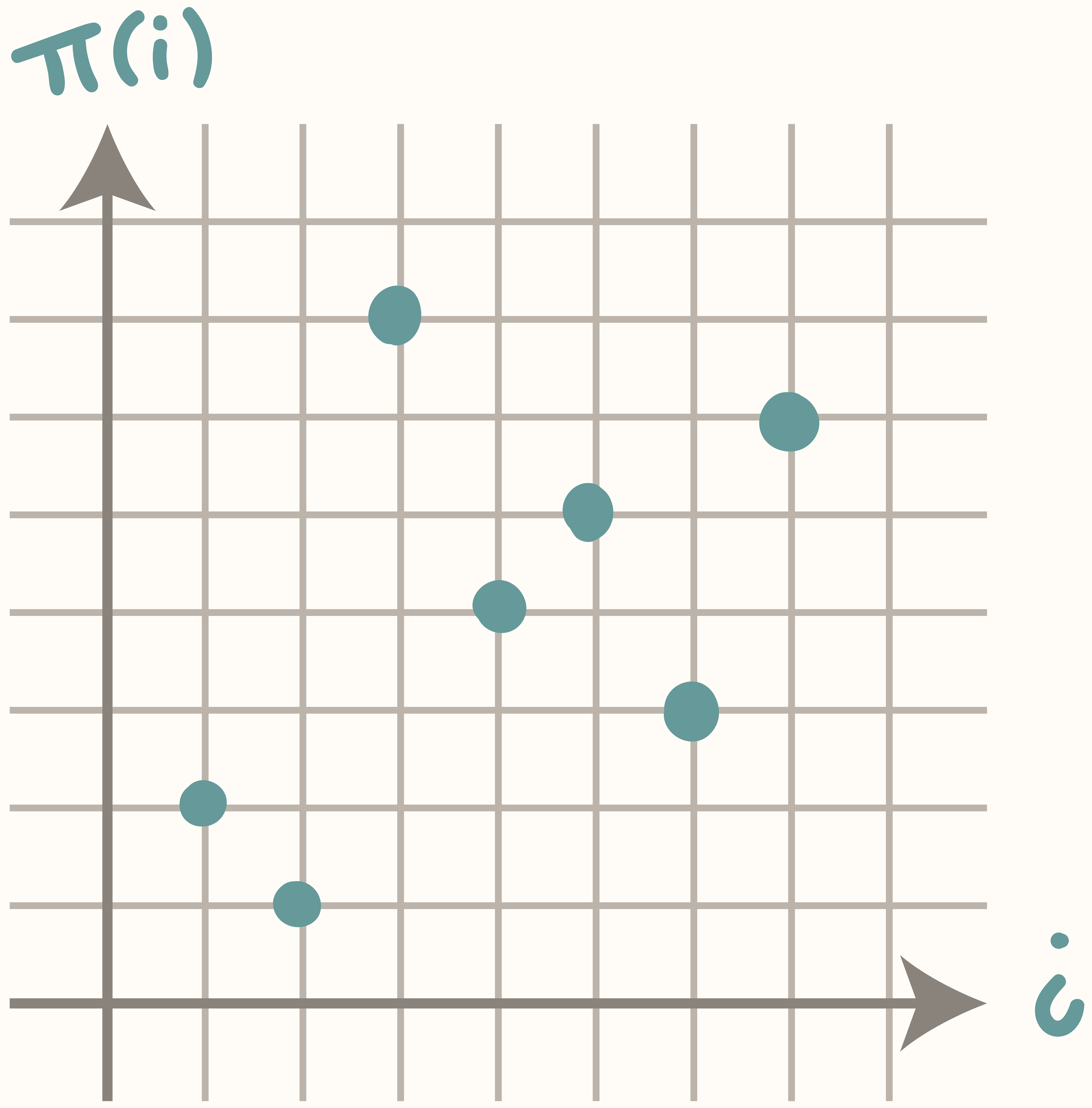
Link with plane permutations

Plane permutation $\longrightarrow$ *Poset*



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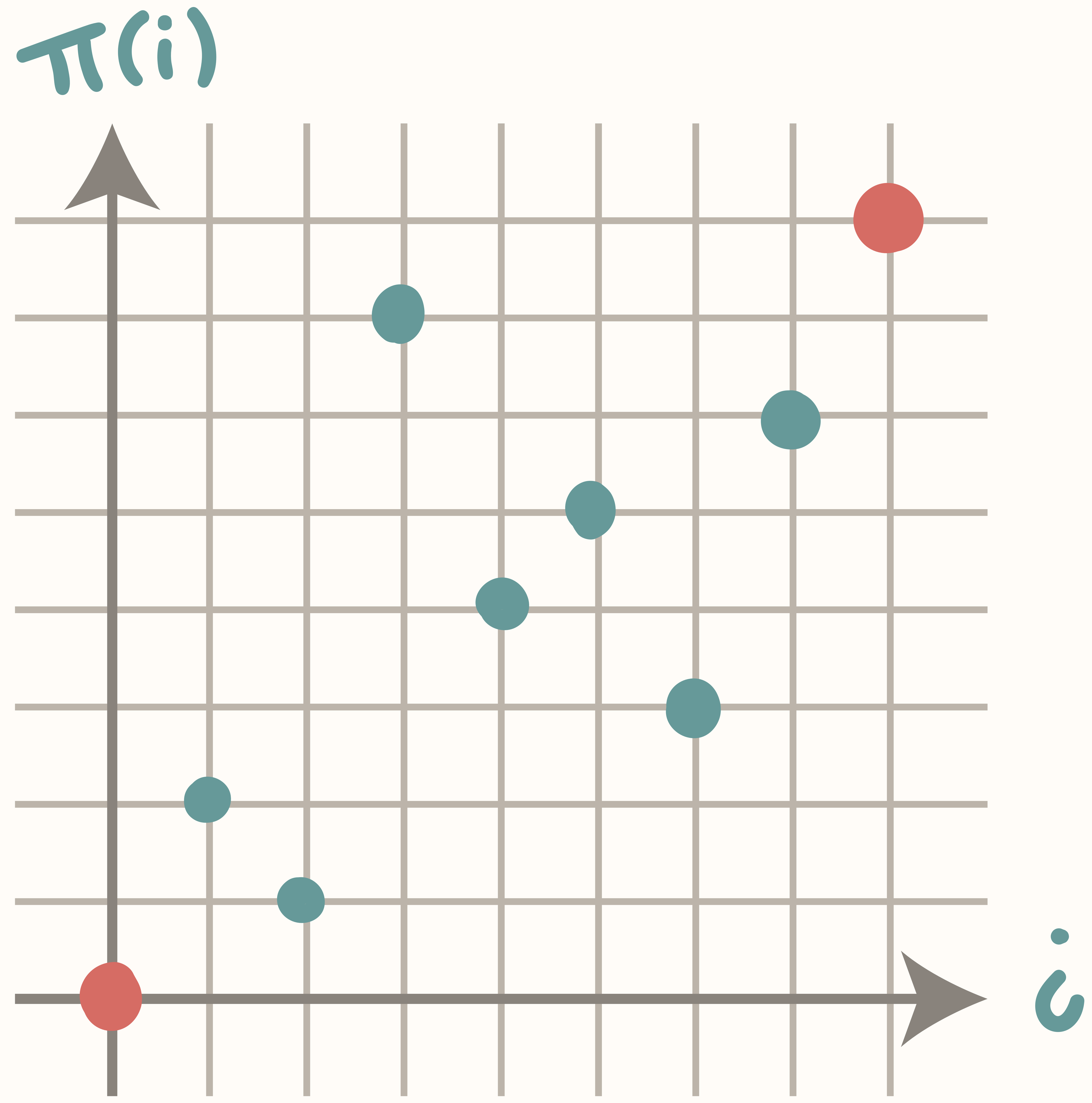
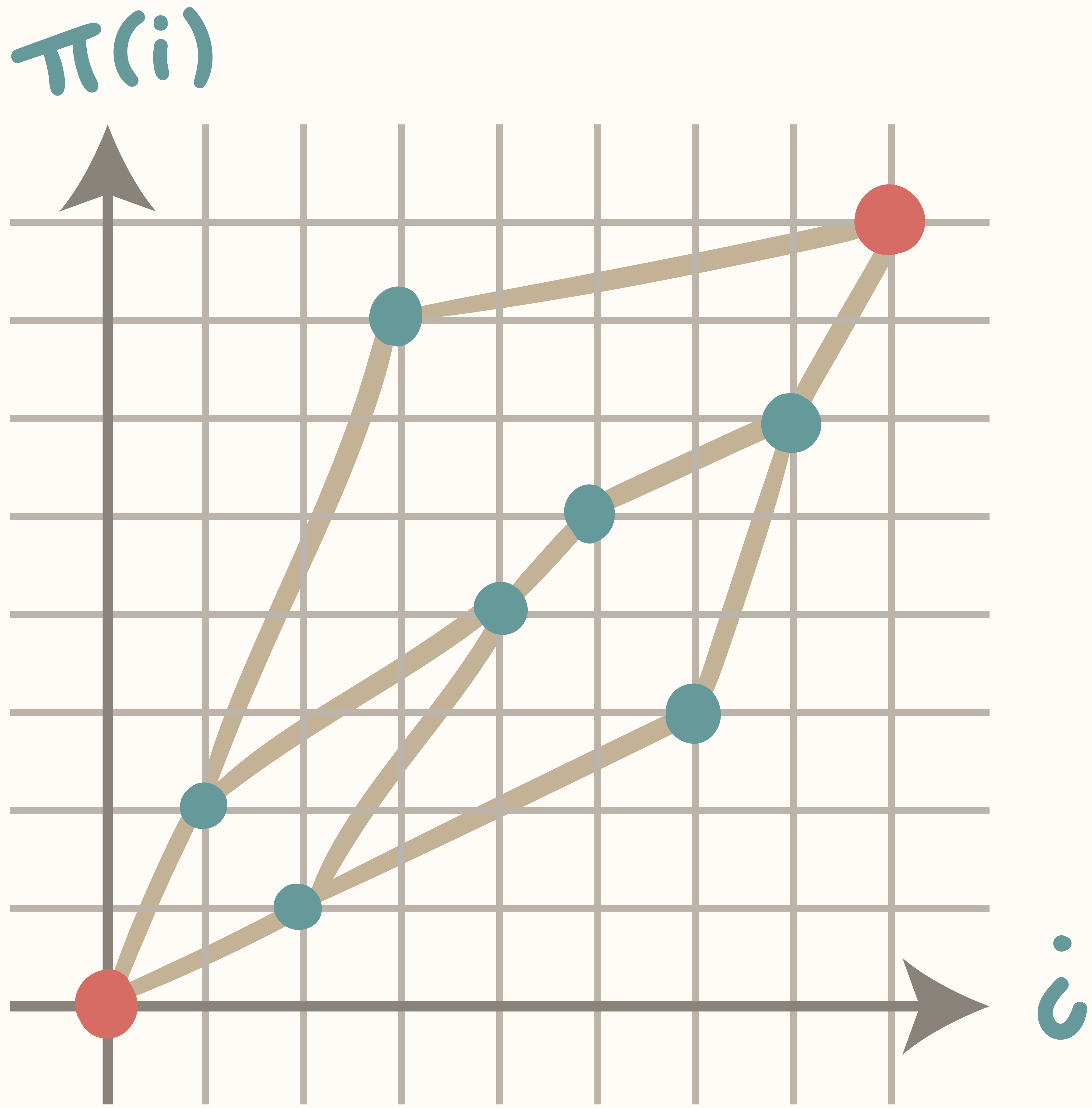


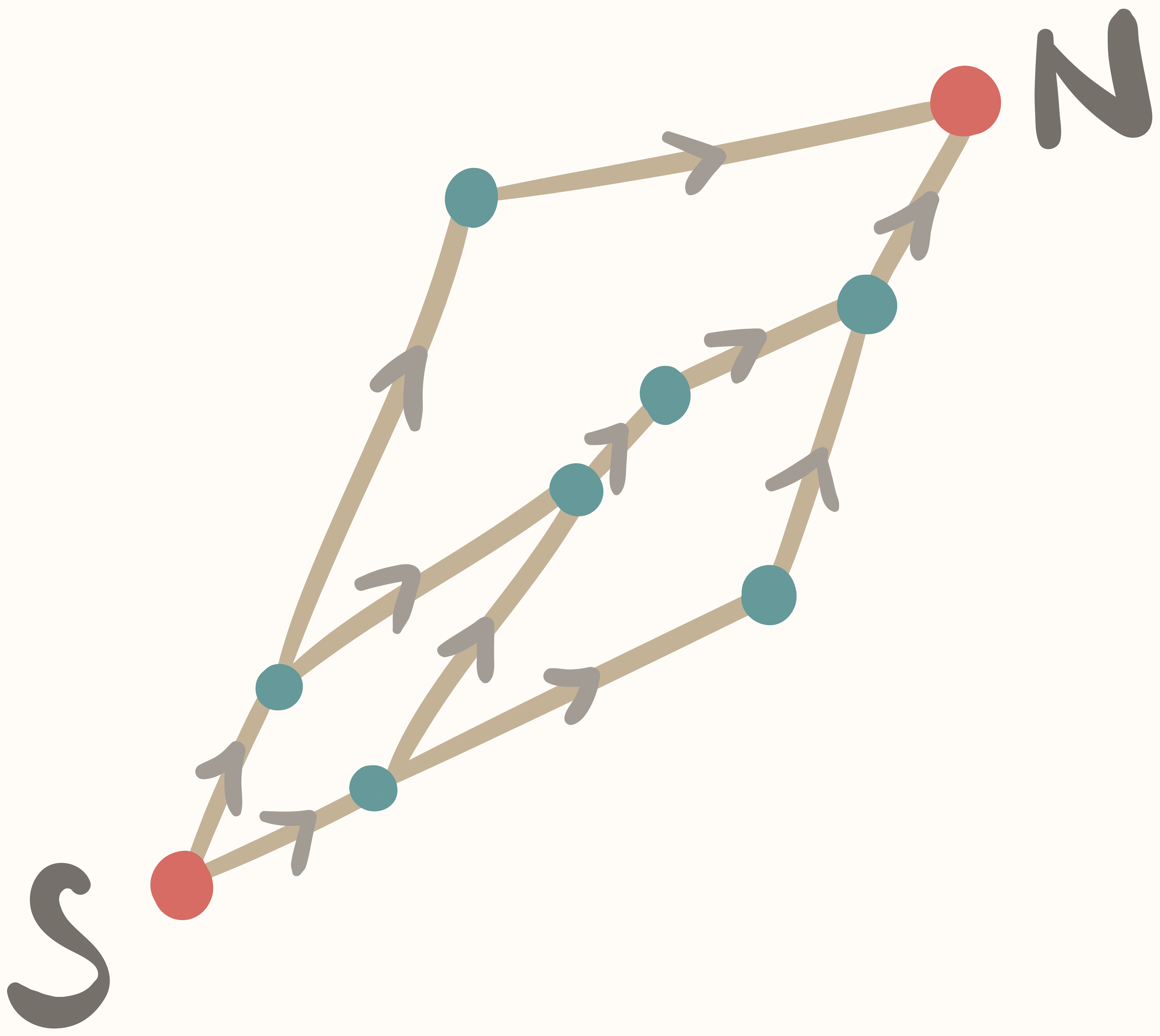
Link with plane permutations

Plane permutation ⟶ *Poset*



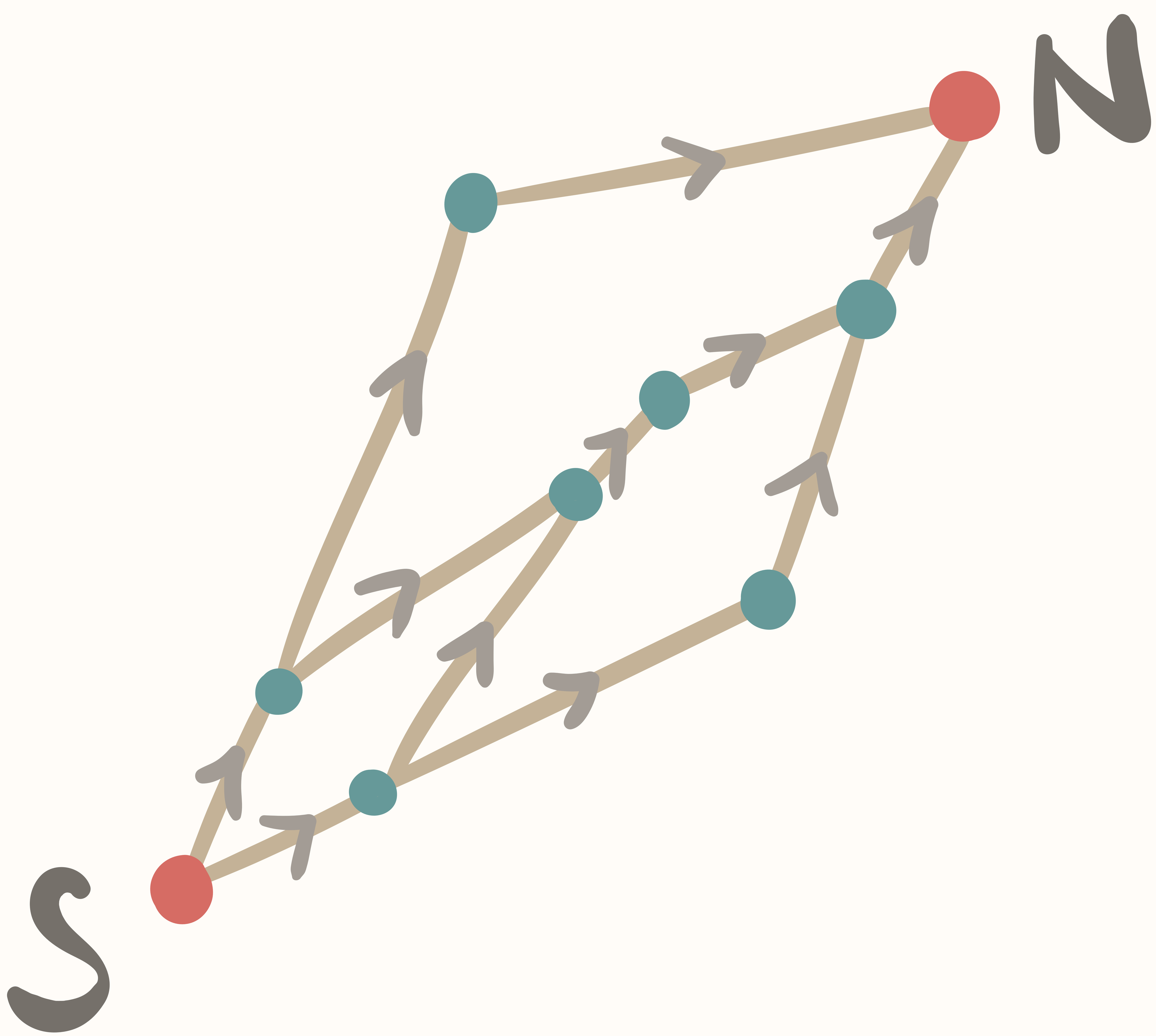
Link with plane permutations

Plane permutation → *Poset*



Link with plane permutations

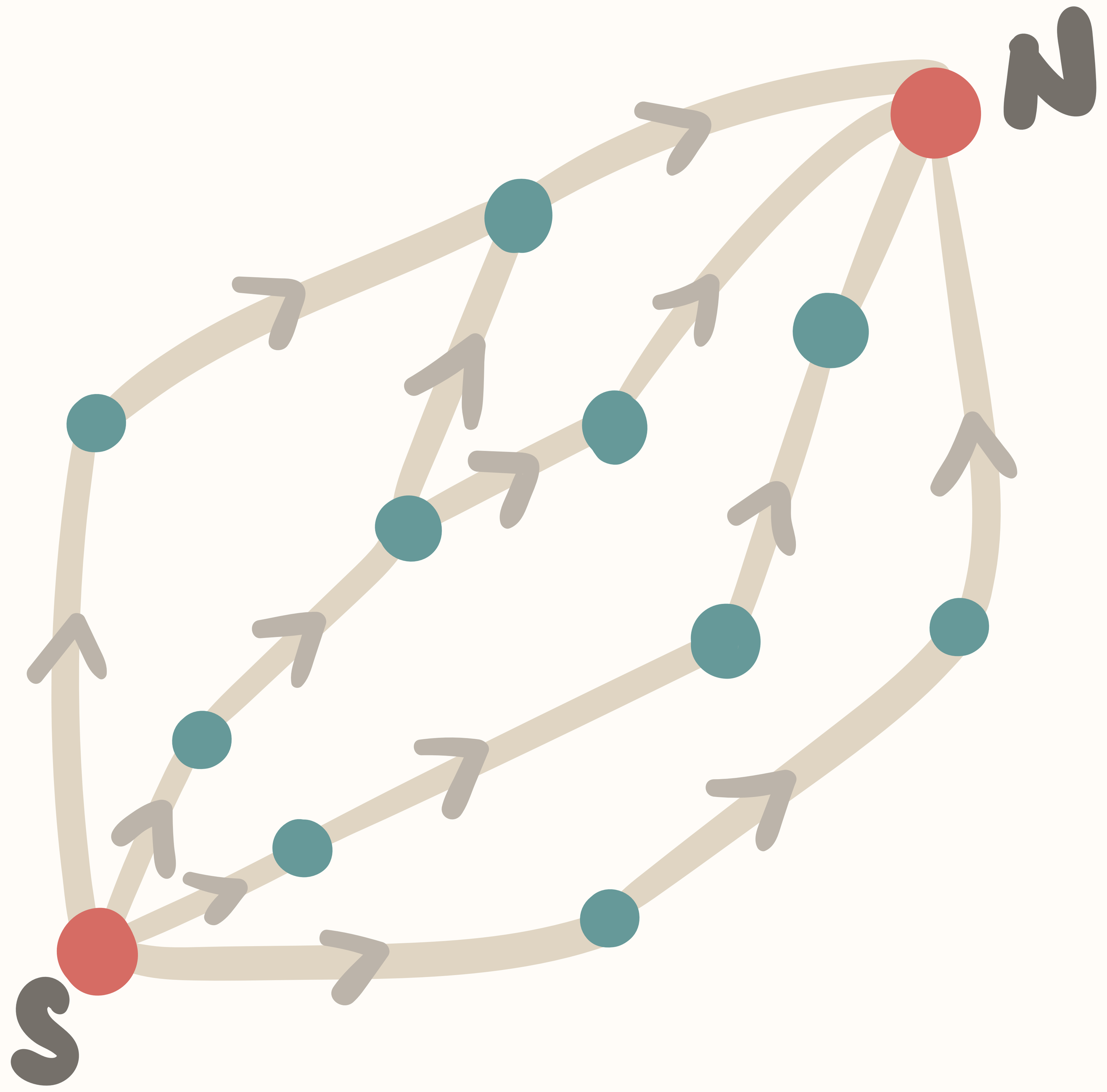
Plane permutation ⟶ *Poset*



Baxter permutations and plane bipolar orientations,
N. Bonichon, M. Bousquet-Mélou, & E. Fusy (2010)

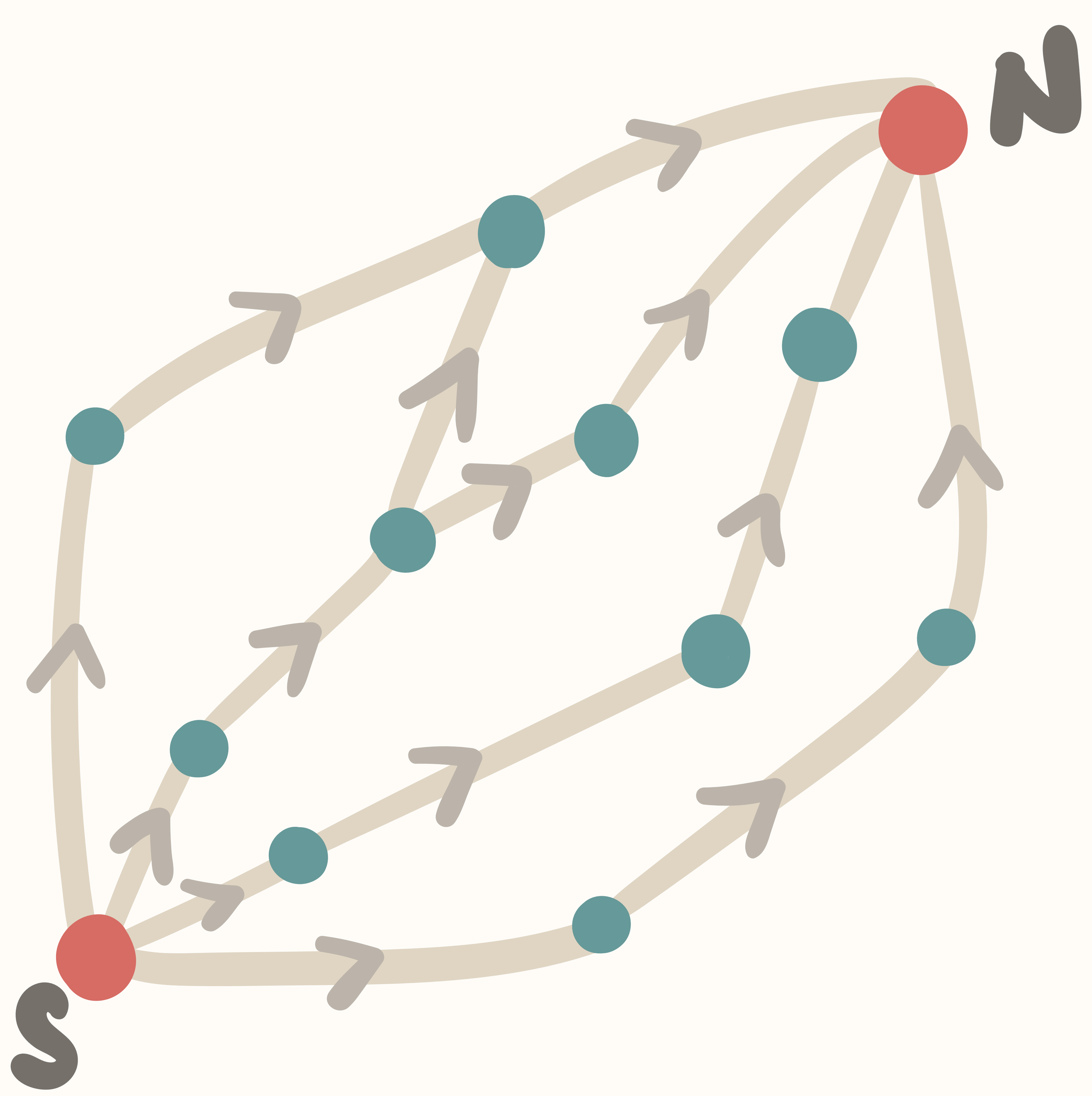
Link with plane permutations

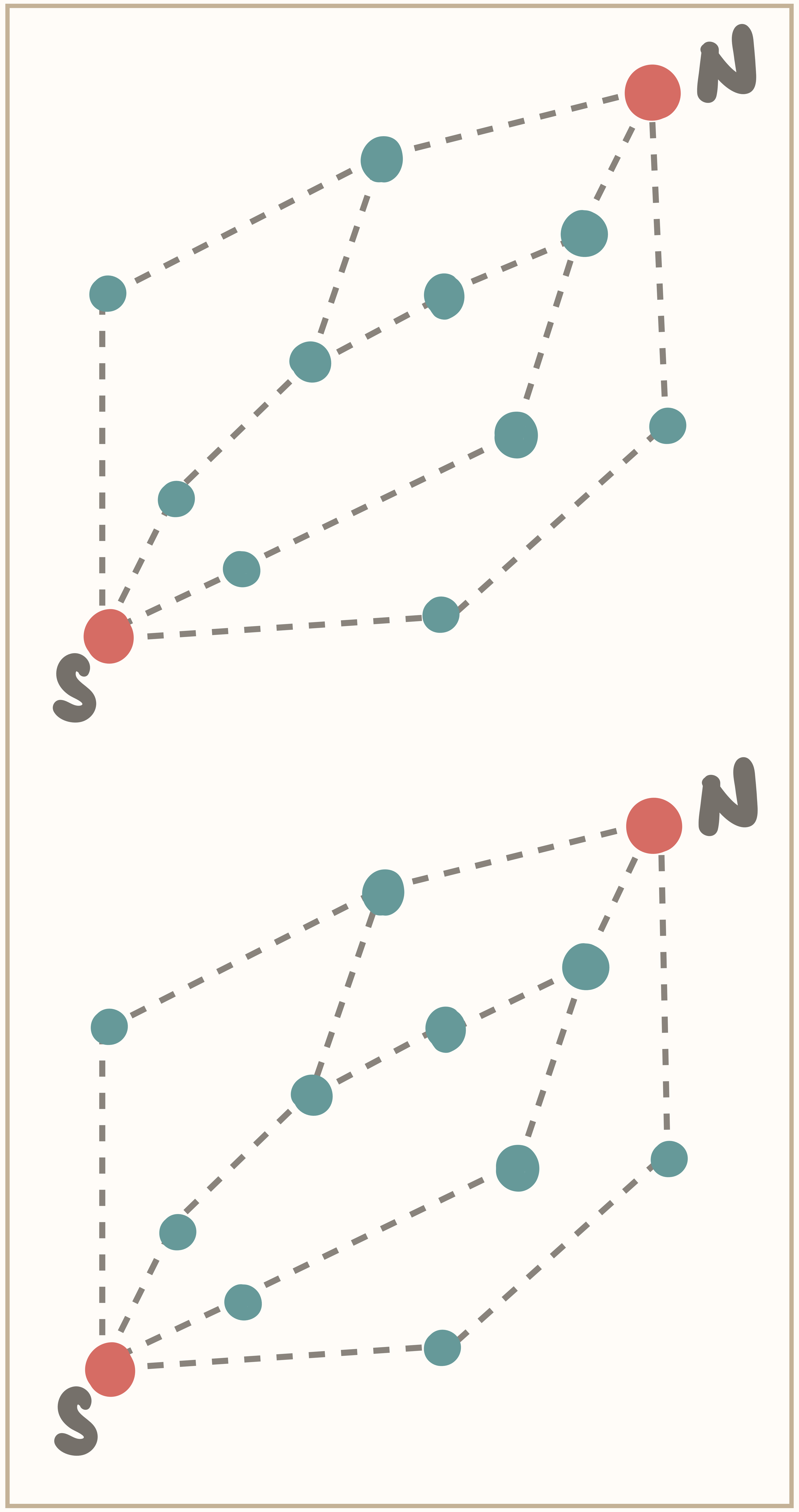
Poset $\longrightarrow$ *Plane permutation*



Link with plane permutations

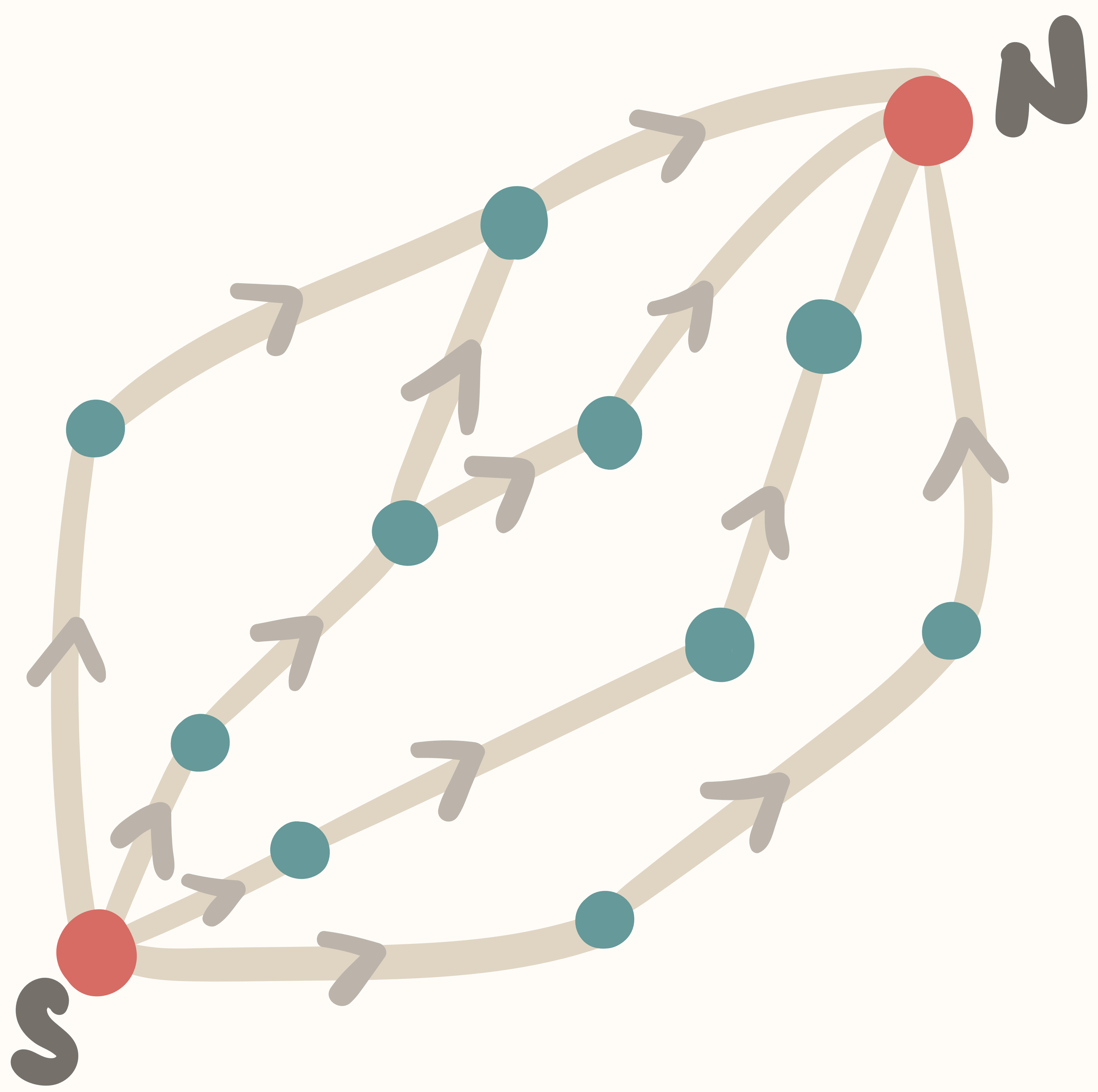
Poset ⟶ *Plane permutation*

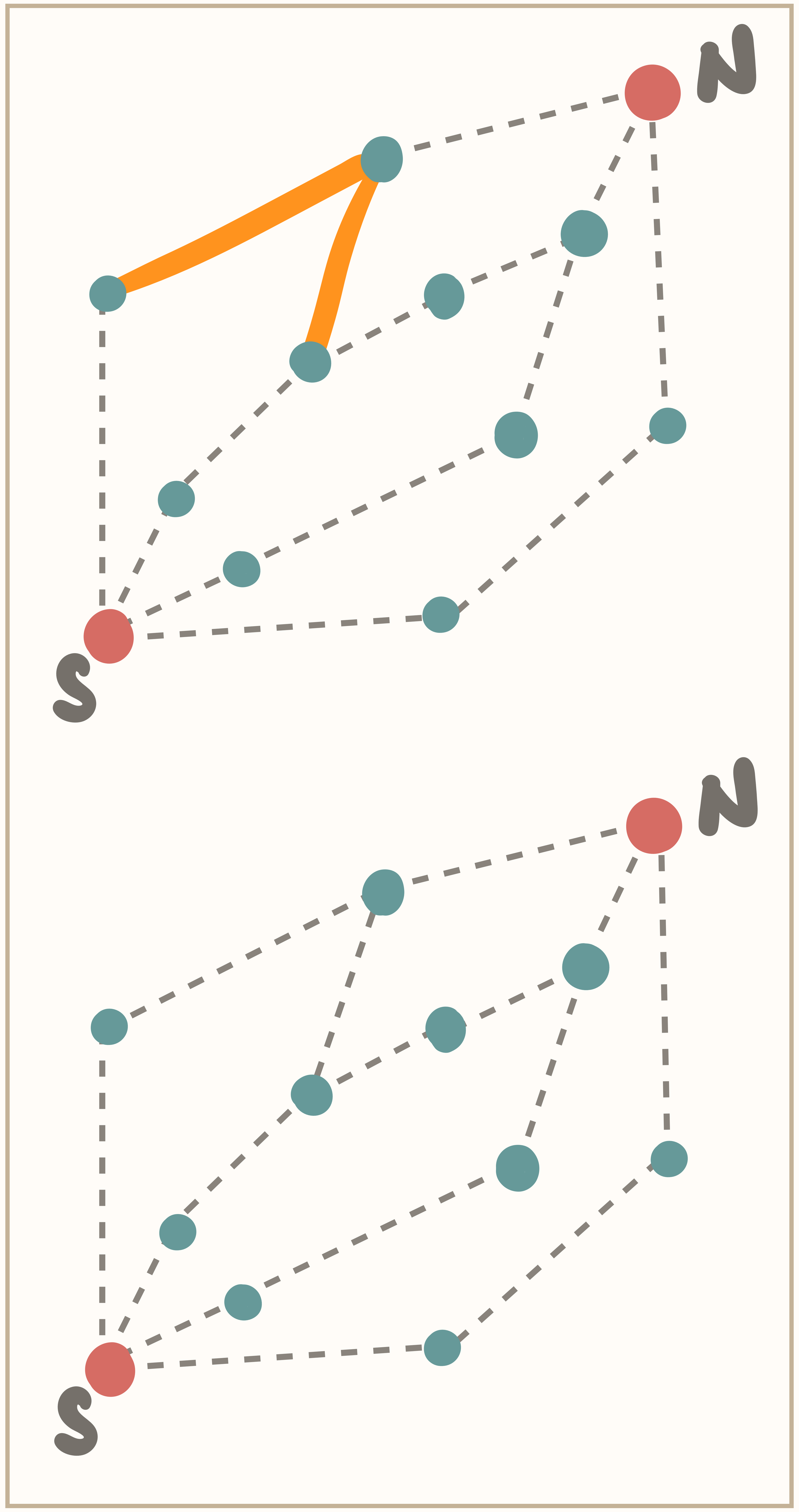




Link with plane permutations

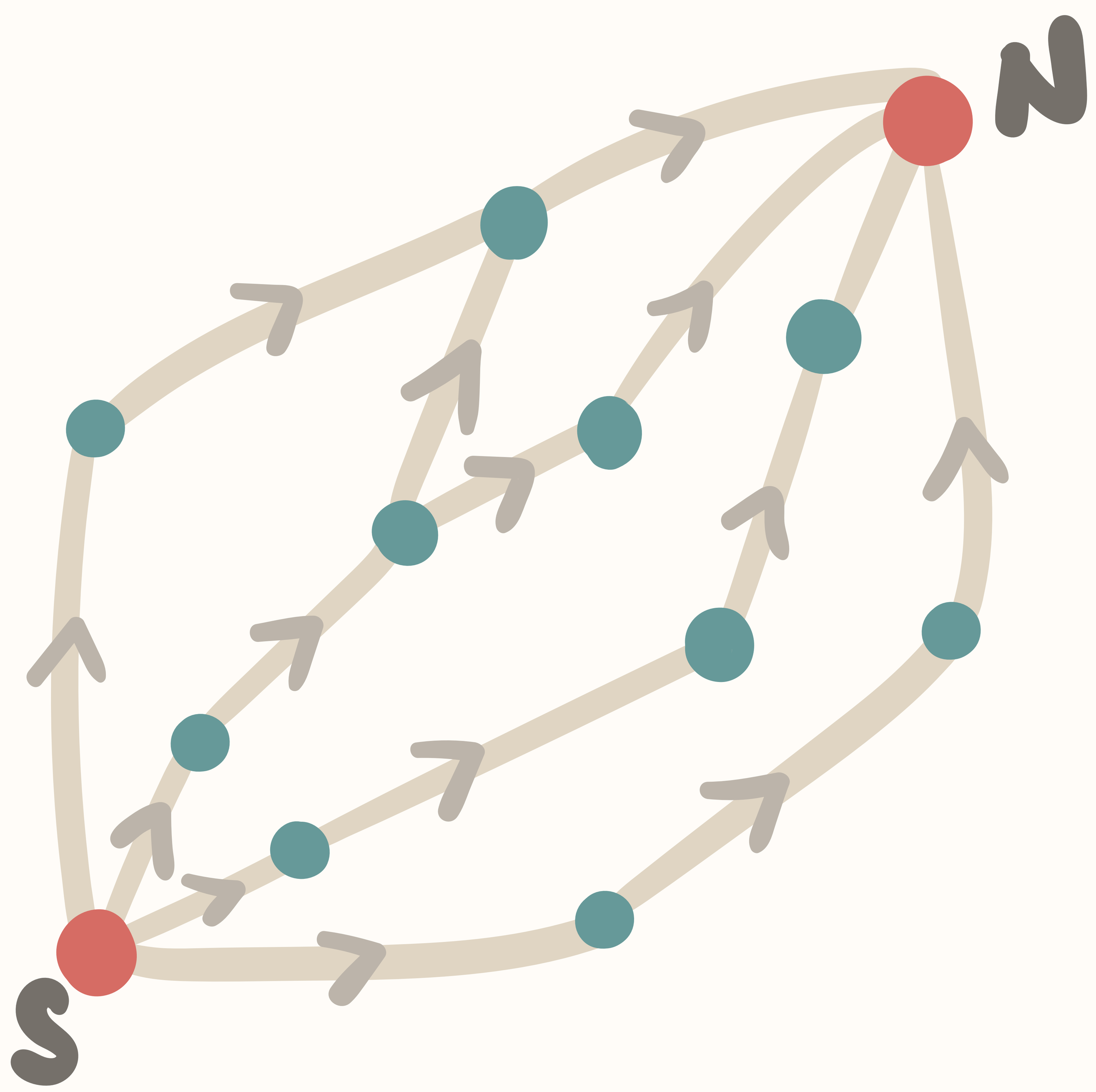
Poset ⟶ *Plane permutation*

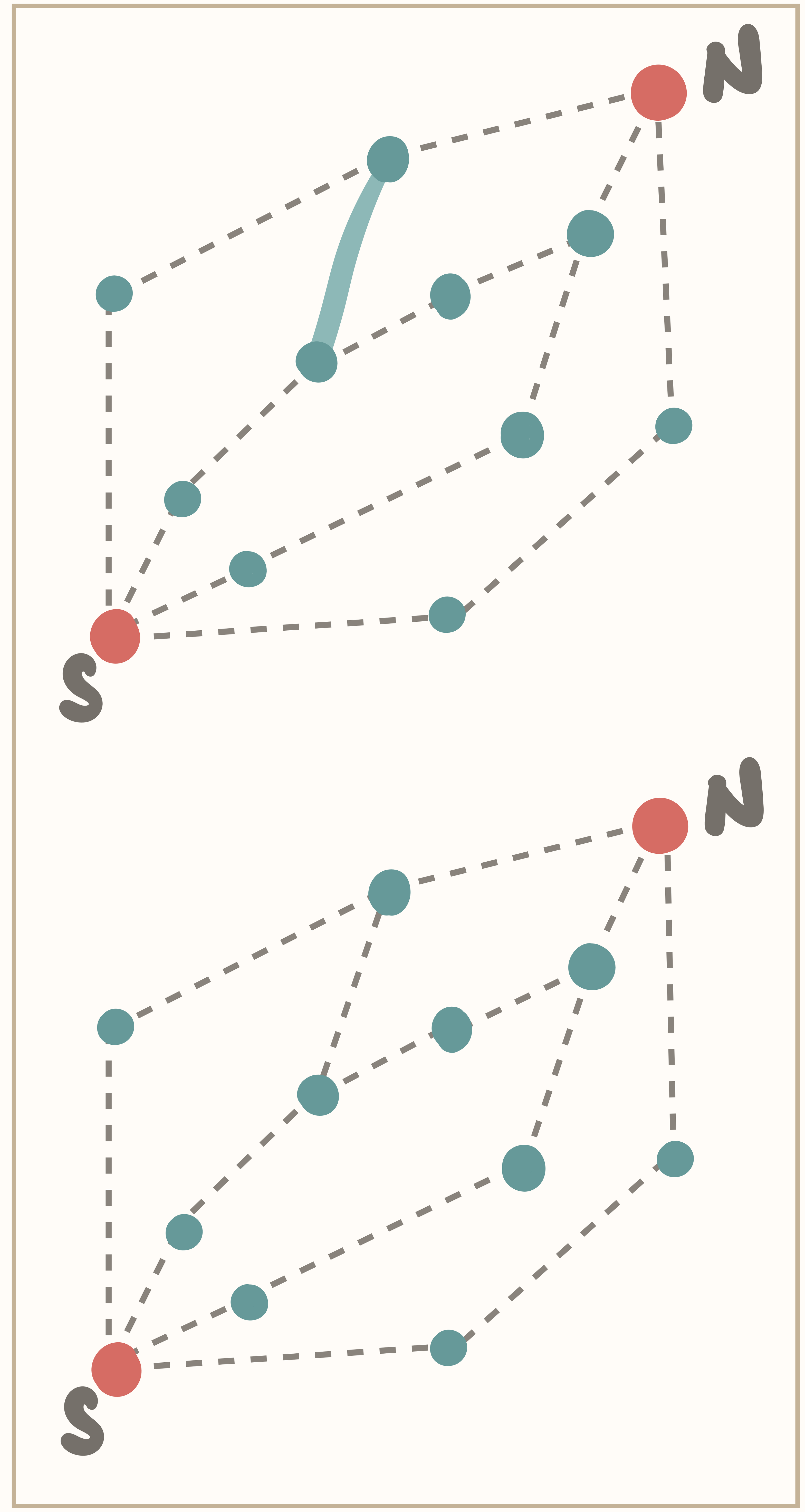




Link with plane permutations

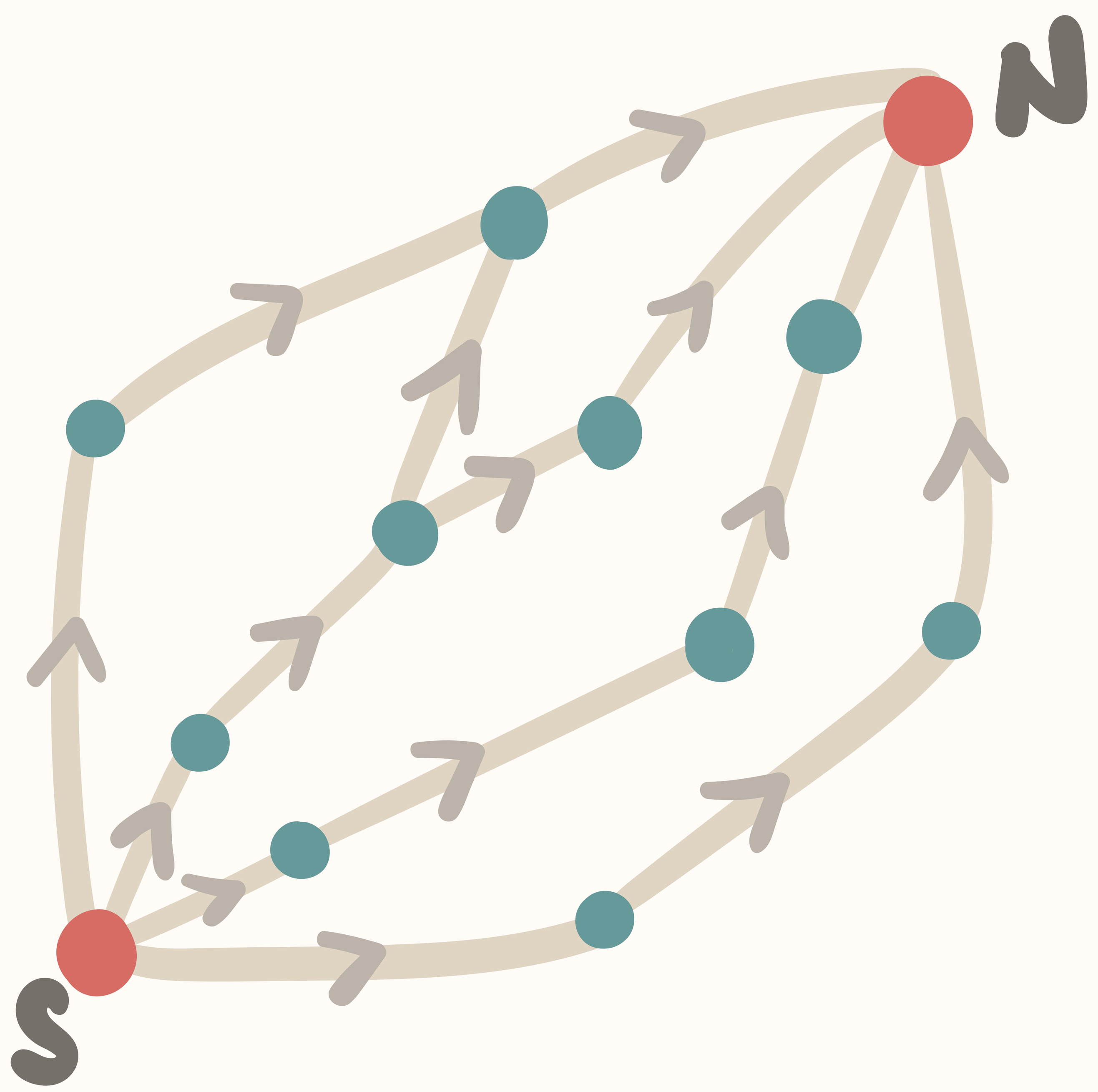
Poset ⟶ *Plane permutation*

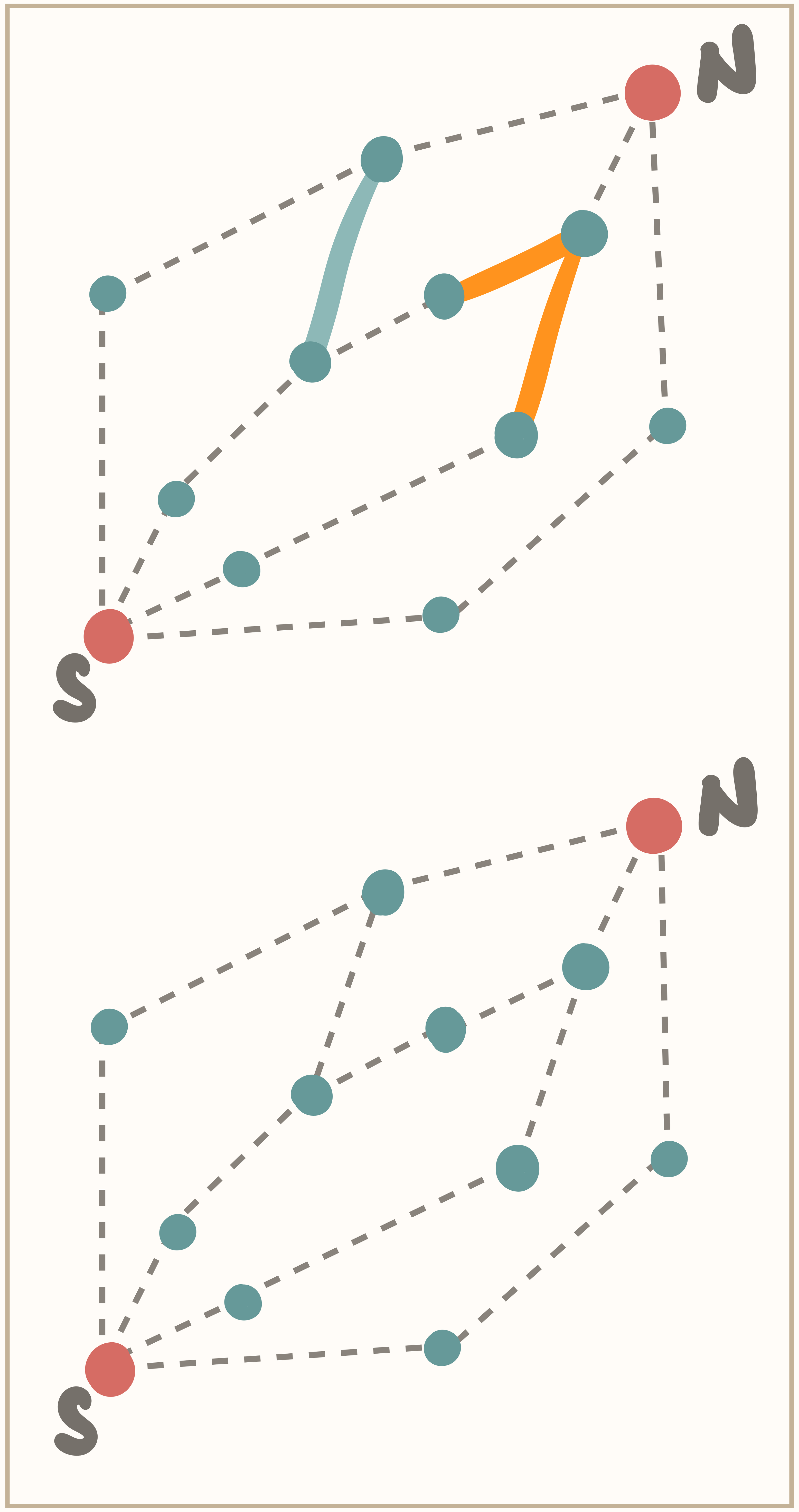




Link with plane permutations

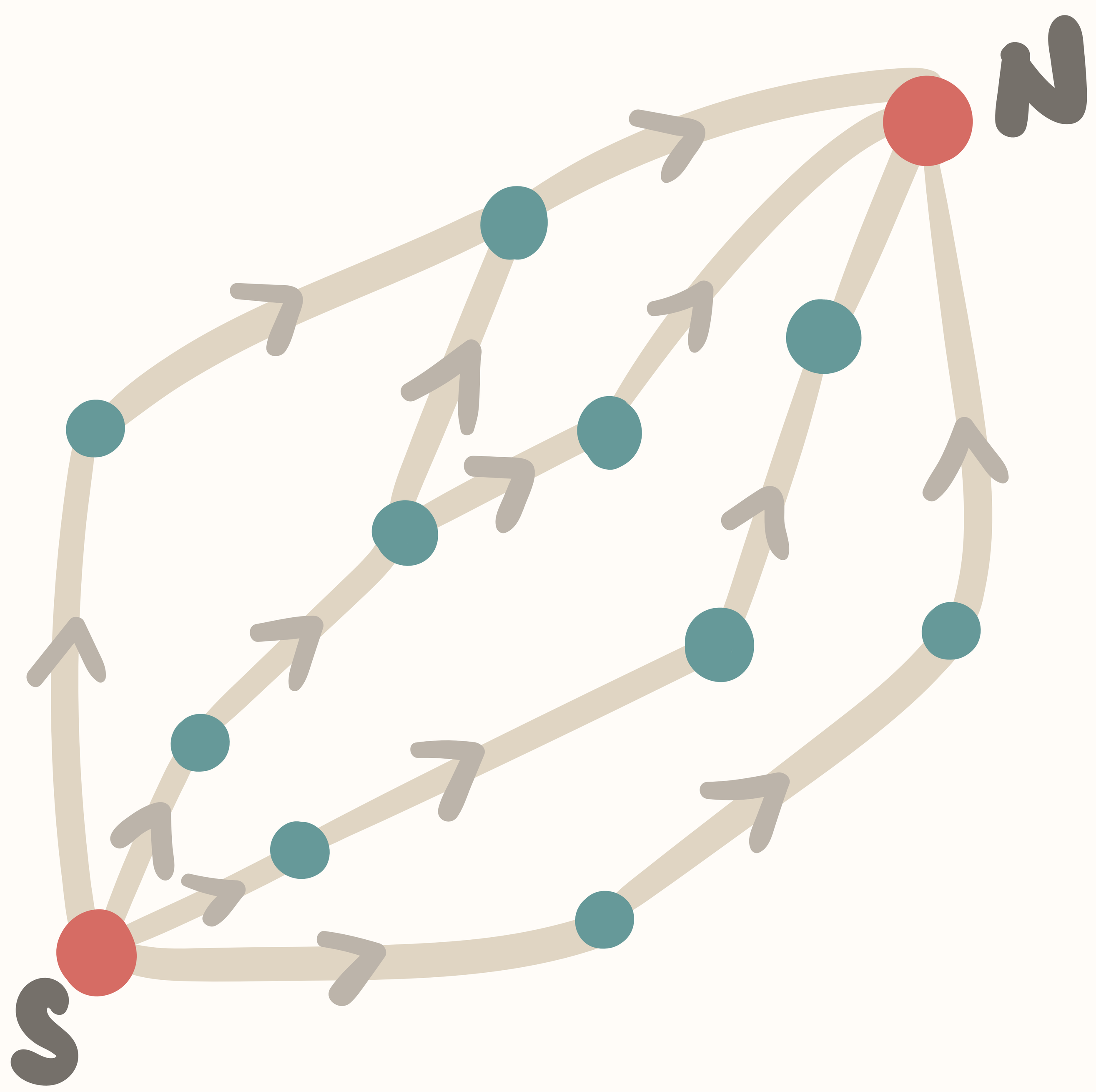
Poset ⟶ *Plane permutation*

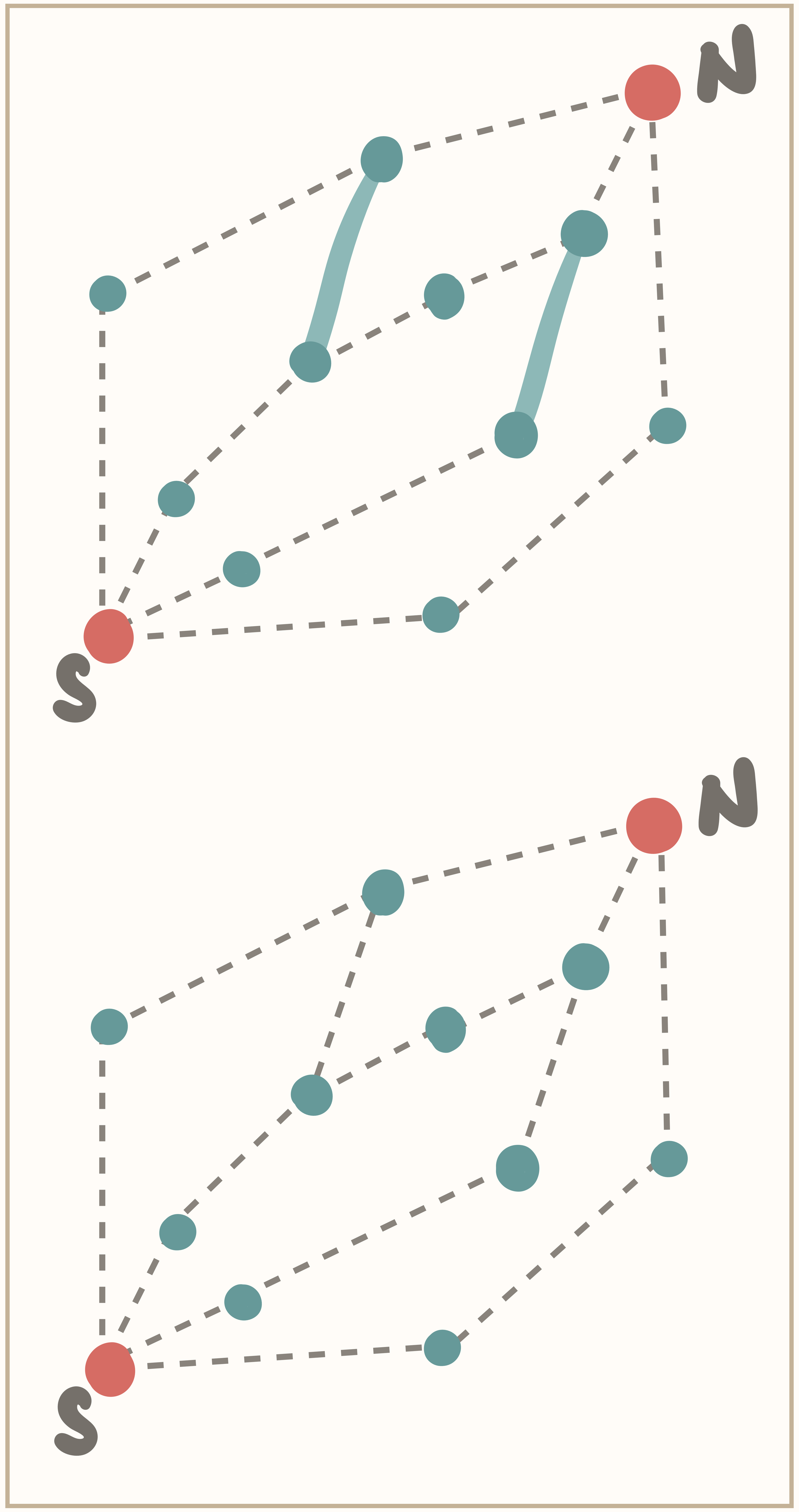




Link with plane permutations

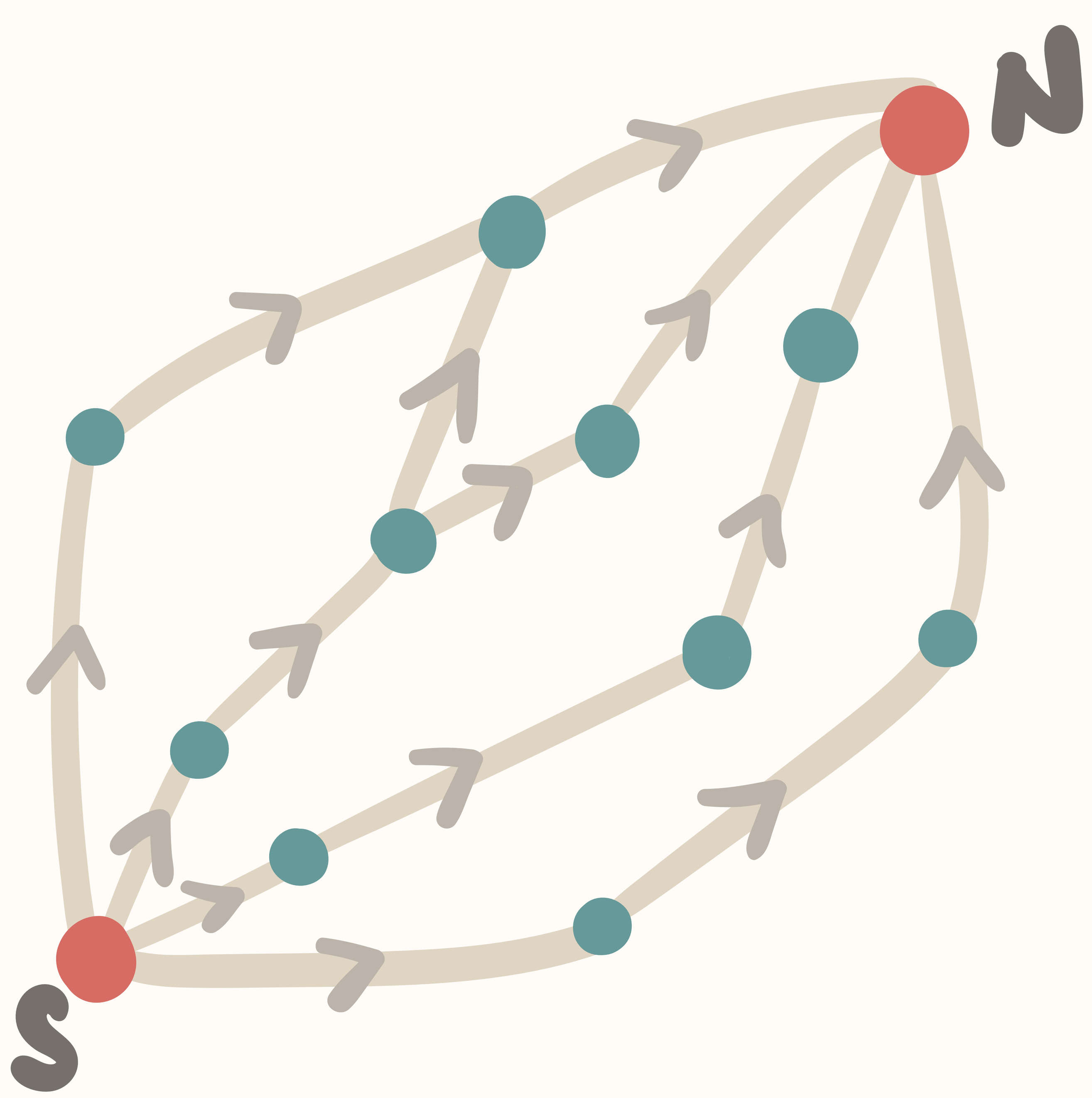
Poset ⟶ *Plane permutation*

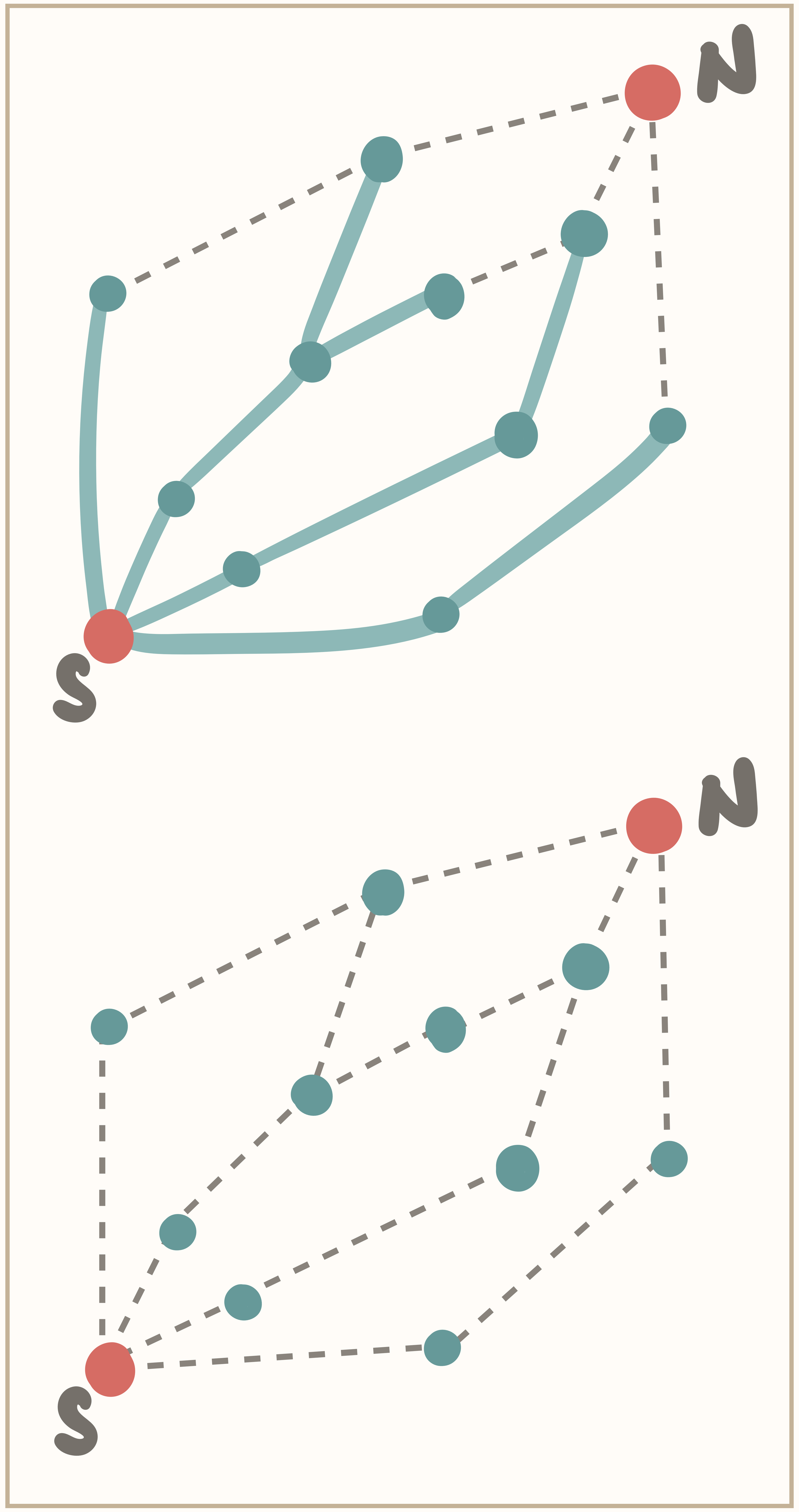




Link with plane permutations

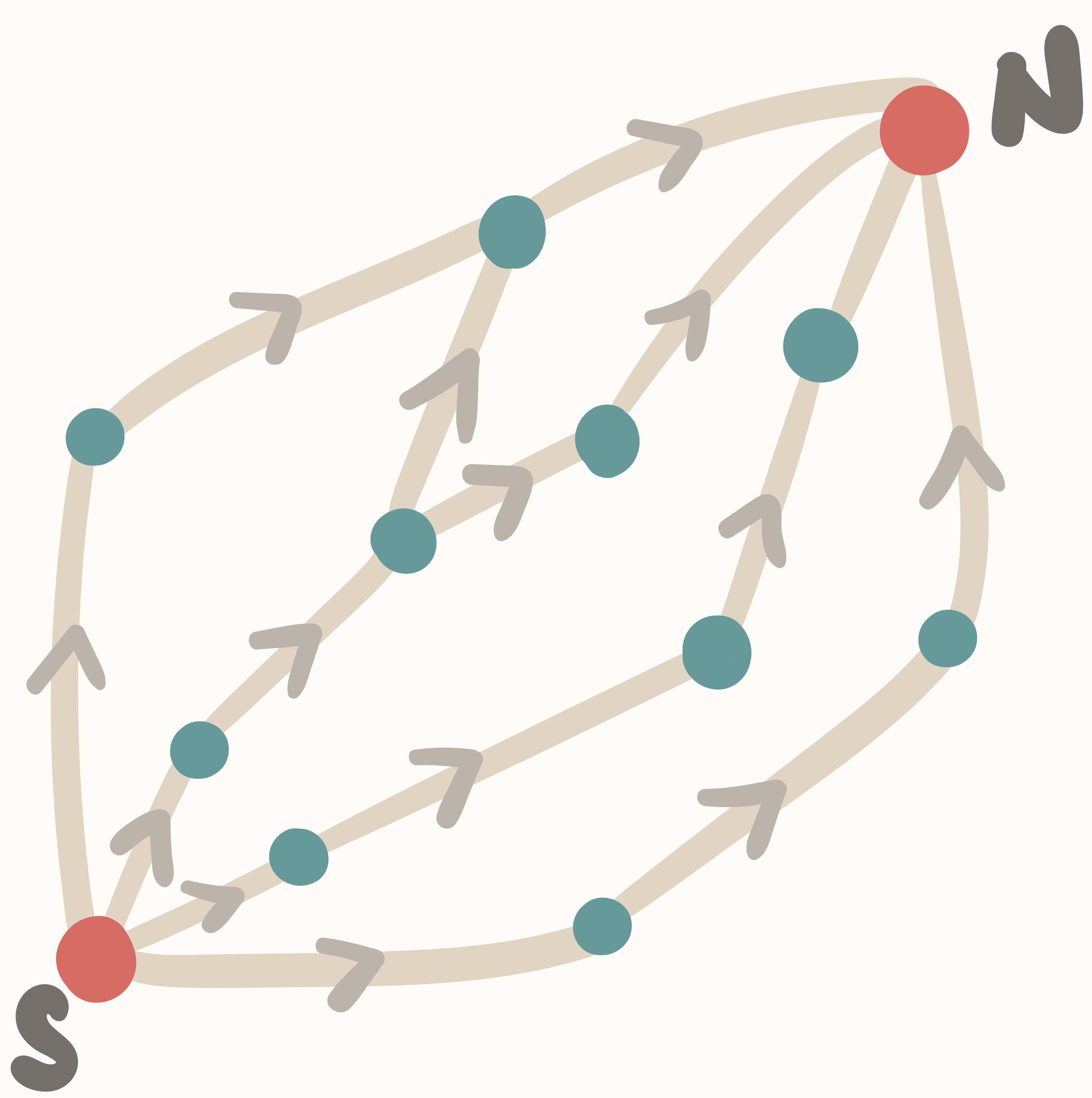
Poset → *Plane permutation*

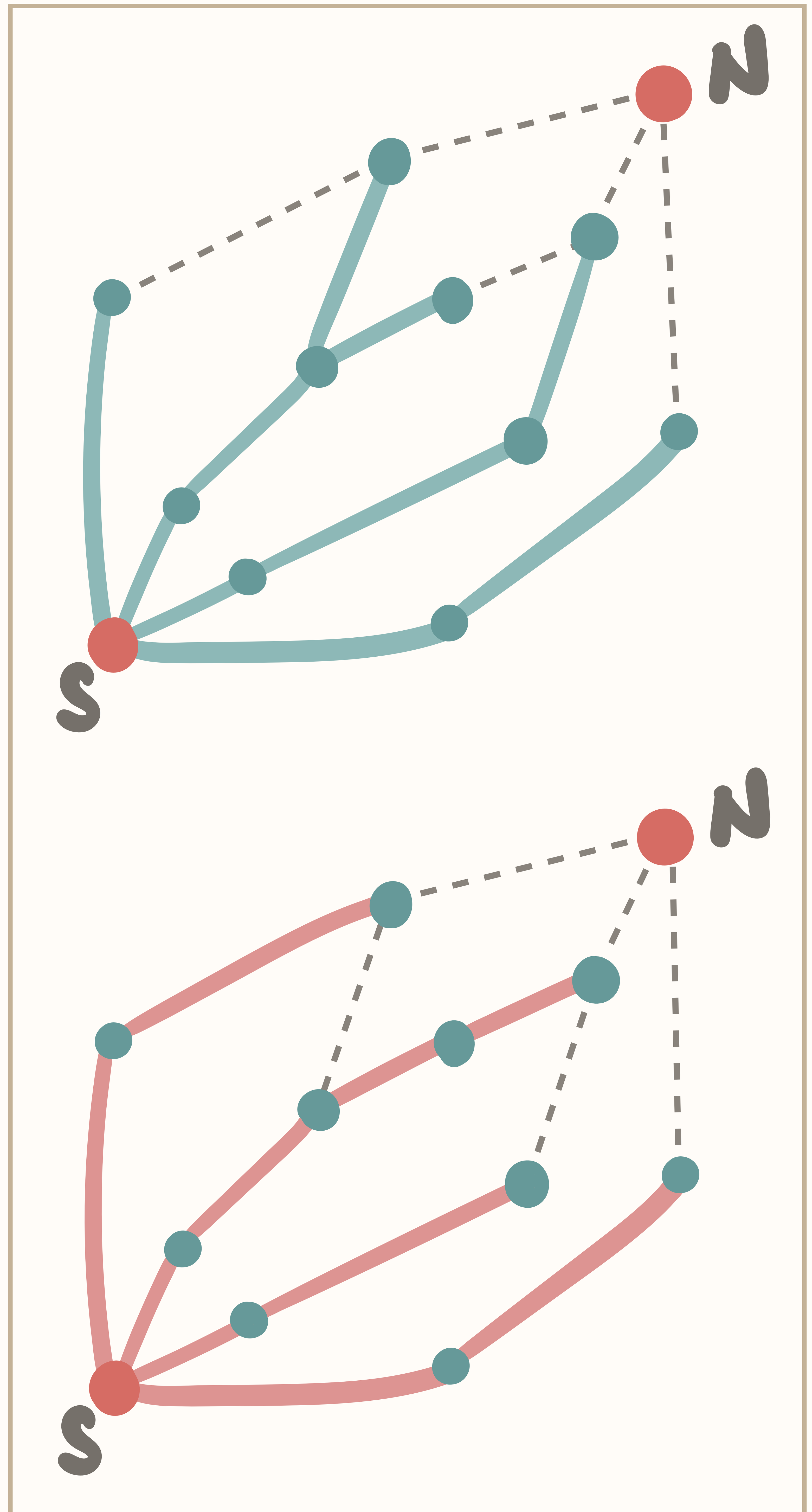




Link with plane permutations

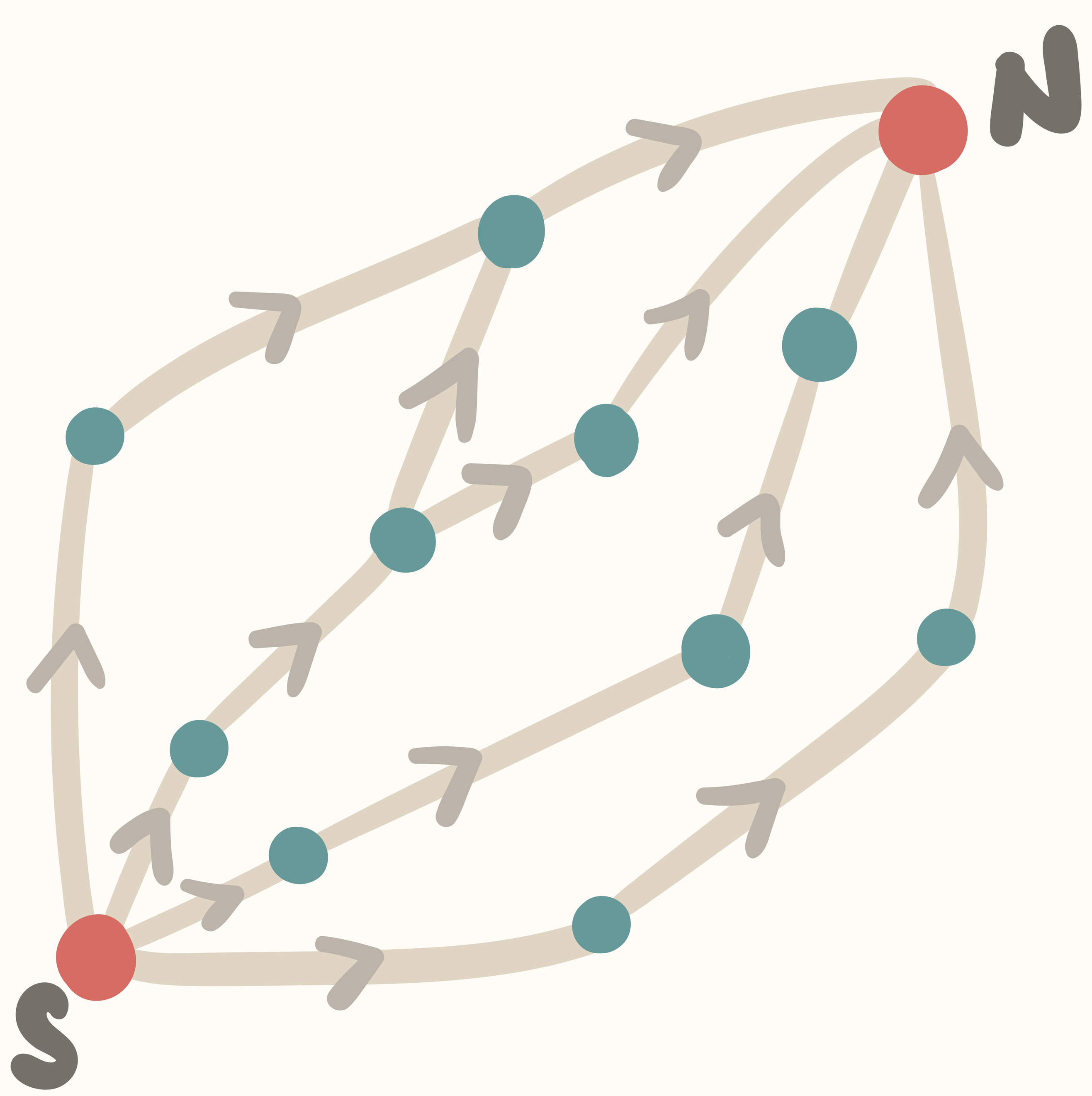
Poset ⟶ *Plane permutation*

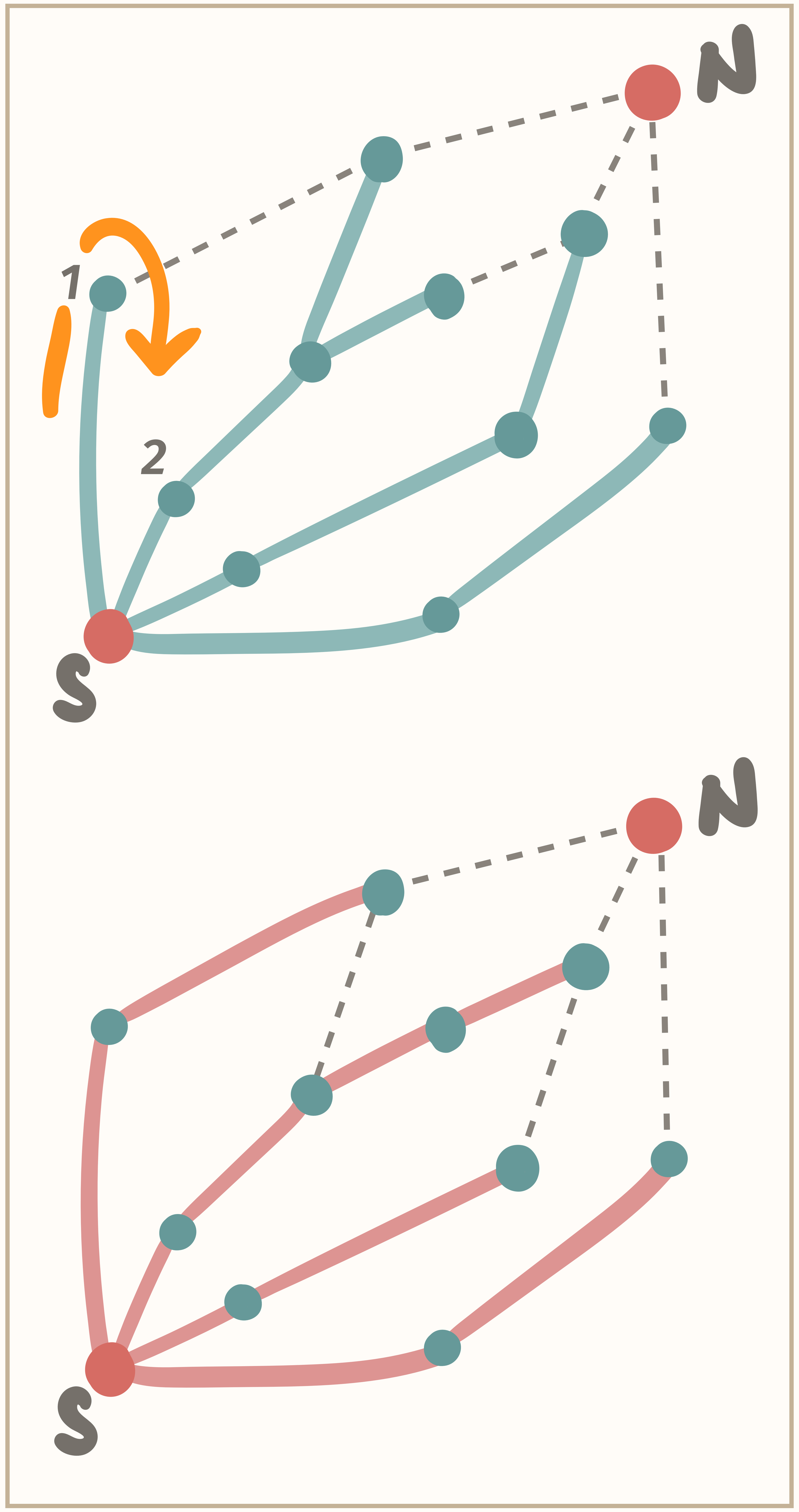




Link with plane permutations

Poset ⟶ *Plane permutation*





Link with plane permutations

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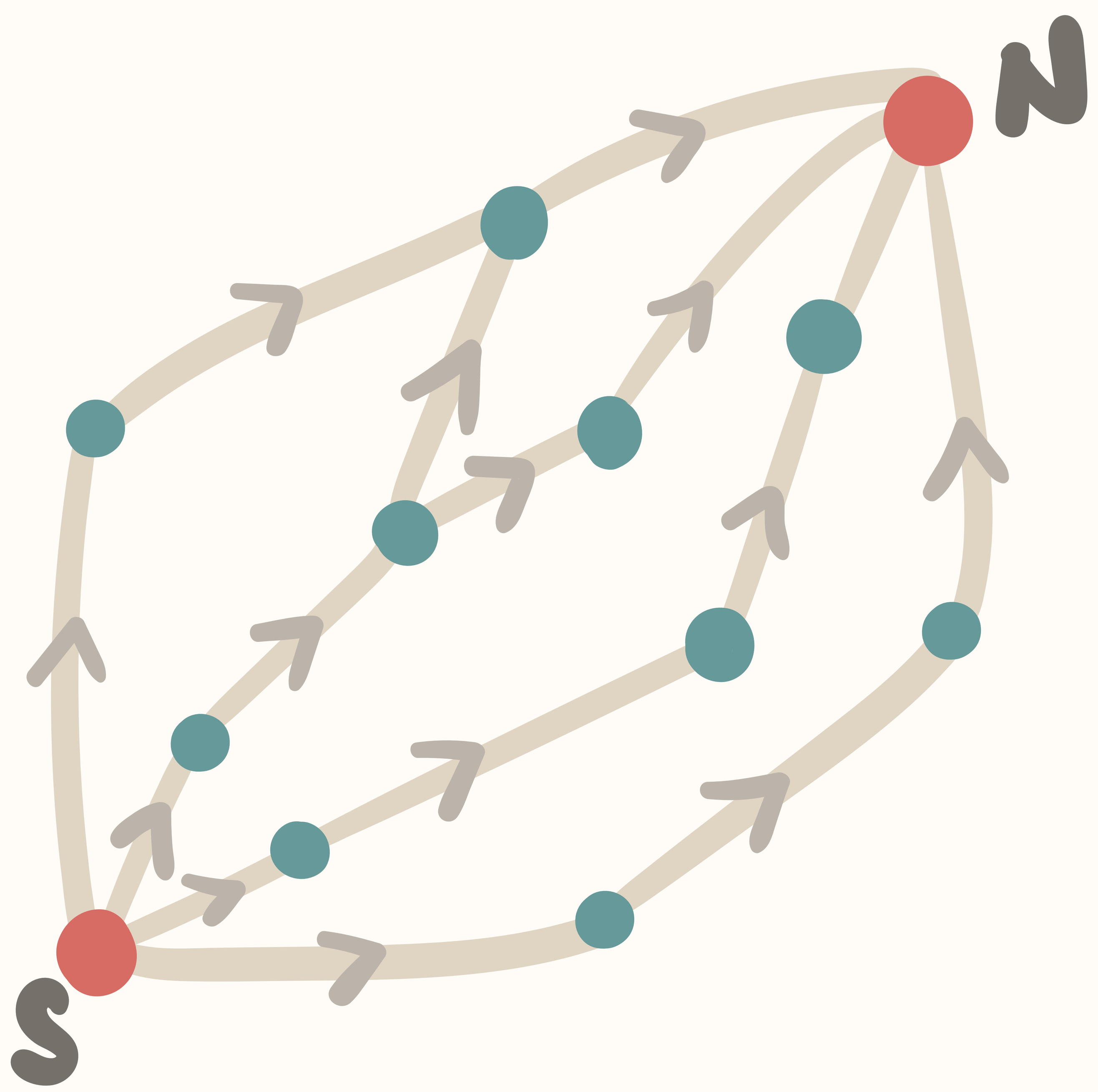

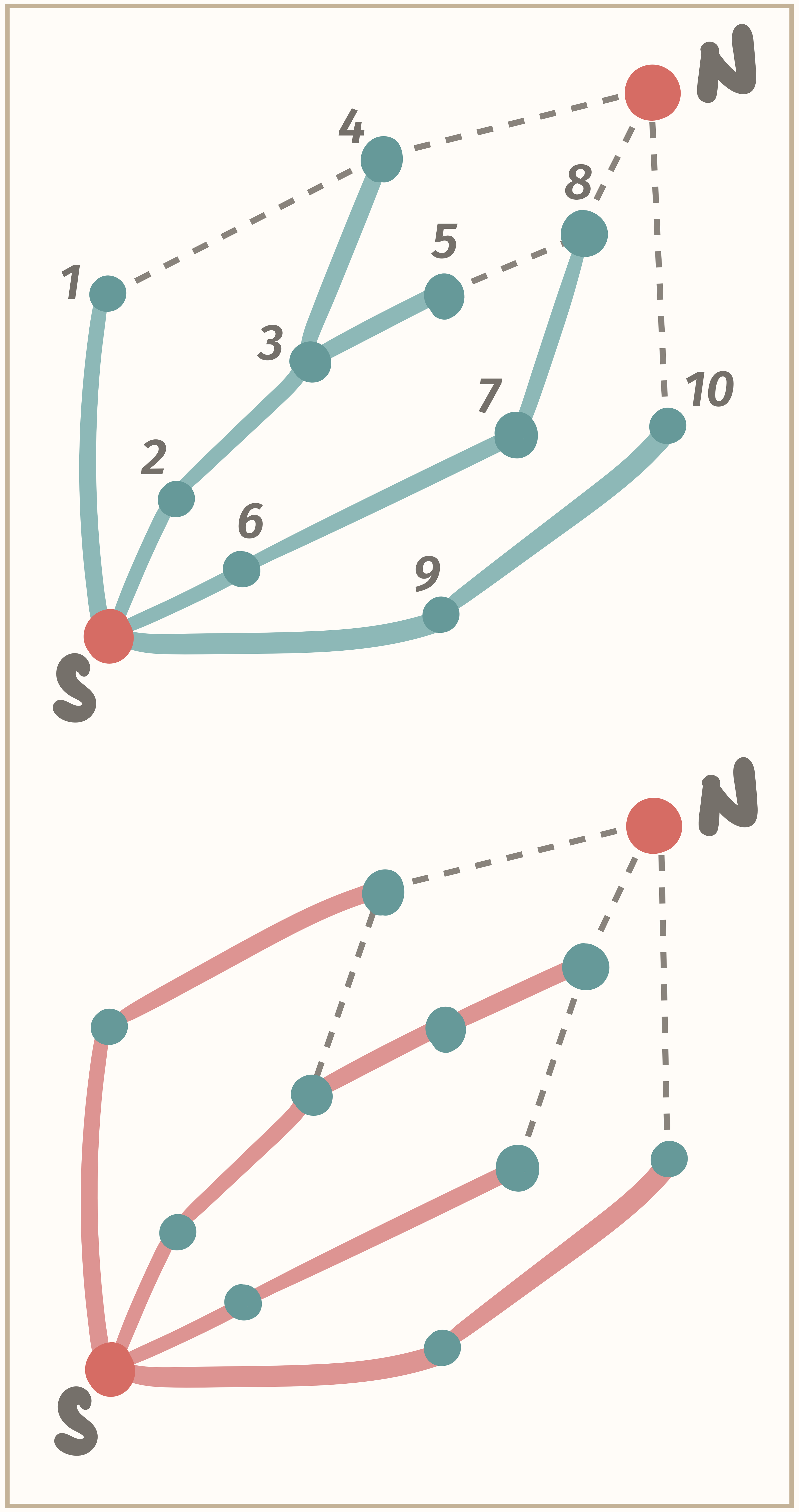

Link with plane permutations

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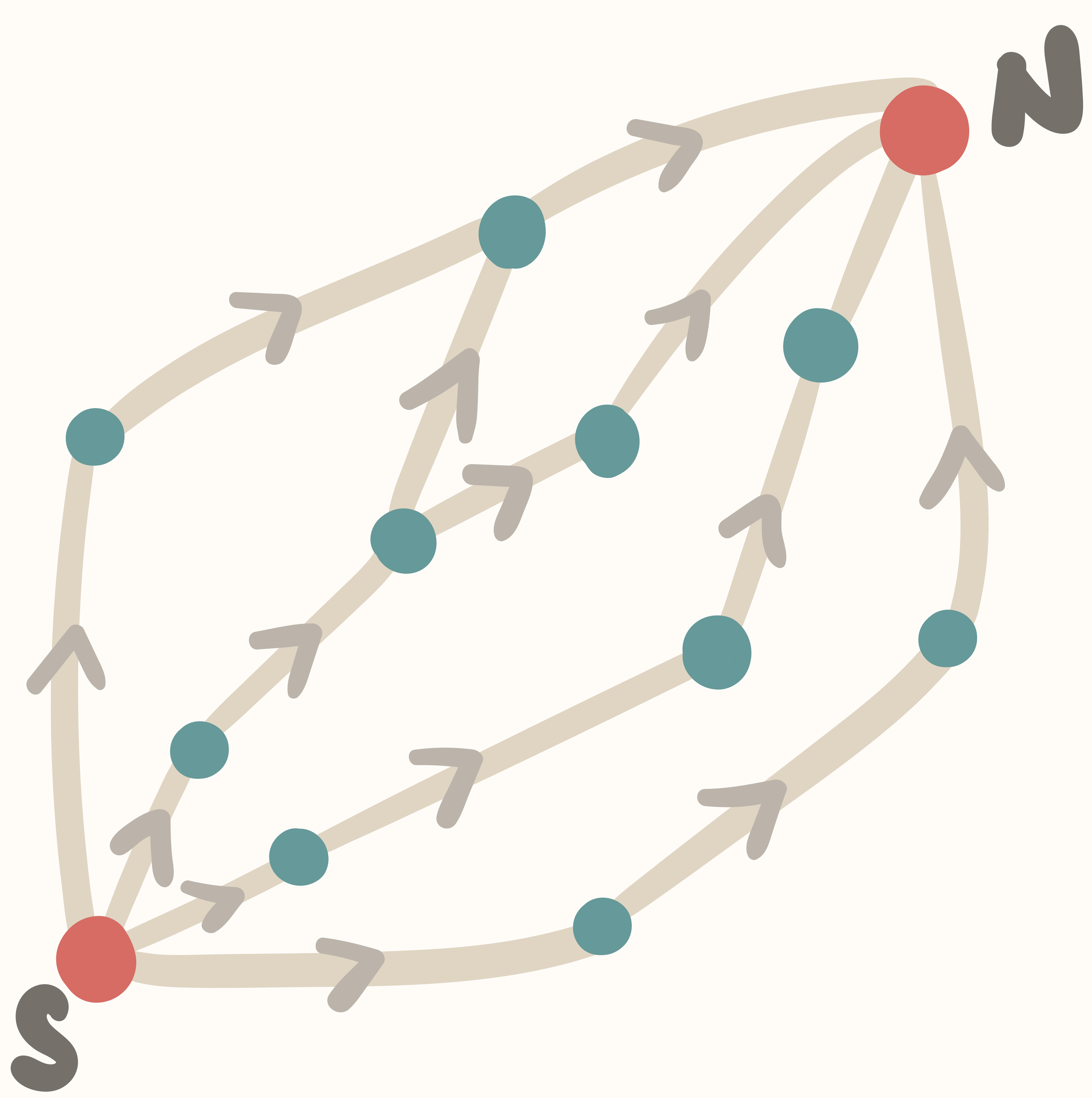
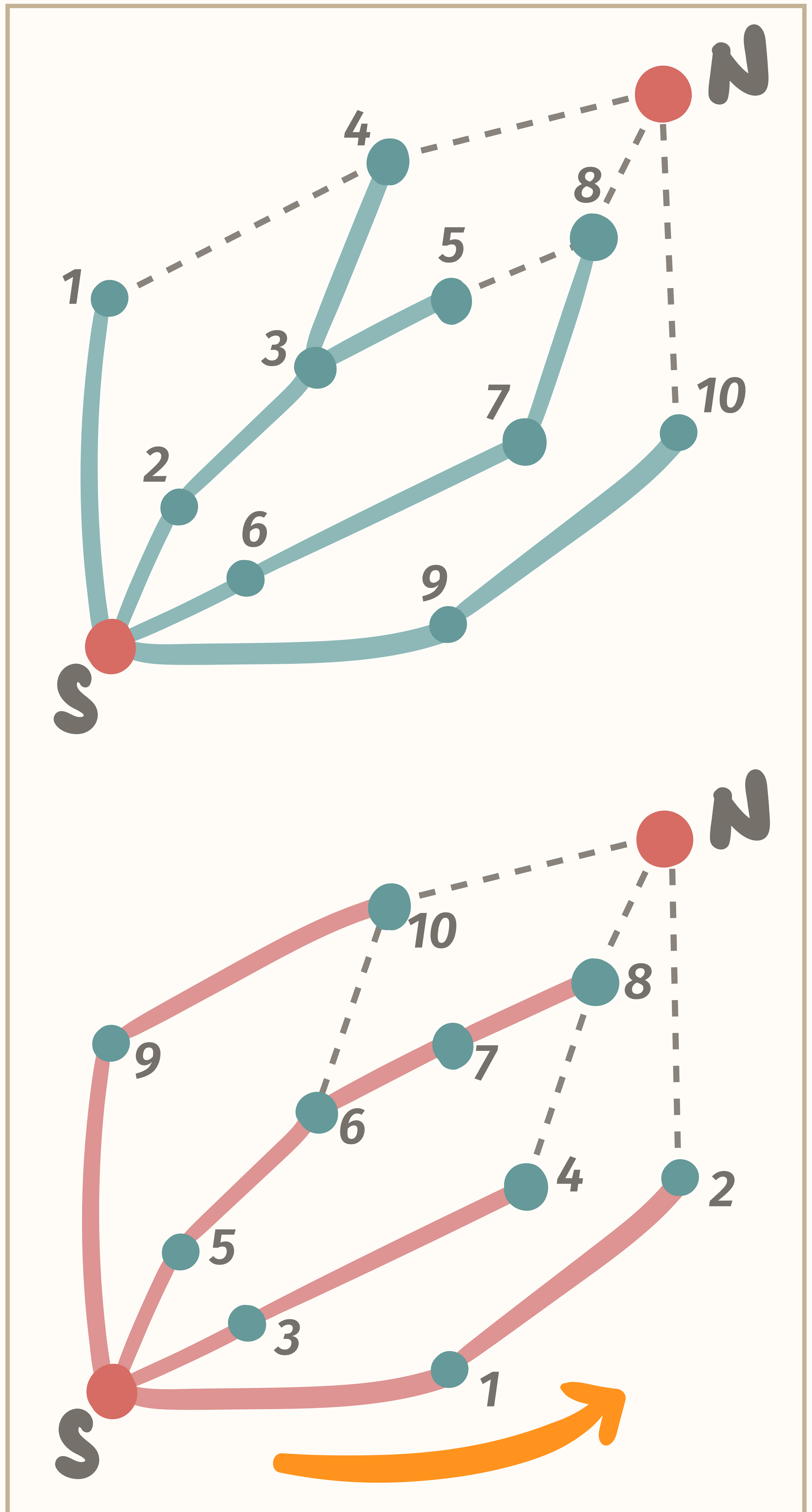
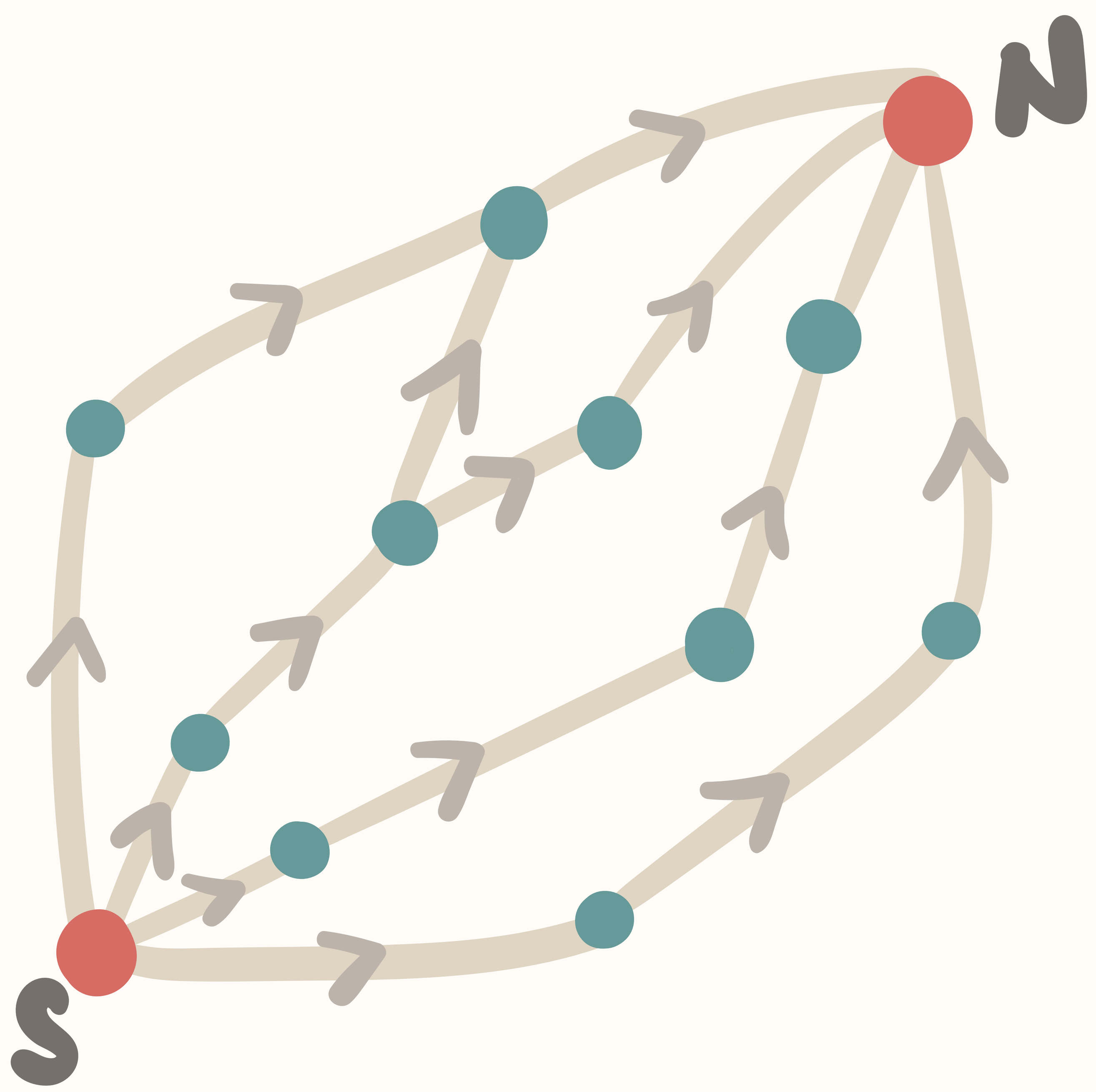
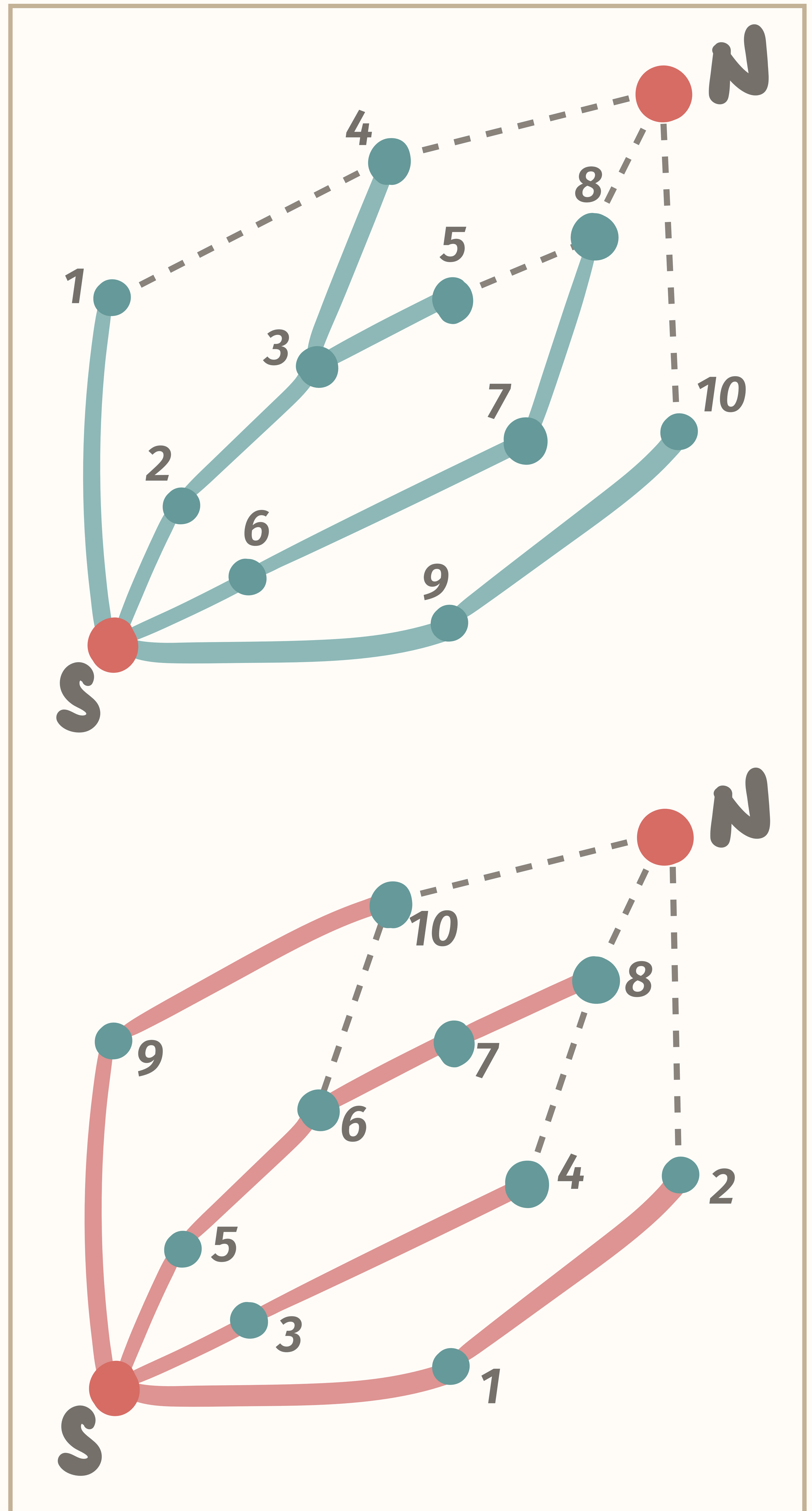


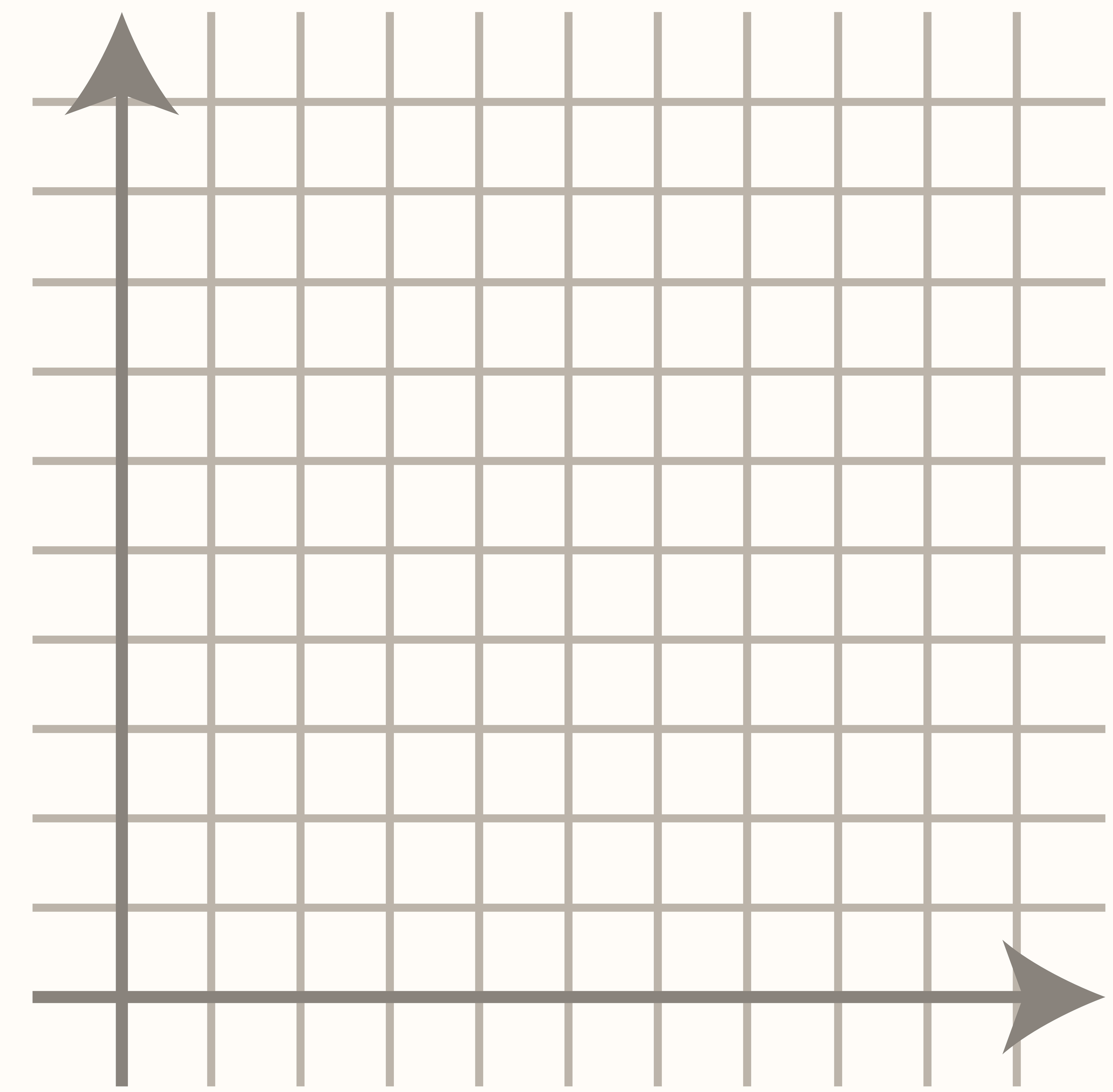


Link with plane permutations

Poset ⟶ *Plane permutation*

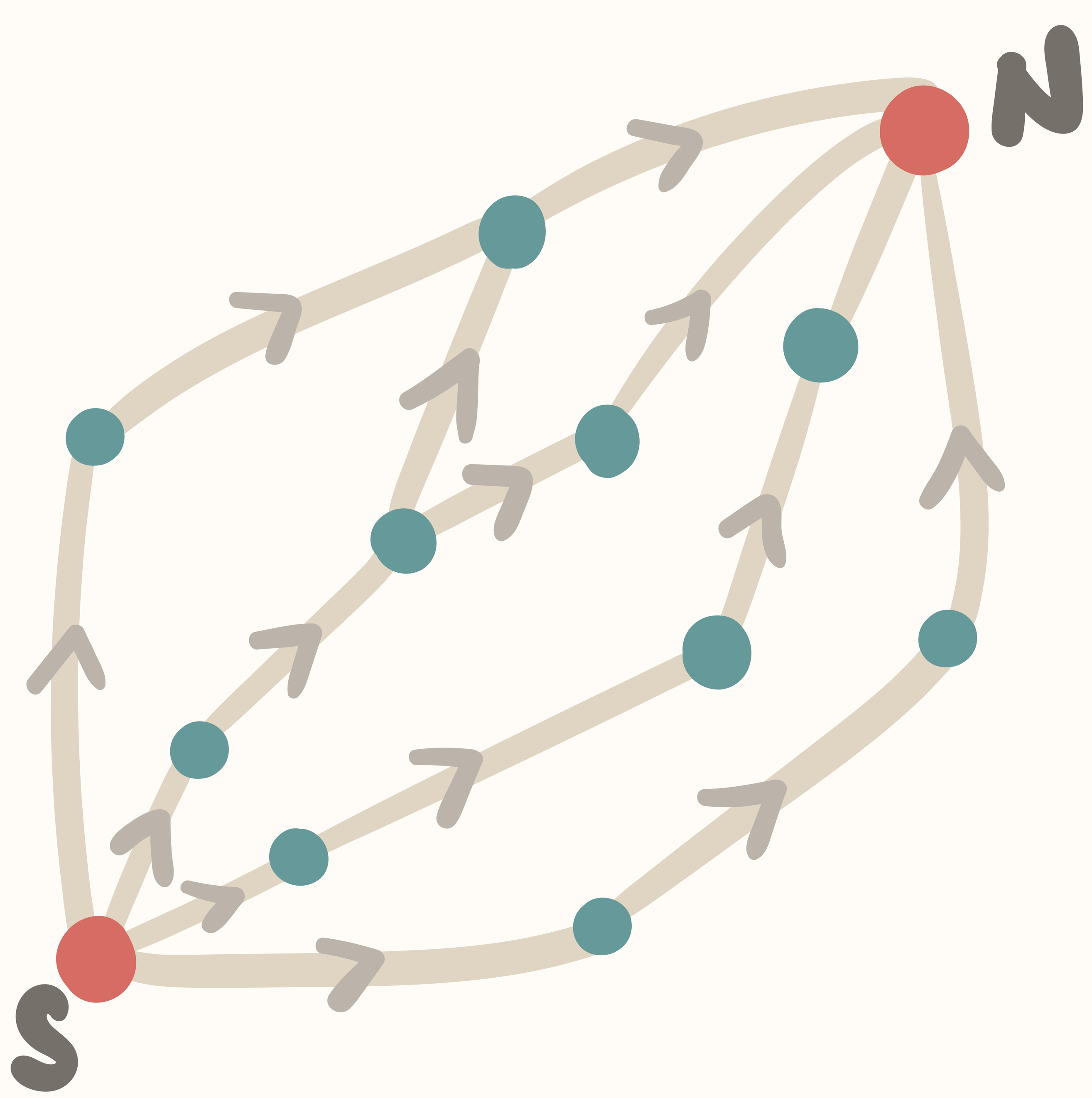


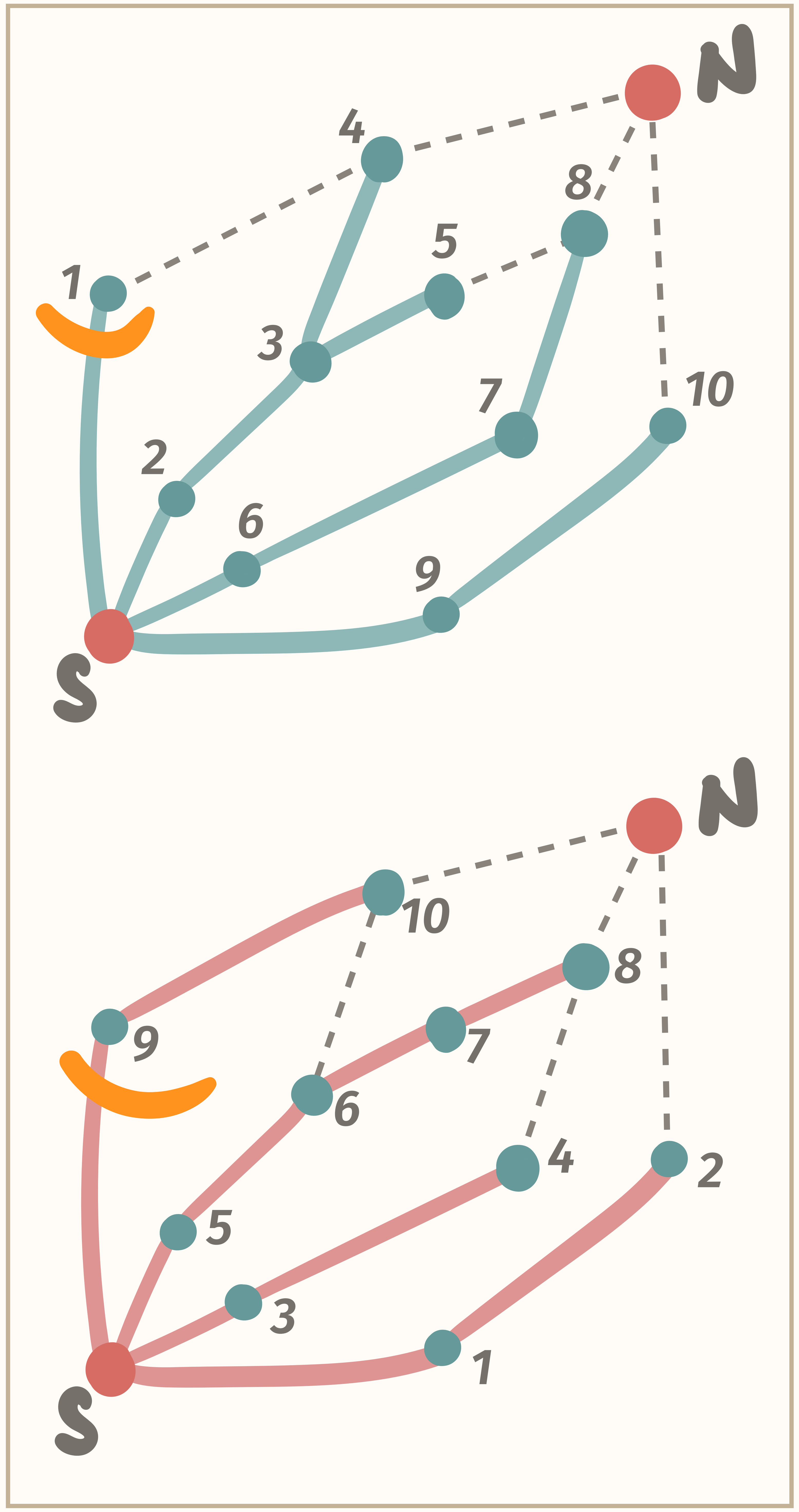


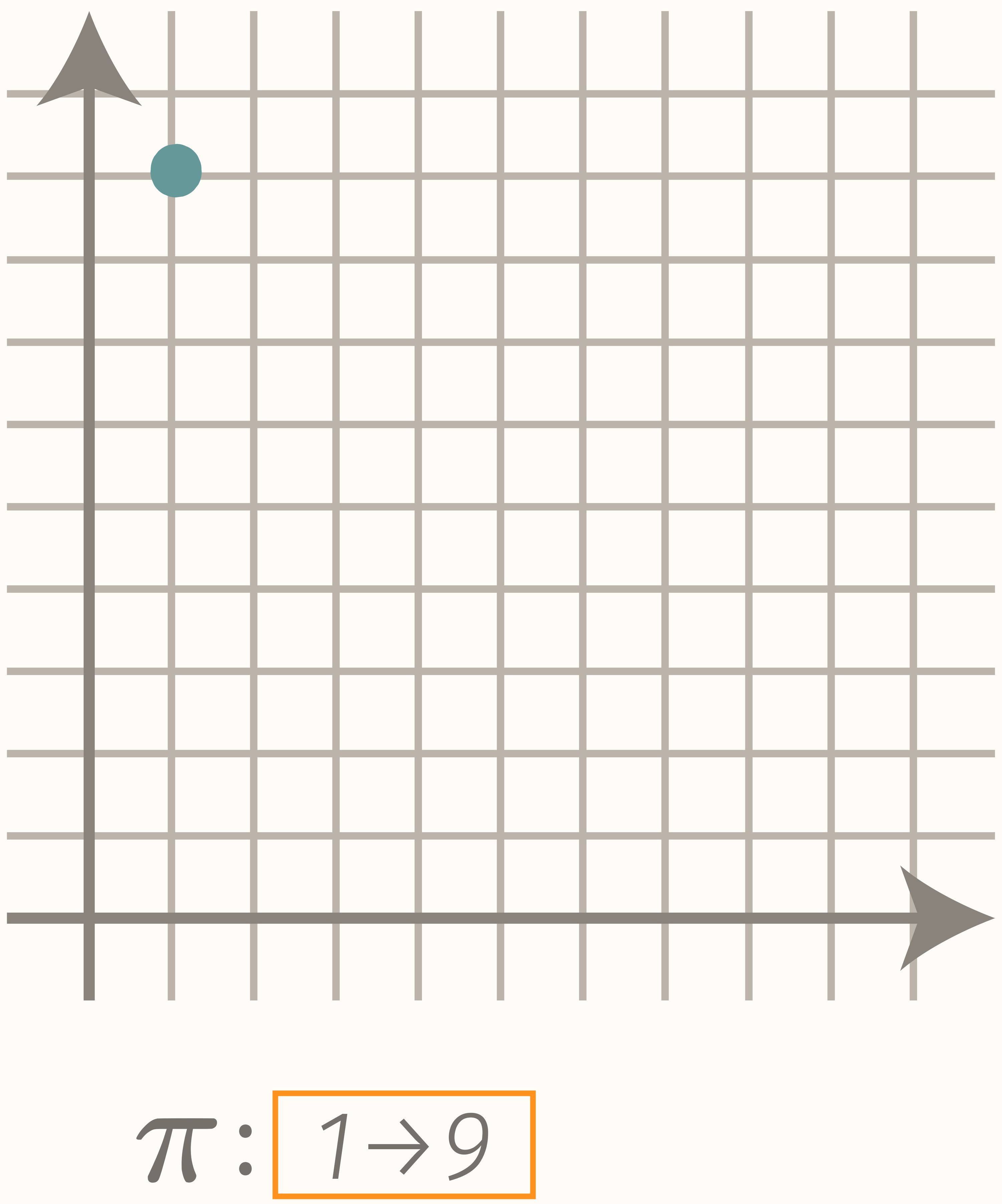


Link with plane permutations

Poset ⟶ *Plane permutation*

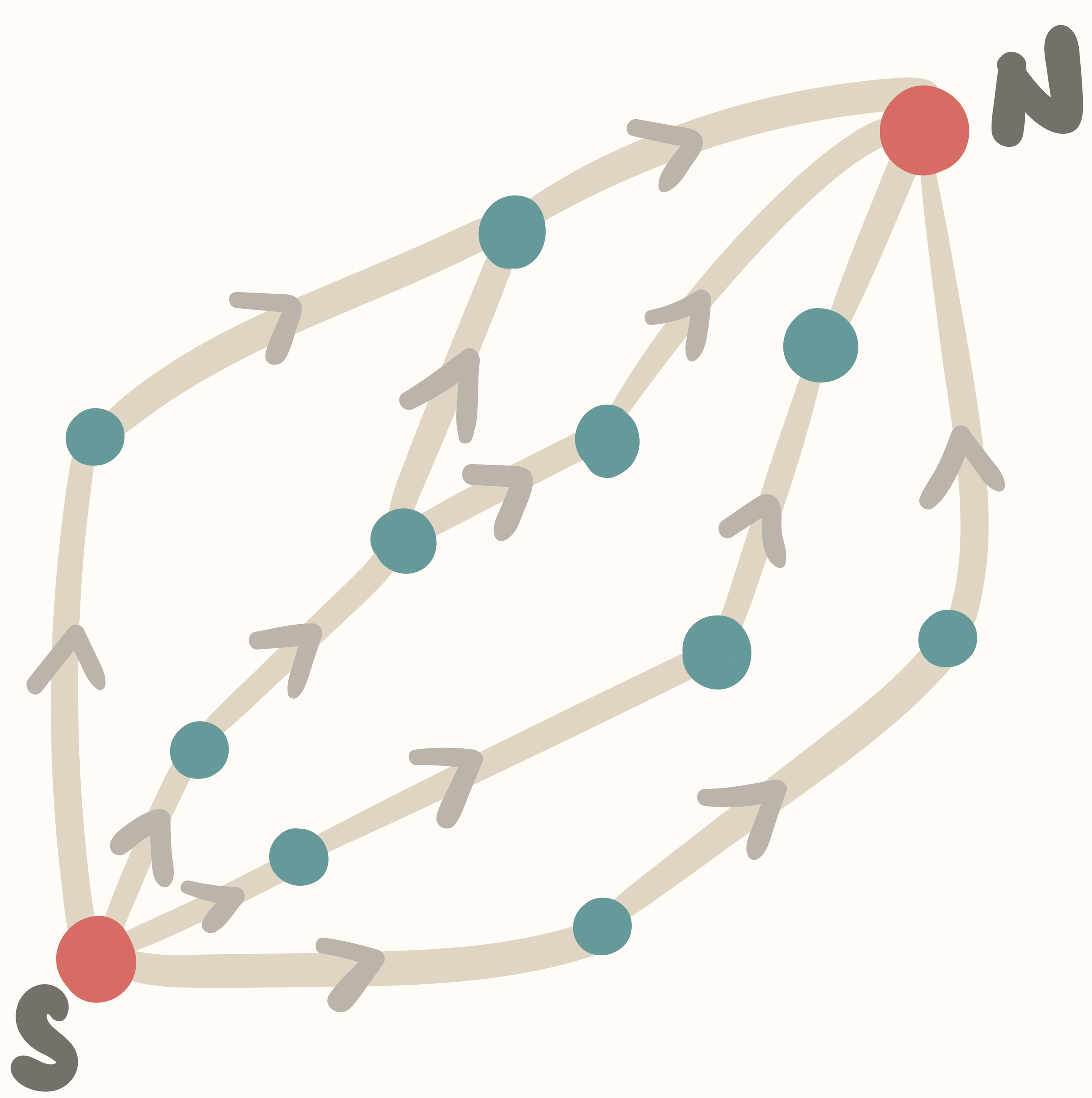


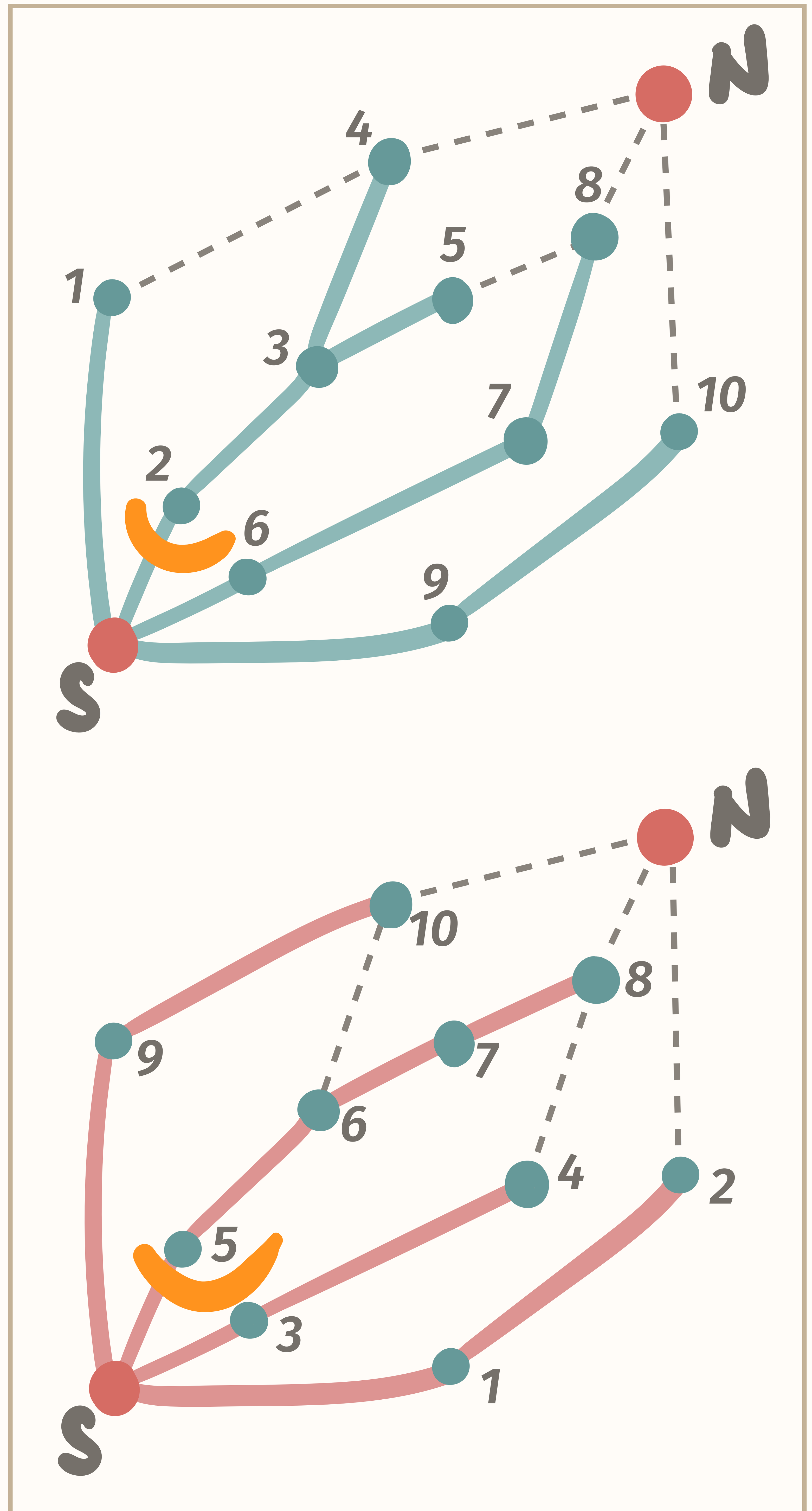


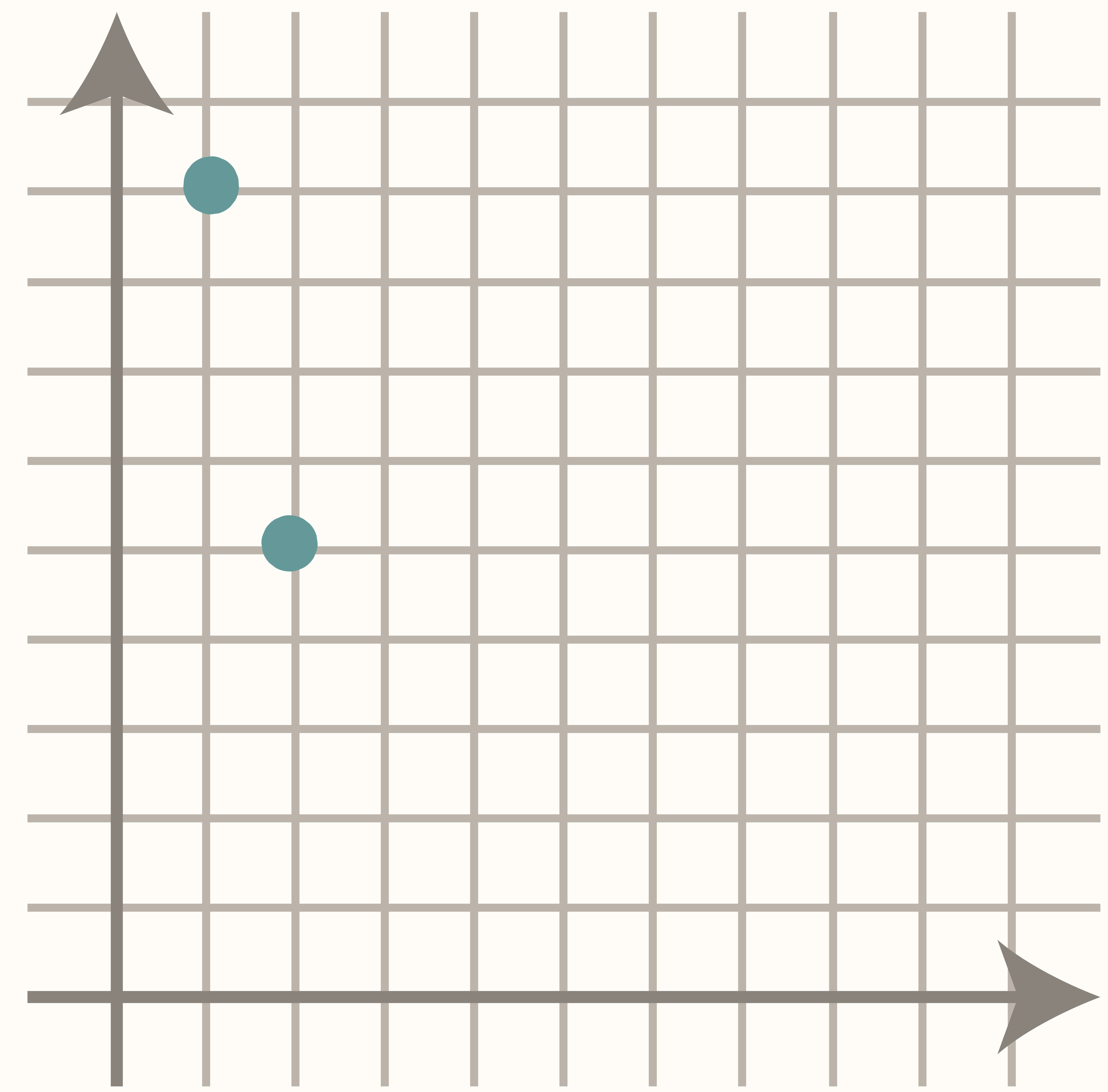


Link with plane permutations

Poset ⟶ *Plane permutation*





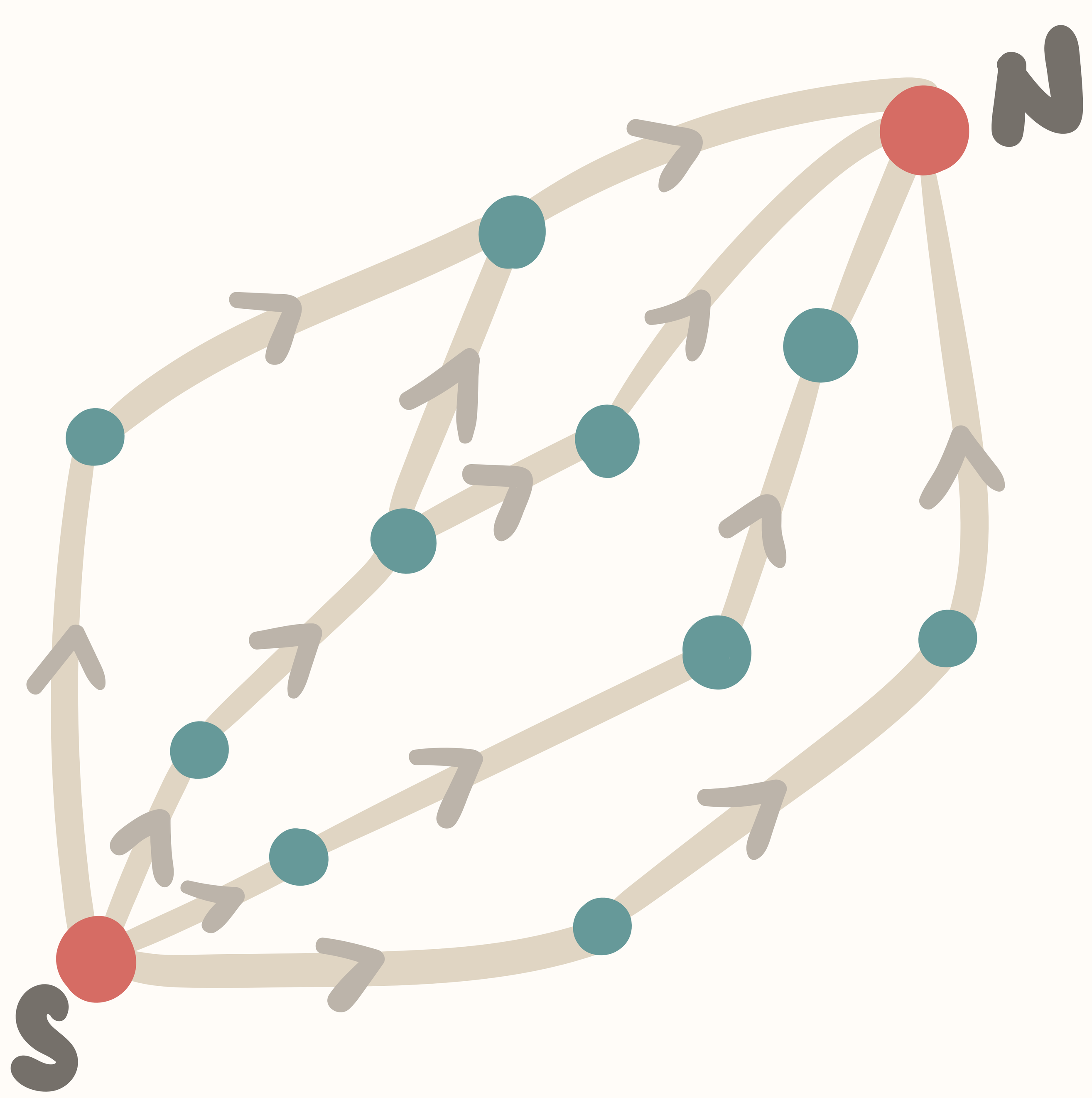


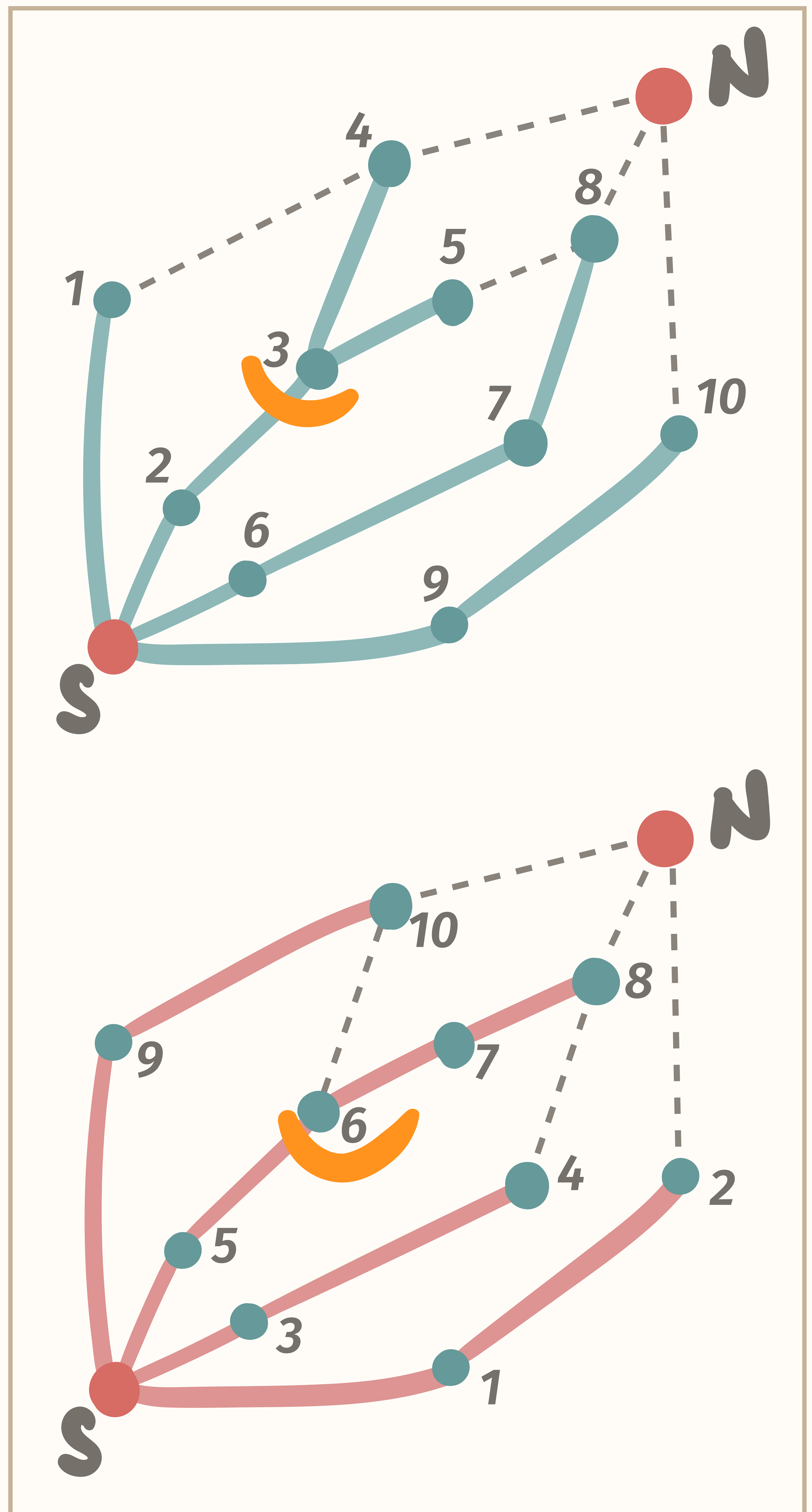
$\pi: \; 1 \to 9$

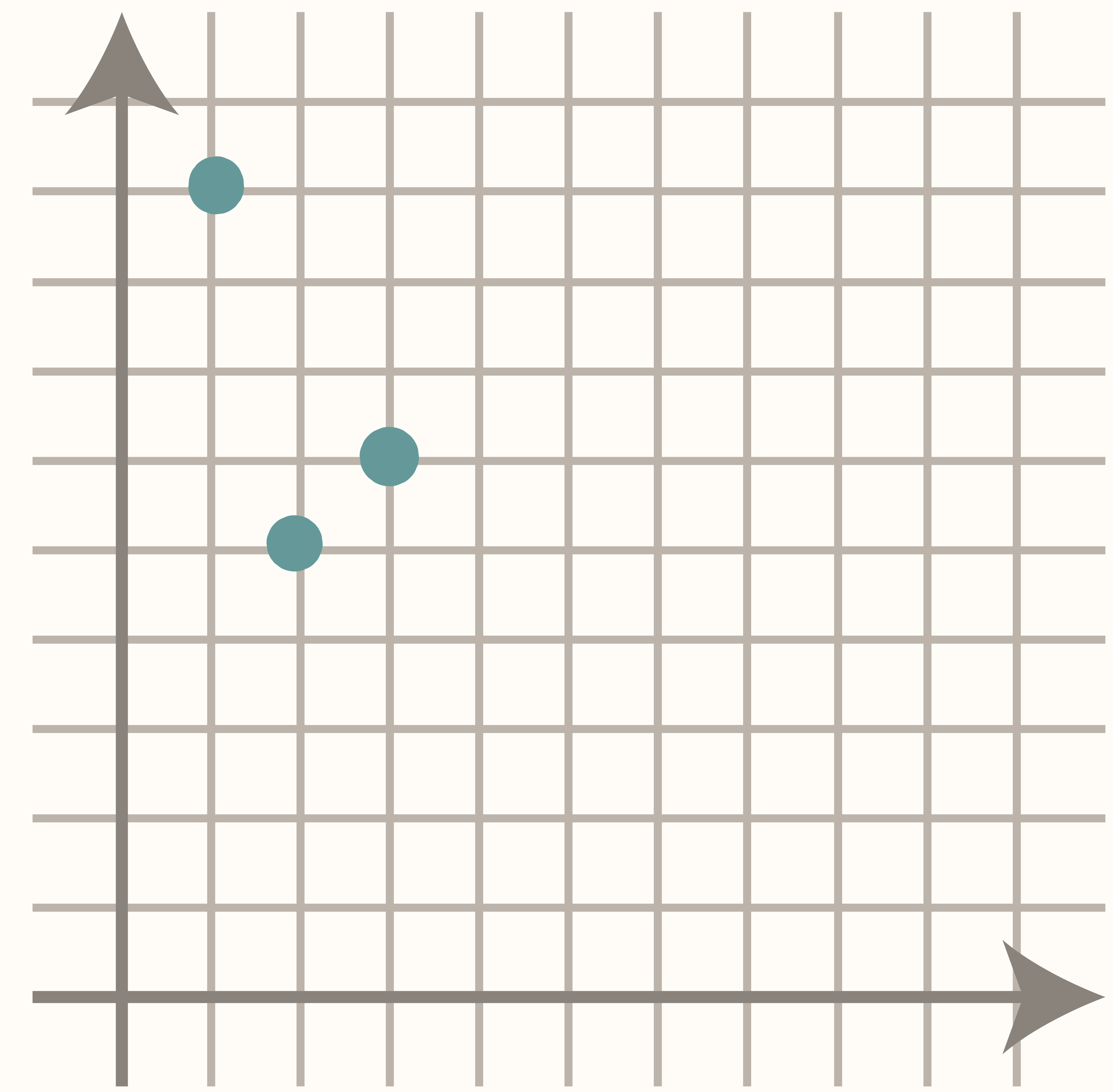
$\boxed{2 \to 5}$

Link with plane permutations

Poset ⟶ *Plane permutation*



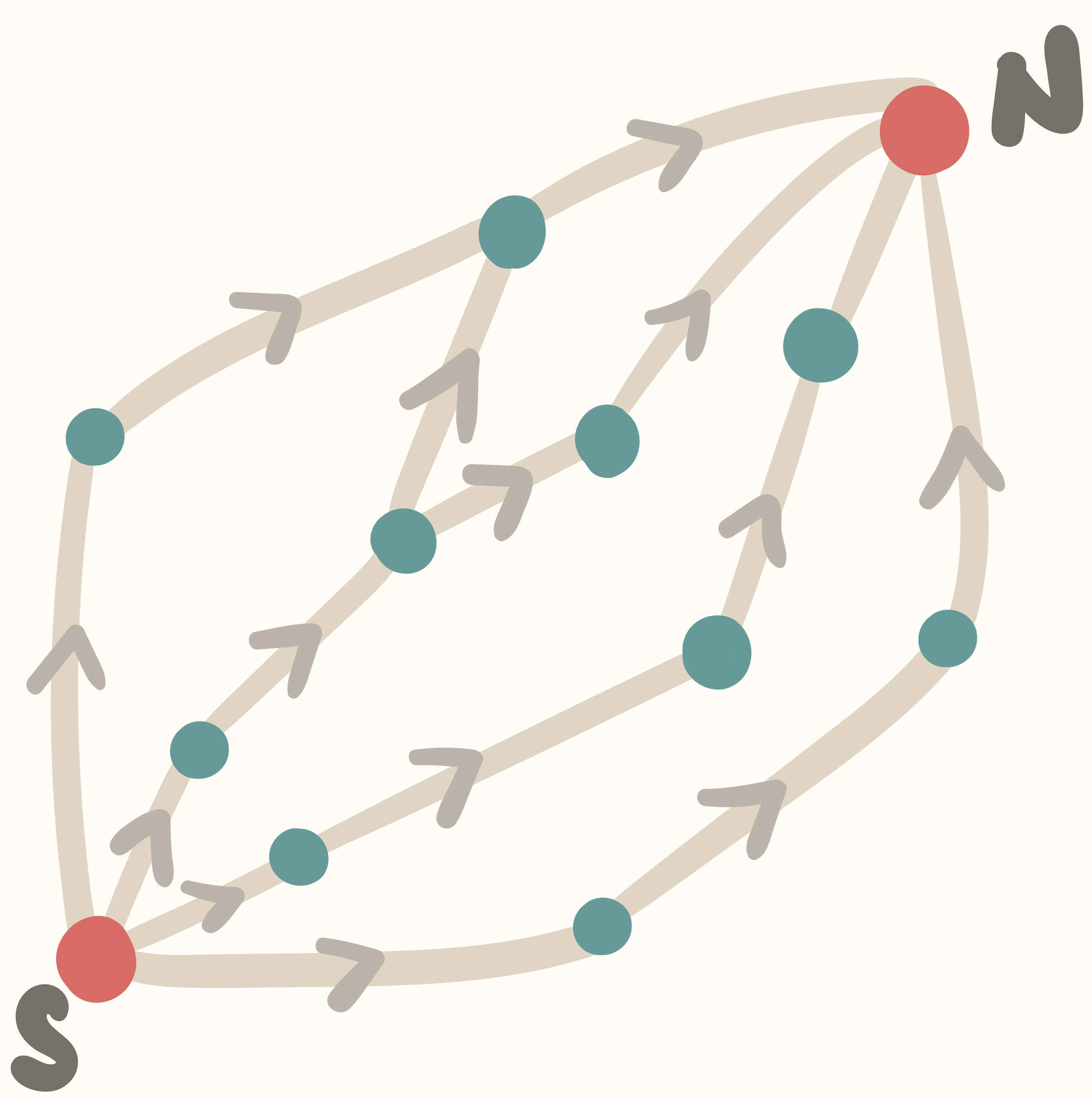


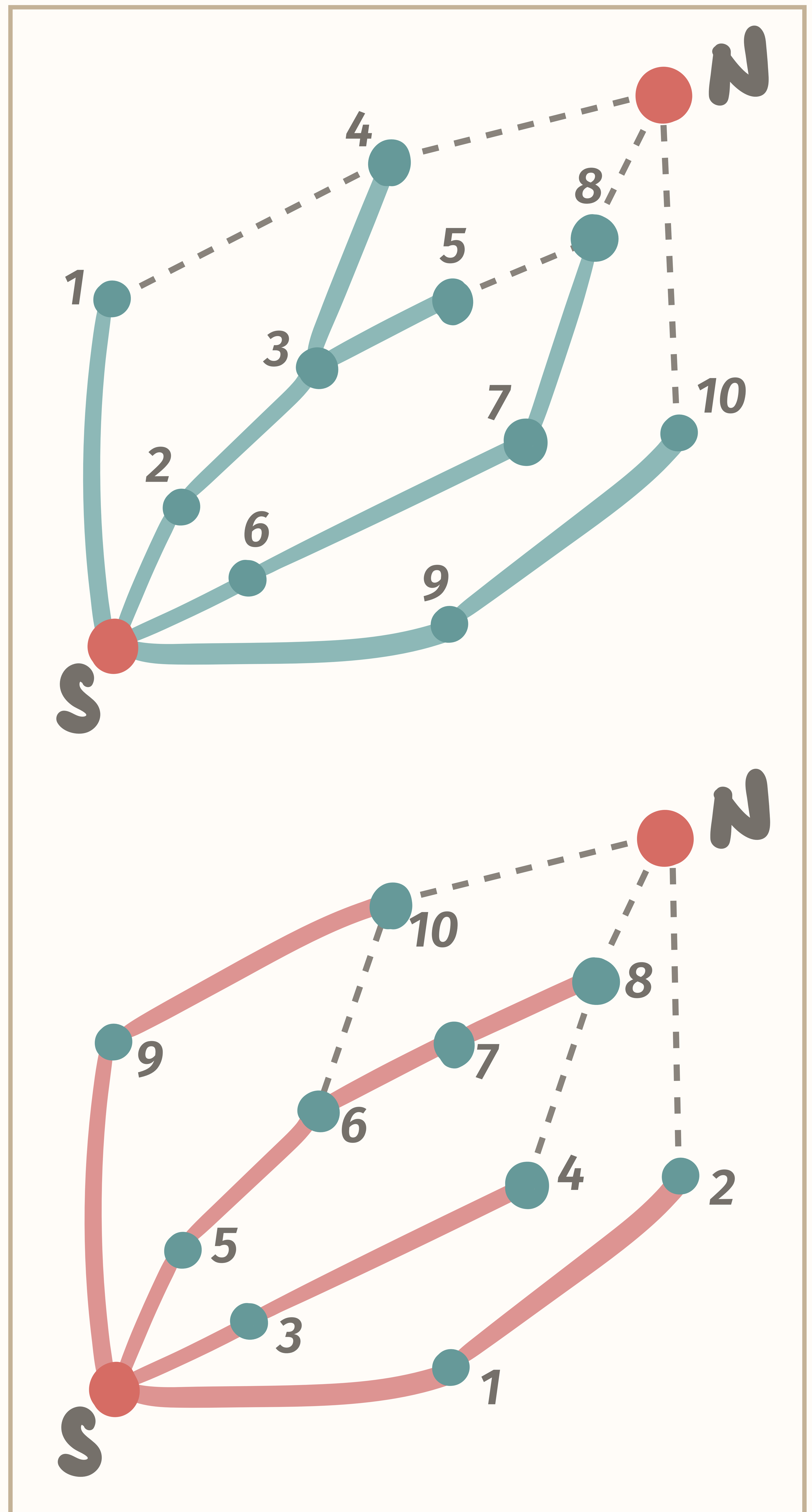


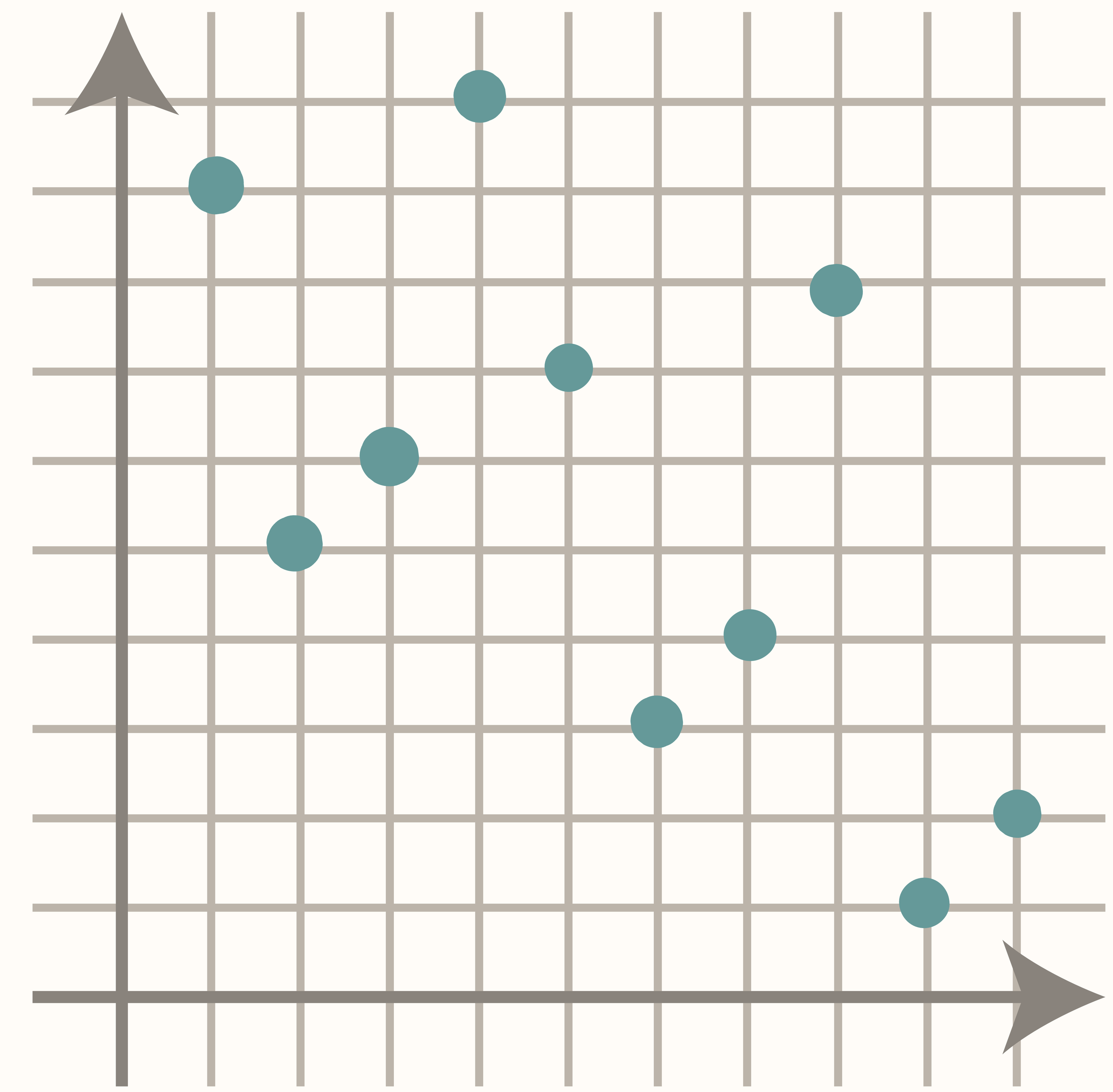
π : $1 \to 9$
$2 \to 5$
$\boxed{3 \to 6}$

Link with plane permutations

Poset $\longrightarrow$ *Plane permutation*





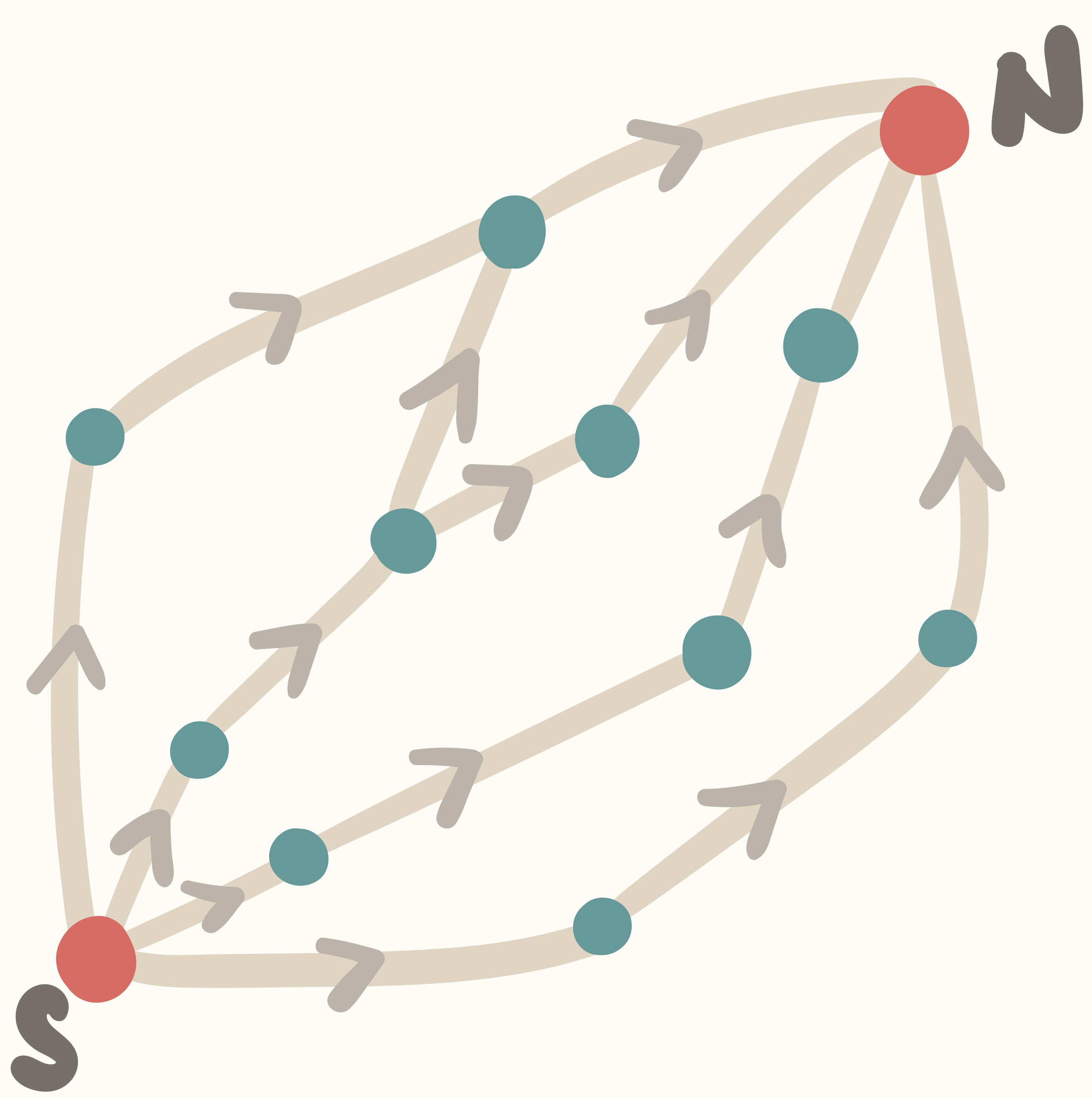


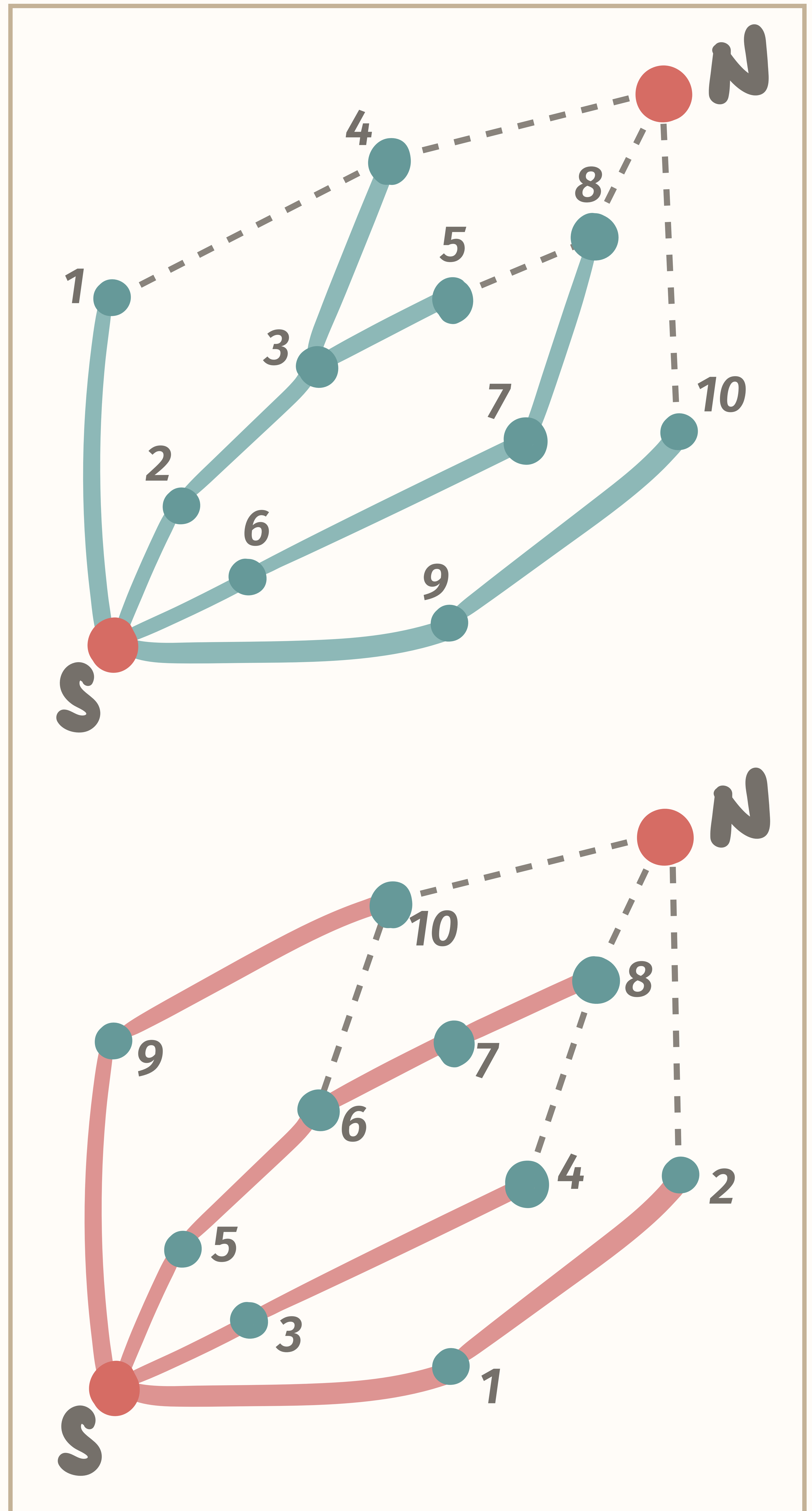
$\pi:$

$1 \to 9$	$6 \to 3$
$2 \to 5$	$7 \to 4$
$3 \to 6$	$8 \to 8$
$4 \to 10$	$9 \to 1$
$5 \to 7$	$10 \to 2$

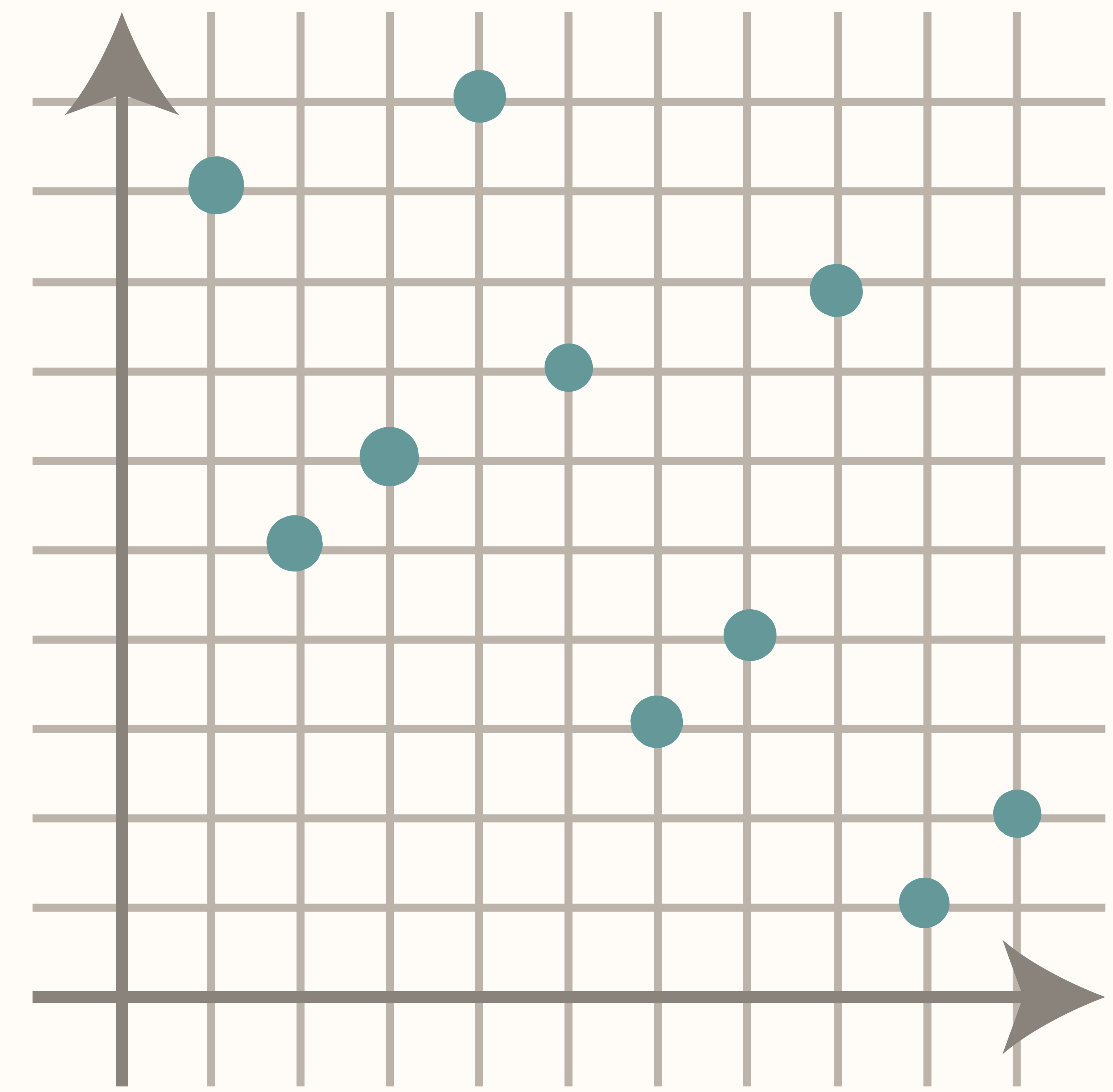
Link with plane permutations

Poset ⟶ *Plane permutation*





≫ *Area requirement and symmetry display of planar upward drawings,* G. Di Battista, R. Tamassia, and I. G. Tollis (1992)

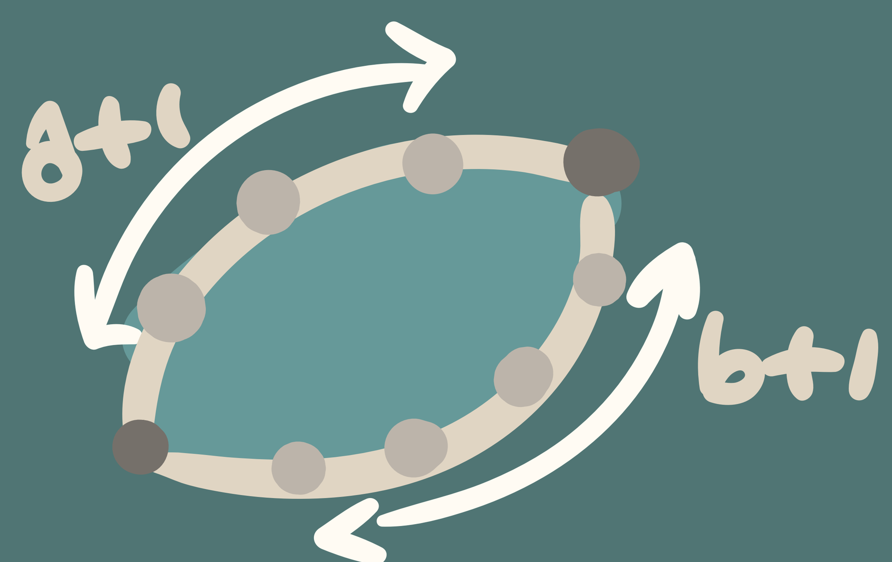


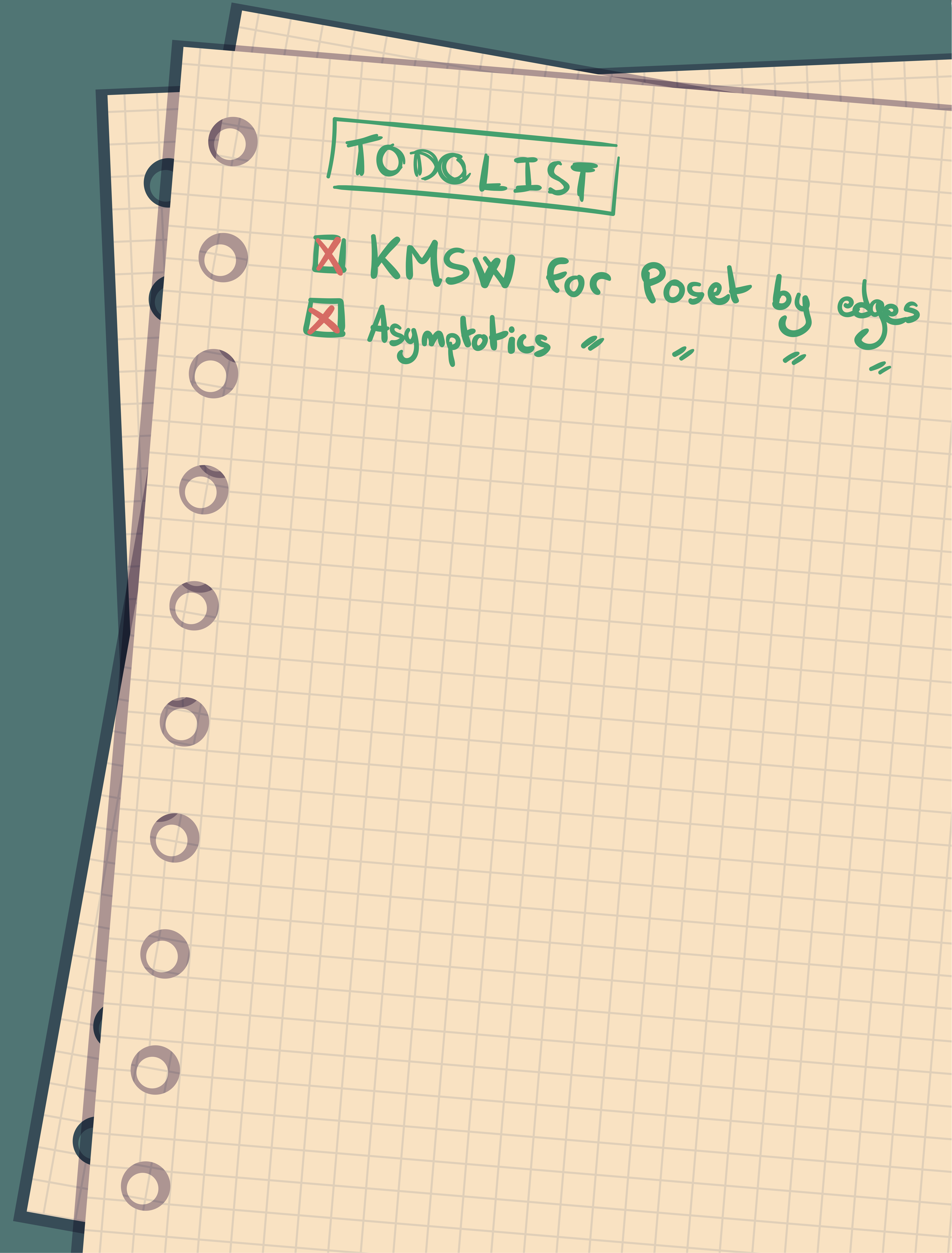
π: 1→9   6→3
2→5   7→4
3→6   8→8
4→10   9→1
5→7   10→2

Conclusion

Conclusion

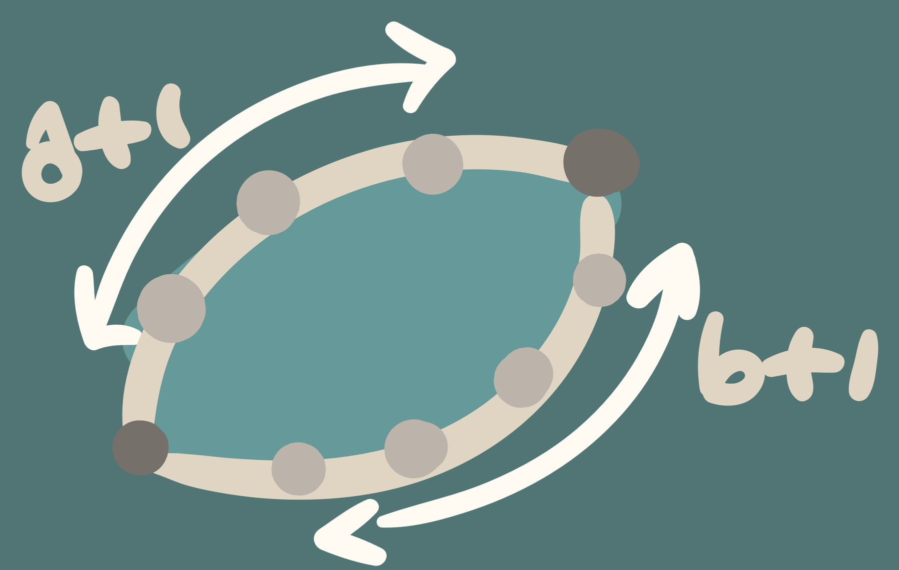
Posets



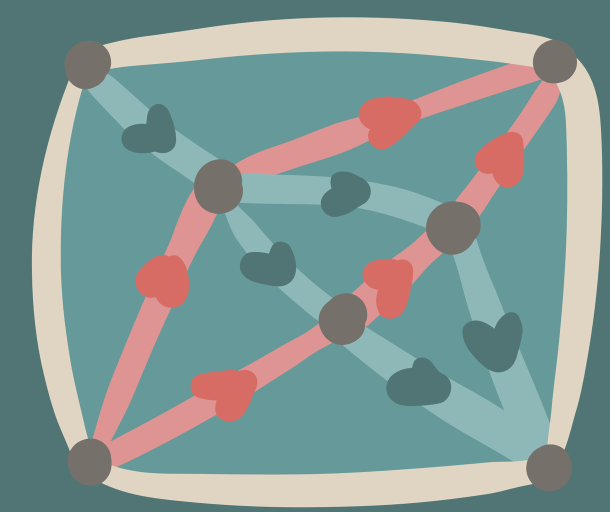


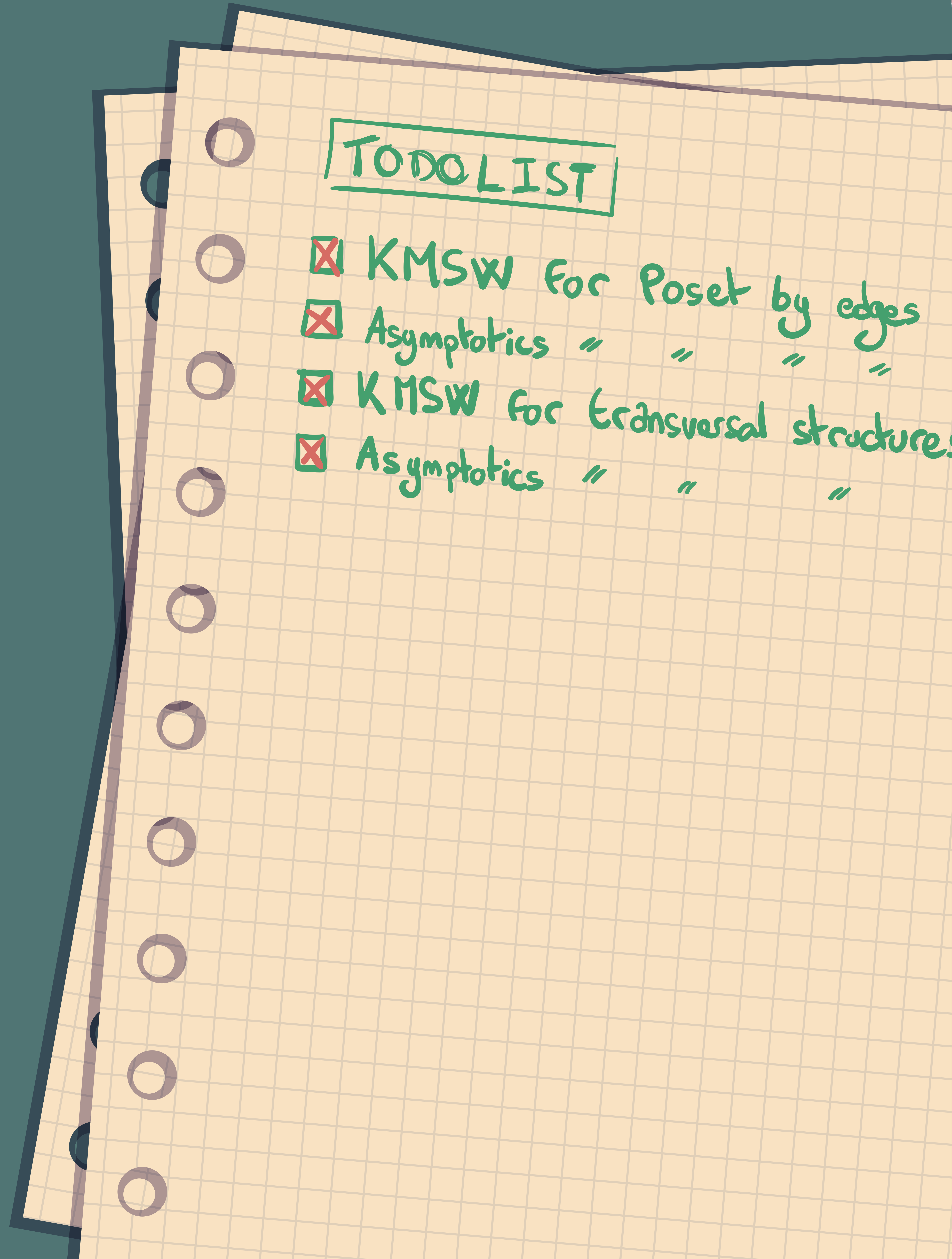
Conclusion

Posets



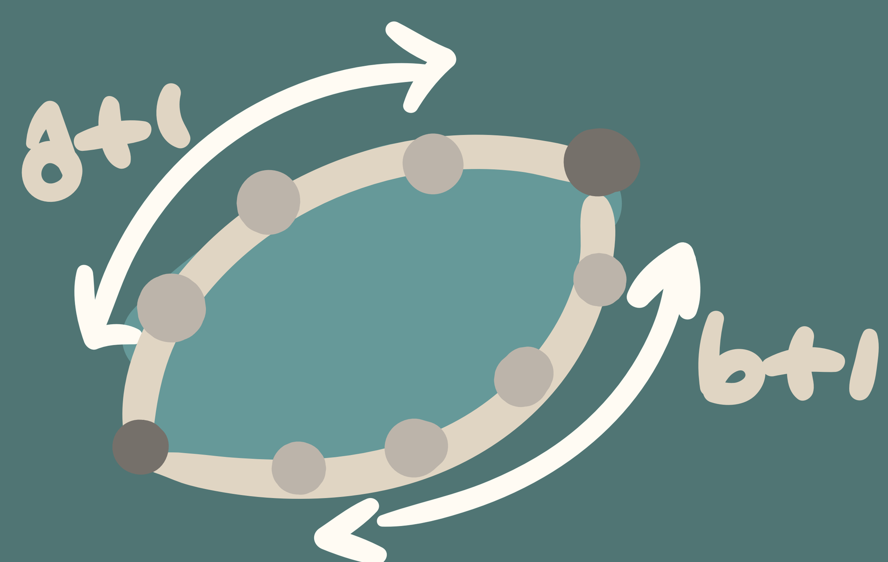
Transversal structures



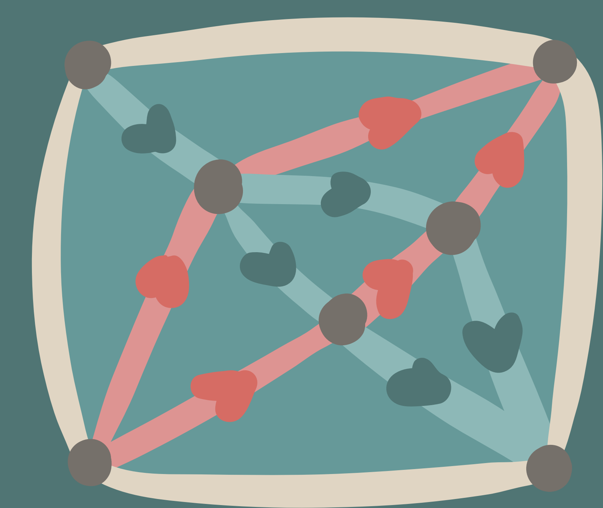


Conclusion

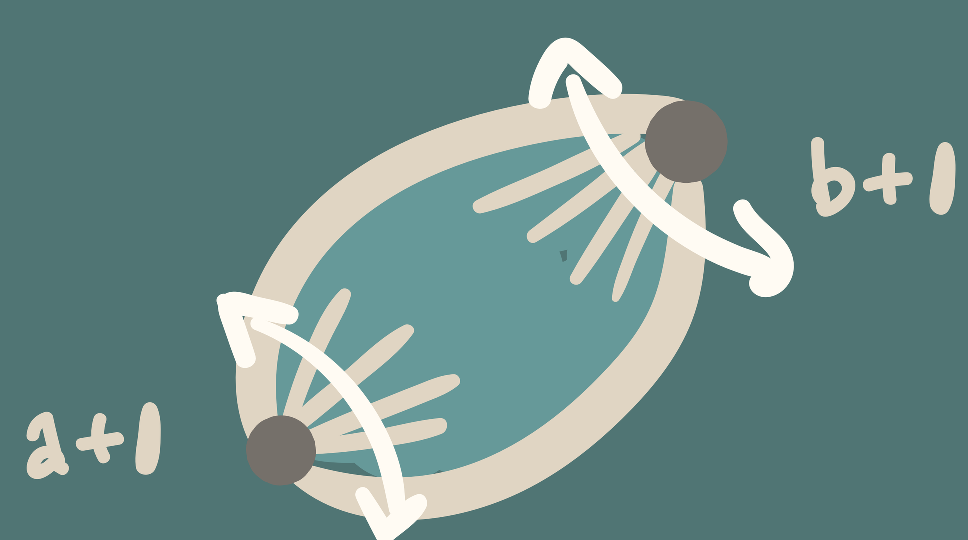
Posets

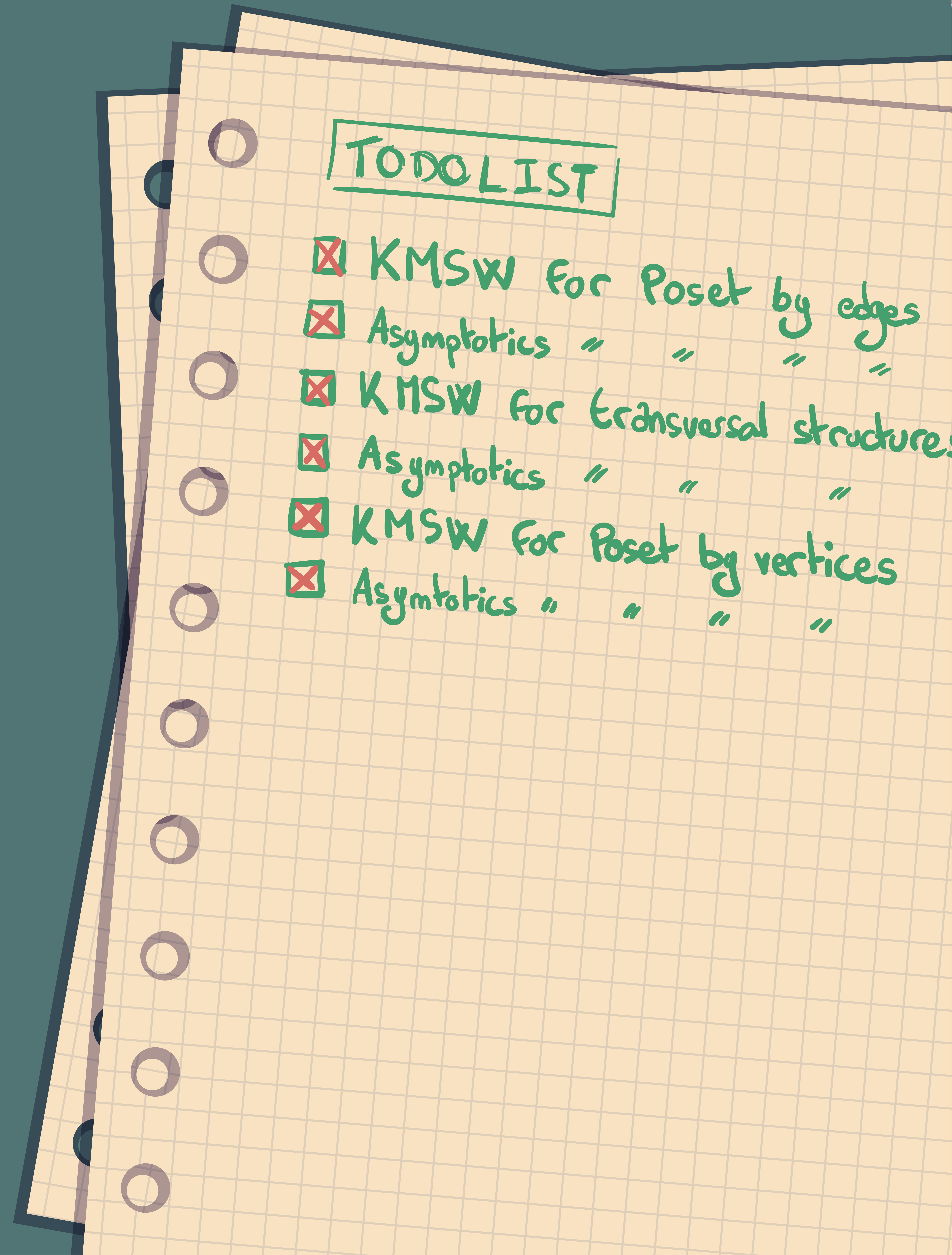


Transversal structures



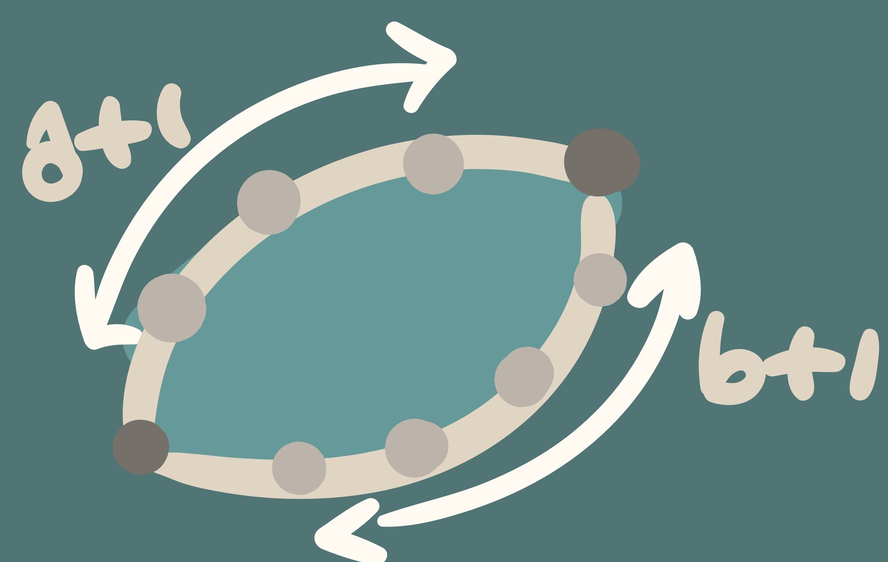
Posets
by vertices



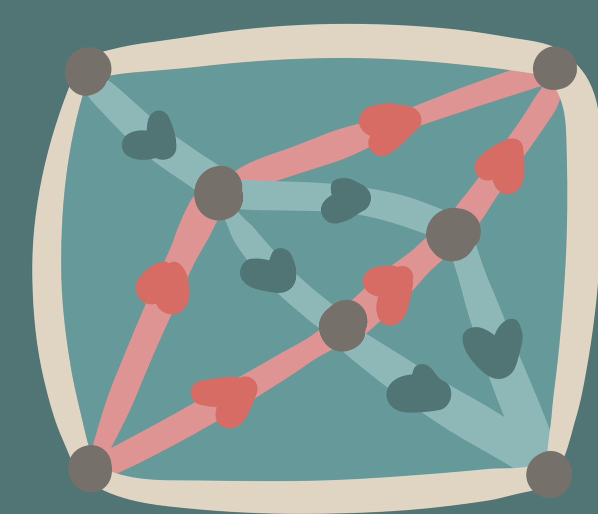


Conclusion

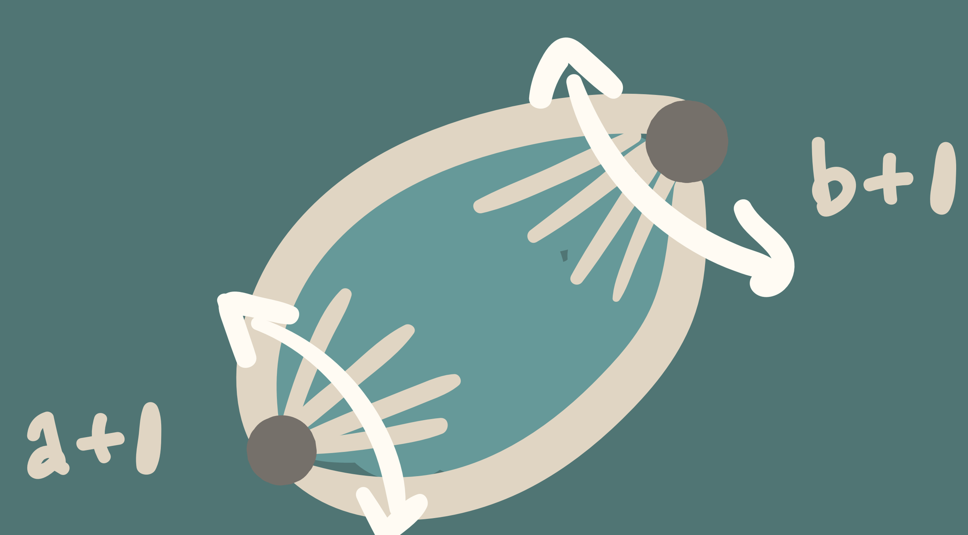
Posets



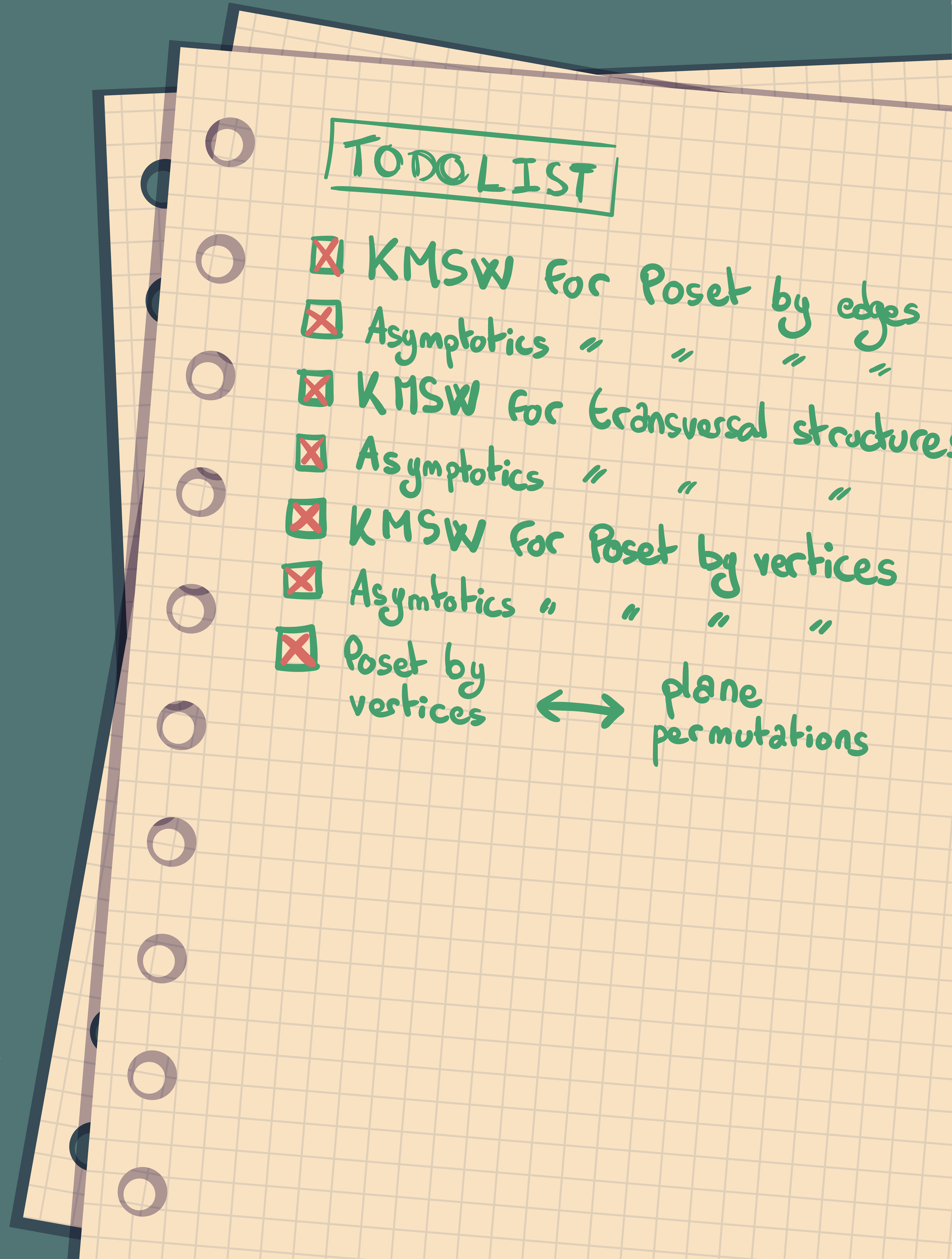
Transversal structures



Posets by vertices

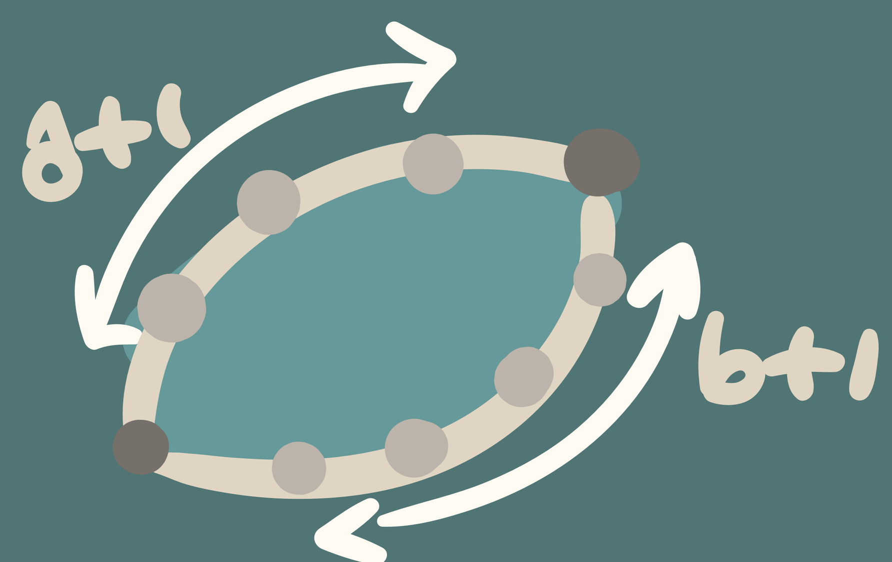


Posets by vertices $\longleftrightarrow$ Plane permutations

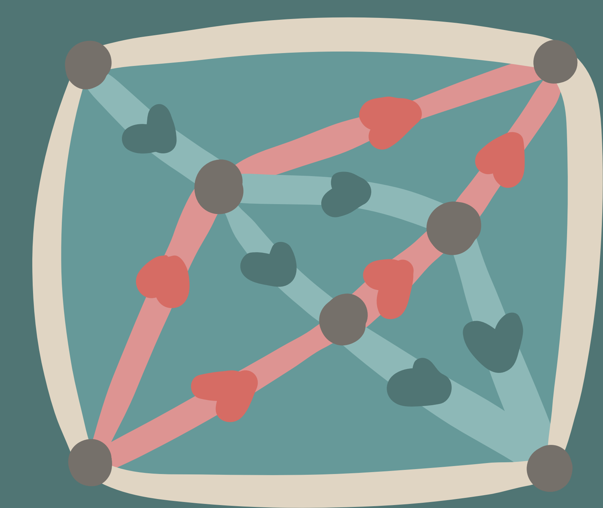


Conclusion

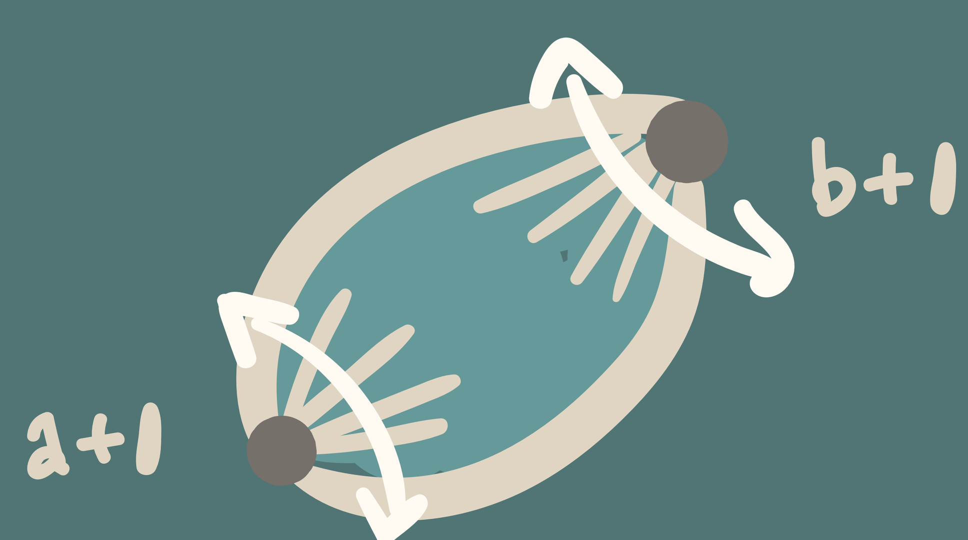
Posets



Transversal structures

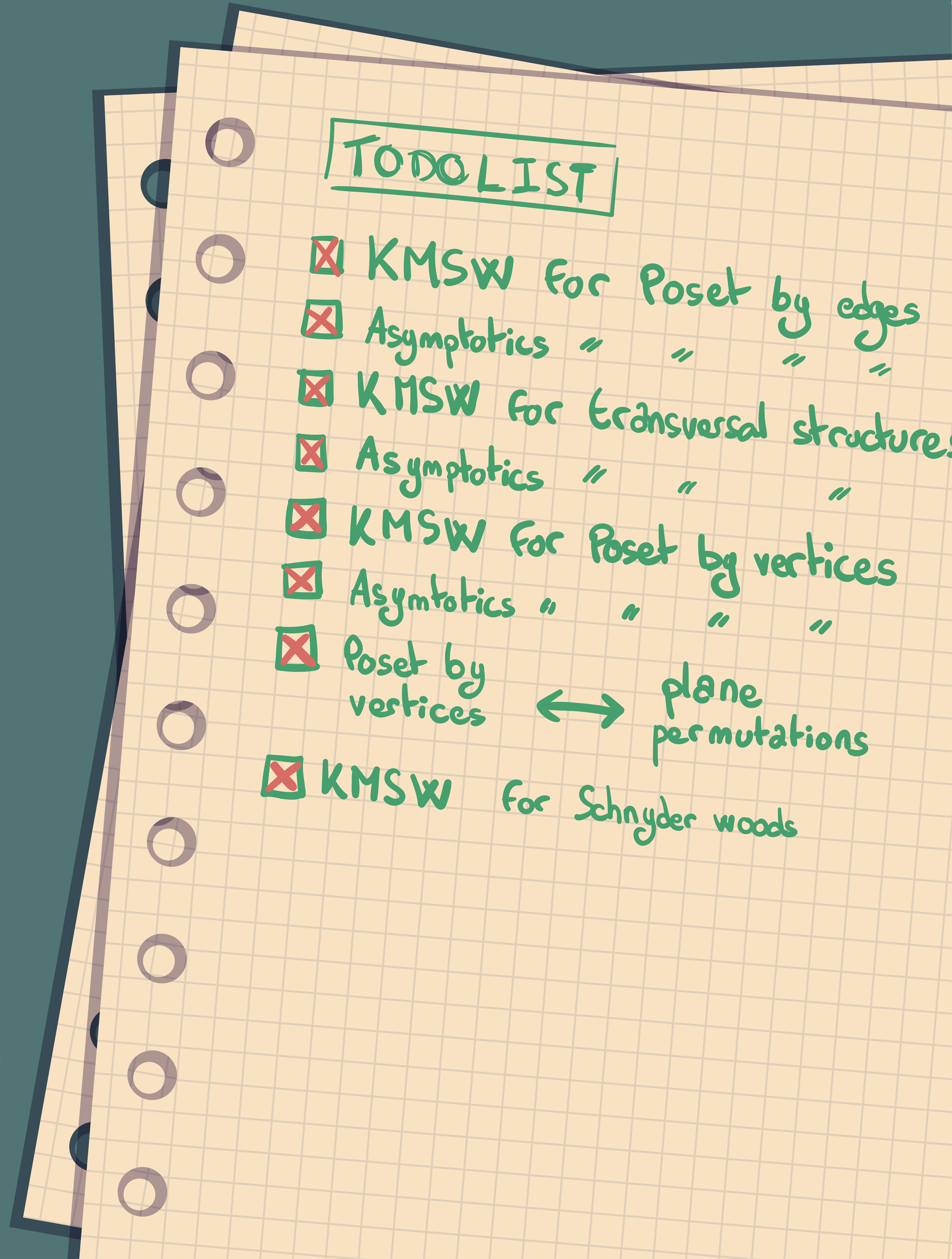


Posets by vertices



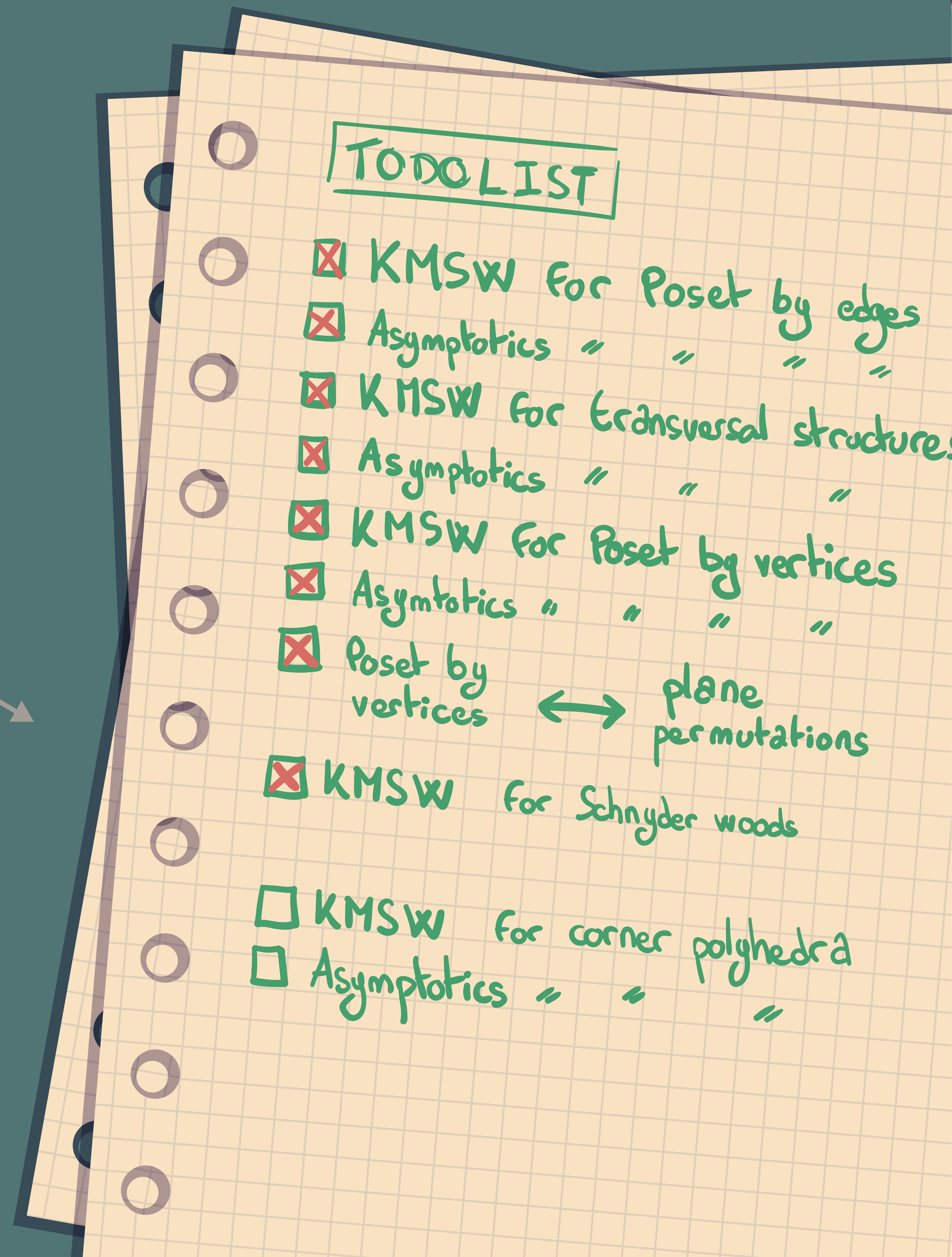
Schnyder woods

Posets by vertices ⟷ Plane permutations

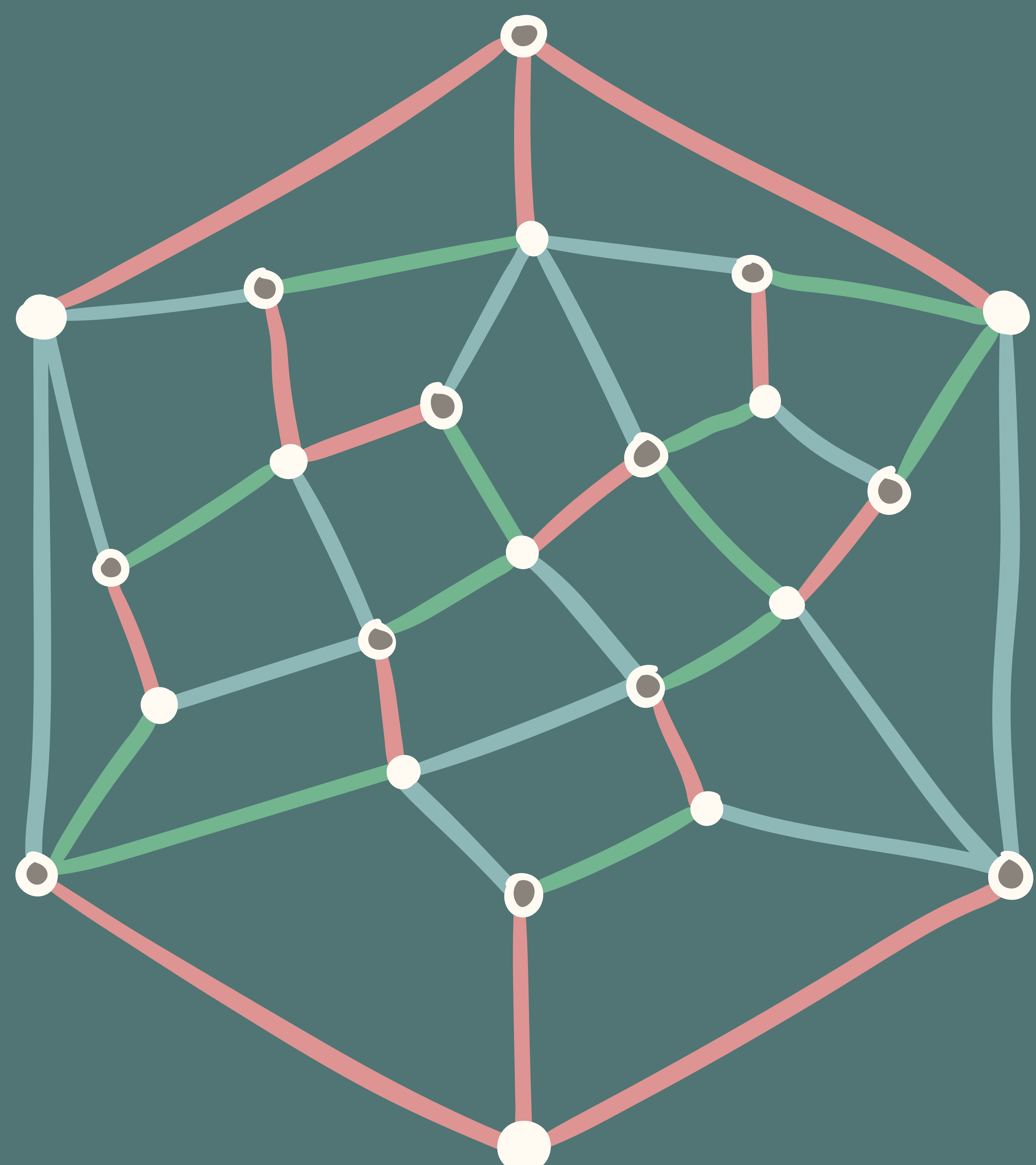


Conclusion

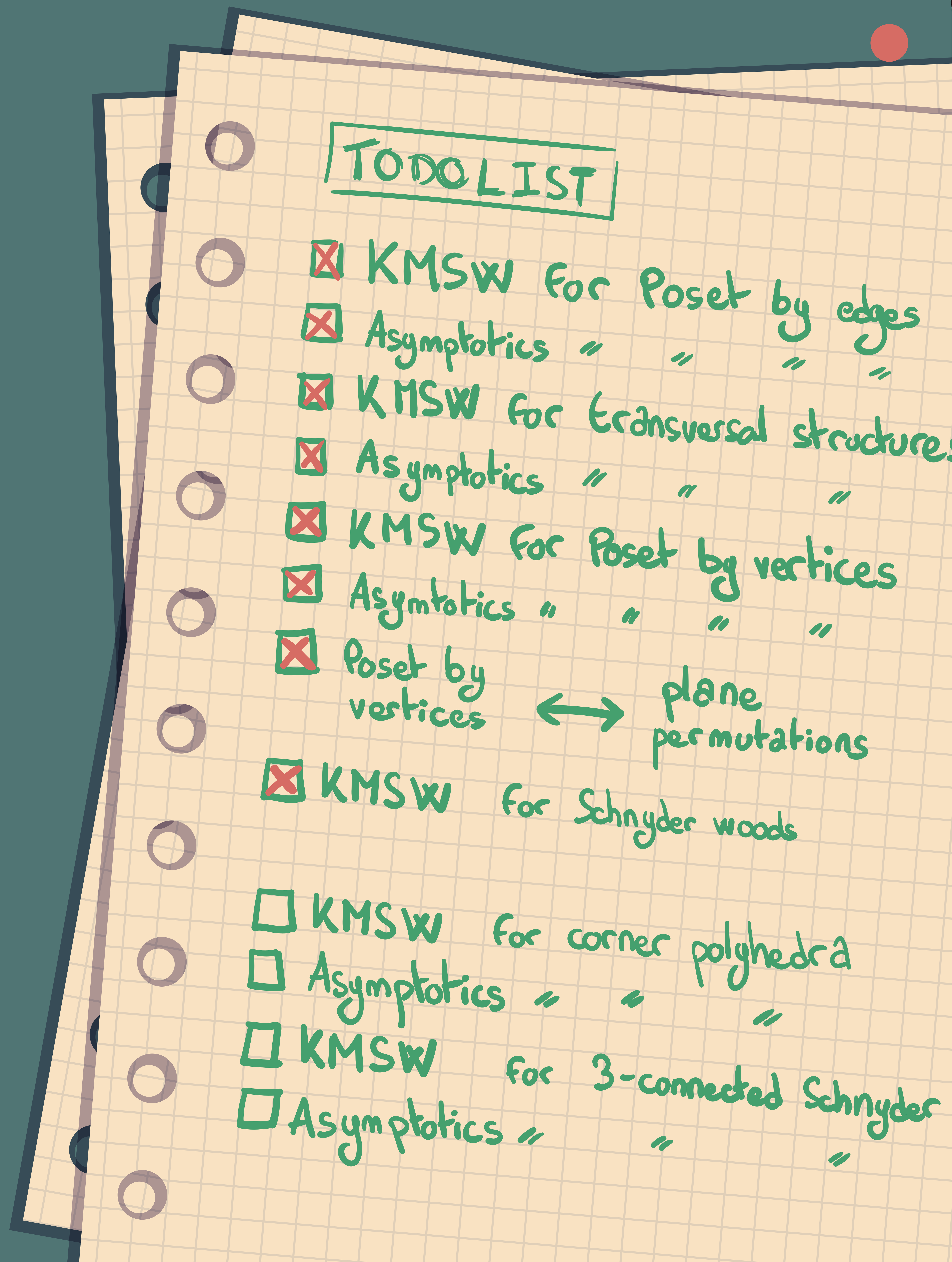
→ *Enuemration of oriented planar maps,* C. Dervieux (2018)



Conclusion



→ *Lattice structures from planar graphs,* S. Felsner (2004)




thanks
merci
TODO LIST
KMSW for Poset by edges
Asymptotics " " " "
KMSW for transversal structures
Asymptotics " " "
KMSW for Poset by vertices
Asymtotics " " " "
Poset by vertices ↔ plane permutations
KMSW for Schnyder woods
KMSW for corner polyhedra
Asymptotics " " "
KMSW for 3-connected Schnyder
Asymptotics " " "